A Wide Coverage Positioning System (WPS) for Underwater Localization

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***Abstract*—Underwater localization is a challenging task as its efficacy is affected by propagation delays, motion-induced doppler shift, phase and amplitude fluctuations, multipath interference etc that is inherent in underwater acoustic channels. In this paper, we consider a recently proposed Underwater Positioning Scheme, which offers unique localization only in a *finite* region. We quantify the conditions for unique localization and propose a variant that offers unique localization with high probability regardless of the reference and unknown node deployment. We demonstrate the trade-offs between both schemes in terms of localizability space, localization latency and energy consumption.**

I. INTRODUCTION

Underwater acoustic sensor networks (UASNs) are envisaged to fulfill the needs of a multitude of applications such as early warning systems for natural disasters, ecosystem monitoring, oil drilling and military surveillance. The data derived from such sensor networks is typically interpreted with reference to a sensor's location, e.g., reporting the occurrence of an event, tracking of a moving object or monitoring the physical conditions of a region.

Location discovery for underwater sensors is non-trivial in the oceanic medium as its efficacy is impacted by propagation delays, motion-induced doppler shift, phase and amplitude fluctuations, multipath interference etc. Moreover, since GPS signals do not propagate through water, underwater nodes need to rely on position references for localization, obtained either through spatial (multiple, fixed references) or time diversity (single, mobile reference). Since underwater acoustic devices are expensive and deployment is costly [1], we expect UASNs to be deployed for long durations.

Underwater Positioning System (UPS) [2] is a promising scheme for underwater localization as it (i) requires no time synchronization, (ii) provides silent positioning, (iii) has low computation overhead and (iv) has been shown to exhibit low positioning error. However, it suffers from the following drawbacks: (i) it relies on reactive beaconing from a fixed set of reference nodes; (ii) its feasible space is finite, i.e., *blind spots* exist where nodes within coverage area cannot be uniquely localized, and (iii) it assumes that the reference nodes cover (in terms of communication range) the entire network, thus limiting the area of interest. In [3], we illustrated the impact of (i) under realistic underwater channel conditions and proposed an Enhanced UPS scheme (E-UPS) to address this deficiency. In this paper, we propose a Wide Coverage Positioning System (WPS) to address the limitations of feasible space with UPS. The constraints imposed by finite communication range are not the focus of this paper, and can be overcome with multi-stage [4] and hierarchical localization [5], [6].

The paper is organised as follows: we review related works on underwater localization in Section II. We describe the generalized UPS algorithm, highlight the issue of finite feasibility region in the original UPS algorithm, and how we address this in WPS in Section III respectively. We evaluate the performance of WPS against UPS numerically in Section V. Finally, we provide some concluding remarks and directions for future research in Section VI.

II. RELATED WORK

Underwater localization techniques have been classified as infrastructure-based vs infrastructure-less respectively [7]. Techniques proposed since then can be further categorized as single-stage [2], [8]–[10] vs hierarchical / multi-stage [4]–[6].

In infrastructure-based localization, reference nodes are deployed on surface buoys (localized using GPS) or at predetermined locations on the seabed. Based on the beaconing signals from the reference nodes, the distance to these reference nodes can be computed at each node using the propagation time. In general, there should exist at least $d+1$ references to uniquely localize a network in d-dimensional space. In [4], the authors propose a purely distributed localization framework that employs a projection technique that transforms the 3D underwater positioning problem into its 2D counterpart. In [5], the authors divide the localization process into two sub-processes: anchor node localization and ordinary node localization. They propose a distributed localization scheme that integrates 3D Euclidean distance estimation with a recursive location estimation method. This method is enhanced in [6] by introducing mobility prediction based on the predictable mobility patterns of underwater objects.

Infrastructure-less localization is usually implemented by using mobile beacon(s). In [8], the authors proposed Dive-and-rise beacons that get their coordinates from GPS while floating above water, and then dive into water. While sinking and rising, they broadcast their positions. The multi-stage extension of this approach for large-scale networks is given in [11]. The need for synchronization amongst nodes with the above approaches is eradicated with AUV-aided localization using omnidirectional [9] or directional antennae [10].

III. GENERALIZED UPS : UPS(N)

In this section, we begin by generalizing the concept of UPS [2] to N reference nodes, where our goal is to determine the location (x,y,z) of a sensor node s, given the location (x_j, y_j, z_j) of reference node R_j, $1 \leq j \leq N$. We denote the scheme by $UPS(N)$, and let d_{sj} and d_{ij} be the Euclidean distance between s and R_j and R_i and R_j respectively.

Initially, node s sends a short beacon to wake up the reference nodes. Those that hear this beacon (i.e., that are within communication range of, and maintains a good communication link with, s) will respond with their ID, coordinates, the arrival time of s's wake-up beacon and the transmission time of the response beacon. Let R denote this set of reference nodes. We assume that each beaconing signal comprises a fixed-size packet, and the probability of packet transmission failure is p.

Node s then computes the beaconing sequence according to the order in which it receives the responses from R. It notifies the reference nodes and starts the timer. Upon receiving the beaconing sequence, the reference nodes will execute the following procedure:

A. Step 1: Range Difference Computation

When R_1 (master node) receives the notification from s, it initiates a beacon signal at $t_1^{'}$. Let $t_{s,1}$, t_j, be the times when s and the reference nodes R_j, $j = \{2, \cdots, N\}$ receive R_1's signal. After some processing delay, δ_2, at time $t_2^{'}$, R_2 replies to R_1 with a beacon signal conveying $t_2^{'} - t_2 = \Delta t_2$ to s. The signal reaches s at $t_{s,2}$. After receiving beacon signals from R_1 and R_2, at time $t_3^{'}$, R_3 replies to R_1 with a beacon signal conveying information $t_3^{'} - t_3 = \Delta t_3$ to s. The signal reaches s at time $t_{s,3}$. In a similar way, R_j, $j = \{4, 5, \cdots, N\}$ will convey information Δt_j to s. Note, however, that for $j \geq 4$, R_j will transmit a beacon as long as it successfully receives the beacon from R_1 and R_{j-1}.

The above procedure is illustrated in Fig. 1.

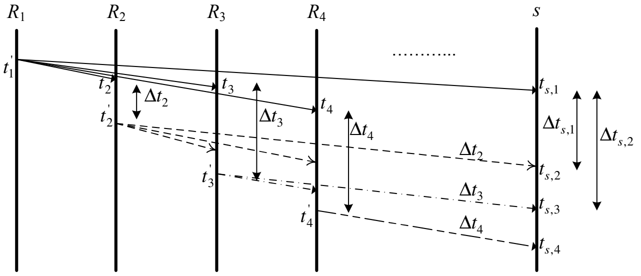

Fig. 1. Illustration of Generalised UPS.

Let v be the speed of sound underwater and let $\Delta t_{s,k} = t_{s,k+1} - t_{s,1}$, for $k = \{1, \cdots, N-1\}$. For $j = \{2, 3, \cdots, N\}$, we have

$$d_{1j} + d_{sj} - d_{s1} + v\Delta t_j = v\Delta t_{j-1},$$

which gives

$$d_{sj} = d_{s1} + k_{j-1}, \tag{1}$$

with

$$k_{j-1} = v\Delta t_{j-1} - v\Delta t_j - d_{1j}.$$

B. Step 2: Location Computation

Expanding Eqn. (1), we obtain the following system of N equations with unknowns x, y, z and d_{s1}:

$$(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = d_{s1}^2 \tag{2}$$

$$(x - x_j)^2 + (y - y_j)^2 + (z - z_j)^2 = (d_{s1} + k_{j-1})^2, \tag{3}$$

where $j = \{2, 3, \cdots, N\}$. Since we need at least 4 equations to solve for 4 unknowns, the necessary condition is $N \geq 4$.

C. Limitations of UPS(4)

Intuitively, the choice of N = 4 offers the best solution since it requires the fewest reference nodes (i.e., lowest infrastructure cost) - this reduces to the Underwater Positioning System [2], which we denote using UPS(4). Without loss of generality, let us assume that the four reference nodes are located at (0,0,0), (x_2,0,0), (x_3,y_3,0) and (x_4,y_4,z_4).

From Eqn. (2), and Eqn. (3) for $j = 2$, it follows that

$$x = A_x d_{s1} + B_x,$$

where

$$A_x = -\frac{k_1}{x_2}, \quad B_x = \frac{x_2^2 - k_1^2}{2x_2}.$$

From Eqn. (2), and Eqn. (3) for $j = 2, 3$, it follows that

$$y = A_y d_{s1} + B_y,$$

where

$$A_y = \frac{k_1}{x_2}\frac{x_3}{y_3} - \frac{k_2}{y_3}$$

$$B_y = \frac{x_3^2 + y_3^2 - x_2 x_3 + \frac{x_3 k_1^2}{x_2} - k_2^2}{2y_3}$$

and

$$z = A_z d_{s1} + B_z,$$

where

$$A_z = \frac{k_1}{x_2}\frac{x_4}{y_4} - \frac{k_3}{z_4} - \frac{y_4(\frac{k_1 x_3}{x_2} - k_2)}{y_3 z_4}$$

$$B_z = \frac{x_4^2 + y_4^2 + z_4^2 - x_2 x_4 + \frac{k_1^2 x_4}{x_2} - k_3^2 - \frac{y_4 x_3^2}{y_3}}{2z_4} + \frac{-y_3 y_4 + \frac{x_2 x_3 y_4}{y_3} - \frac{k_1^2 x_3 y_4}{x_2 y_3} + \frac{k_2^2 y_4}{y_3}}{2z_4}.$$

If we now replace in Eqn. (2) the expressions of x,y and z found above, we find that d_{s1} has to satisfy the following second degree equation:

$$d_{s1}^2(\Sigma_A - 1) + 2(A_x B_x + A_y B_y + A_z B_z)d_{s1} + \Sigma_B = 0, \tag{4}$$

where $\Sigma_A = A_x^2 + A_y^2 + A_z^2$ and $\Sigma_B = B_x^2 + B_y^2 + B_z^2$.

Denote by Δ the discriminant of Eqn. (4), where $\Delta > 0$, and let $d^{'}_{s1}$ and $d^{''}_{s1}$ be the two real solutions for d_{s1}. The uniqueness of d_{s1} depends on the value of Σ_A as follows:

$$\begin{aligned} d^{'}_{s1} = \; & d^{''}_{s1}, \quad \Sigma_A = 1; \\ d^{'}_{s1} \cdot d^{''}_{s1} < \; & 0, \quad \Sigma_A < 1; \\ d^{'}_{s1} \cdot d^{''}_{s1} > \; & 0, \quad \Sigma_A > 1. \end{aligned} \tag{5}$$

The conditions stated in Eqn. (5) suggest that if $\Sigma_A > 1$ for a given deployment of reference nodes, and location of s, then node s cannot be uniquely localized since $d^{'}_{s1}, d^{''}_{s1} > 0$.

To illustrate the extent of this infeasible region, let us deploy $\{R_i\}_{i=1:4}$ at (0,0,0), (D,0,0), (0,D,0) and (0,0,D) respectively, where $\sqrt{3}D$ is the communication range of each node (including node s). We consider a 3-dimensional space, S, of size $D \times D \times D$ that contains the reference nodes as well as node s. The deployment is shown in Fig. 2.

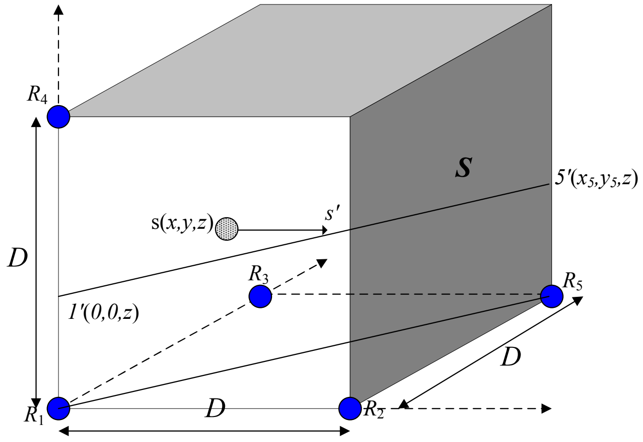

Fig. 2. Deployment of reference nodes and computation of TO_S.

We plot the feasible region for s (shaded), i.e., locations for which s can be uniquely localized in Fig. 3 for z=0, 10 and D, where D = 50. To quantify this, we plot the proportion of feasible region in S as a function of z in Fig. 4 for various values of D. We observe that up to 16% of the plane containing R_1, R_2 and R_3 is not localizable, which is quite significant.

The problem of infeasible regions has been pointed out in [2], where the authors claim that the correct position for s can be computed as long as it resides in the enclosed space by the four reference nodes, even when it is close to a reference node. According to our investigations, this is not true. For example, in Fig. 3, for z=10, we observe that infeasible locations exist.

D. UPS(5)

In this section, we address the limitations of UPS(4) by considering a fifth reference node, located at (x_5, y_5, z_5) in the space S. We denote this scheme as UPS(5).

Substituting for x,y,z found in Section III-C into Eqn. (3) for $j = 5$, we get:

$$d_{s1}^2(\Sigma_A - 1) + 2(\Sigma_{AB} - k_4)d_{s1} + \Sigma_{BB} - k_4^2 = 0, \tag{6}$$

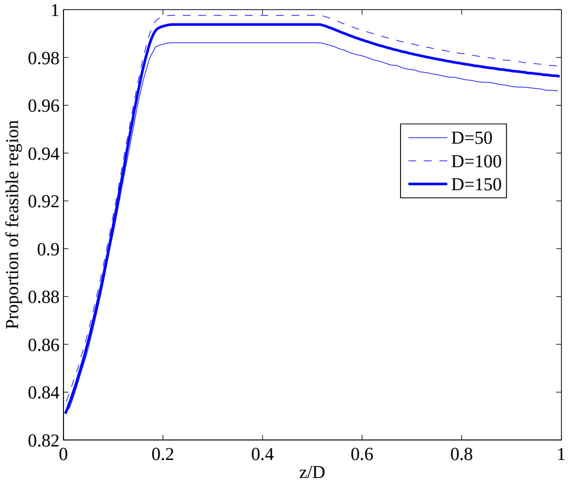

Fig. 4. Proportion of feasible region with UPS(4).

where B_{aa} = B_a-a_5, $a \in \{x, y, z\}$ and

$$\begin{aligned} \Sigma_{BB} &= B_{xx}^2 + B_{yy}^2 + B_{zz}^2, \\ \Sigma_{AB} &= A_x B_{xx} + A_y B_{yy} + A_z B_{zz}. \end{aligned}$$

Eqn. (4) and (6) have the same solutions if the sum and product of the two solutions are identical. This happens under the following conditions:

$$\begin{aligned} \Sigma_{AB} - k_4 &= A_x B_x + A_y B_y + A_z B_z \\ \Sigma_{BB} - k_4^2 &= \Sigma_B. \end{aligned}$$

The above conditions can be rewritten as follows:

$$\begin{aligned} A_x x_5 + A_y y_5 + A_z z_5 &= k_4 \\ B_x x_5 + B_y y_5 + B_z z_5 &= \frac{x_5^2 + y_5^2 + z_5^2 - k_4^2}{2}. \end{aligned} \tag{7}$$

Suppose $\Sigma_A > 1$, i.e., s cannot be uniquely localized with UPS(4). Then, adding R_5 will not achieve unique localization only if the *additional* conditions in (7) are satisfied; otherwise, the value of d_{s1} is given by the solution of the following first degree equation:

$$2d_{s1}(\Sigma_A + k_4) - (d_{05}^2 - 2B_x x_5 - 2B_y y_5 - 2B_z z_5) + k_4^2 = 0, \tag{8}$$

where $d_{05}^2 = x_5^2 + y_5^2 + z_5^2$ and s can be uniquely localized with UPS(5).

IV. WIDE COVERAGE POSITIONING SYSTEM (WPS)

Although UPS(5) *achieves* unique localization to node s w.h.p compared to UPS(4), it may introduce additional latency and communication costs redundantly in cases where UPS(4) suffices. Accordingly, we propose a Wide Coverage Positioning (WPS) system that (i) relies on an infrastructure of 5 reference nodes but (ii) only utilizes beaconing from the fifth reference node when required.

In WPS, the reference nodes perform beaconing according to UPS(5), as described in Section III-A. Node s monitors the

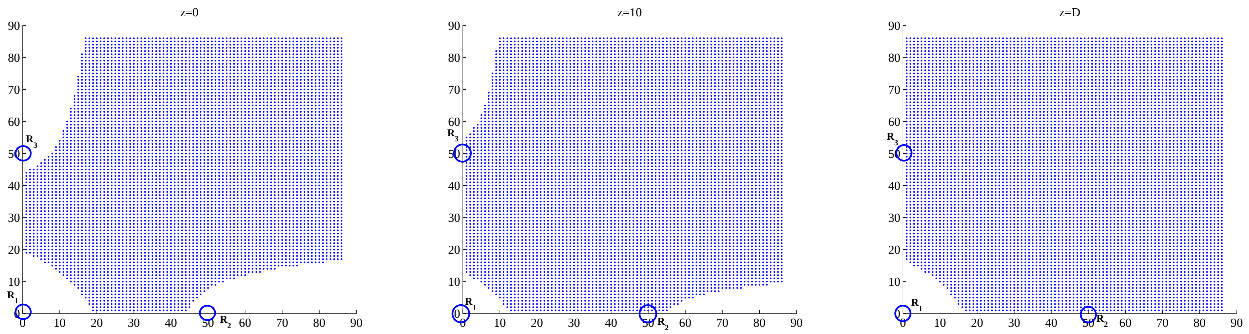

Fig. 3. Feasible region with UPS(4) for z=0 (left), z=10 (centre) and z=D (right) (D = 50).

beacons received from the reference nodes. Upon receiving 4 beacons, it computes Σ_A and checks condition (5): if s cannot be uniquely localized, it will wait for the 5^{th} beacon and check condition (7) before declaring successful localization. The pseudo-code for node s is given in Algorithm 1.

Algorithm 1 WPS: Pseudocode for node s

```
procedure WPS($TO_S$)
    t = 0
    rMSG = 0
    LOCALIZED = 0
    Start timer and notify beaconing sequence
    while t < $TO_S$ & LOCALIZED==0 do
        if (receive new beacon) then
            rMSG += 1
        end if
        if (rMSG == 4 & $\Sigma_A \leq 1$) | (rMSG == 5 & condition (7)==FALSE) then
            LOCALIZED=1
        end if
    end while
    if (LOCALIZED==0) then
        node $s$ times-out
    end if
end procedure
```

Due to harsh underwater acoustic channel conditions, it is possible that s receives less than 4 beacons, in which case, a time-out will be triggered. The design of the time-out value, TO_S, is based on UPS(5) and is described next.

A. *Design of TO_S*

Let t_p be the packet transmission time. The processing delays δ_j can be computed at s based on the time-stamps it receives in response to the wake-up beacon.

If all beacon transmissions are successful, the *maximum* localization time is given by:

$$T_0 = \tau_{s,1} + \sum_{i=1}^{4} \tau_{i,i+1} + \tau_{5,s} + \sum_{i=1}^{5} \delta_i + \delta_s + 6t_p,$$

where $\tau_{a,b}$ is the propagation delay incurred for sending a message from node a to node b. By projecting s onto the plane formed by the line joining nodes 1 and 5 and orthogonal to the x-y plane (denoted by s'), as shown in Fig. 2, and applying the triangular inequality, we obtain the following:

$$\begin{aligned} \tau_{1,s'} + \tau_{5,s'} &\leq \tau_{1,1'} + \tau_{1',5} \\ &= \frac{z}{c} + \sqrt{(\frac{z}{c})^2 + \tau_{1,5}^2}. \end{aligned}$$

Since $d_{1s} \leq d_{1s'} + d_{SS'}$ and $d_{5s} \leq d_{5s'} + d_{ss'}$, we have:

$$\tau_{1,s} + \tau_{5,s} \leq \frac{z}{c} + \sqrt{(\frac{z}{c})^2 + \tau_{1,5}^2} + 2\tau_{ss'}.$$

Since s' is constrained to lie on the line joining 1' and 5', we can write the following:

$$\tau_{ss'} \leq \tau_{3,5}.$$

Hence, we can express T_0 as follows:

$$T_0 \leq \sum_{i=1}^{4} \tau_{i,i+1} + \frac{z}{c} + \sqrt{(\frac{z}{c})^2 + \tau_{1,5}^2} + 2\tilde{\tau}_{15} + \Delta,$$

where

$$\Delta = \sum_{i=1}^{5} \delta_i + \delta_s + 6t_p.$$

Since s is constrained within space S, $z \leq Y$, i.e.,

$$\begin{aligned} T_O &\leq \sum_{i=1}^{4} \tau_{i,i+1} + \frac{Y}{c} + \sqrt{(\frac{Y}{c})^2 + \tau_{1,5}^2} + 2\tilde{\tau}_{15} + \Delta \\ &\equiv T_{min}. \end{aligned}$$

Hence, if s is unable to localize itself by T_{min}, it should trigger a time-out to re-initiate the localization procedure. Accordingly, we can set TO_S = T_{min}, i.e.,

$$TO_S = \sum_{i=1}^{4} \tau_{i,i+1} + \frac{Y}{c} + \sqrt{(\frac{Y}{c})^2 + \tau_{1,5}^2} + 2\tilde{\tau}_{15} + \Delta.$$

Since node s knows the location of all the reference nodes, it will be able to compute TO_S.

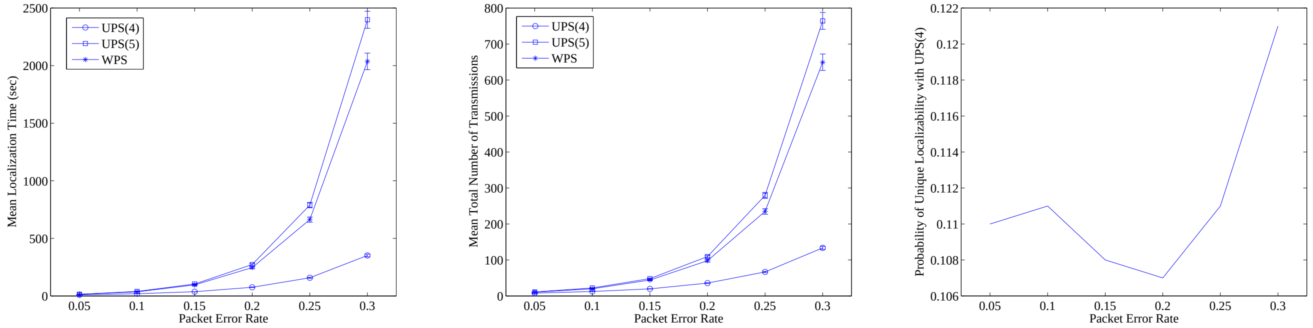

Fig. 5. Localization latency, communication costs and uniqueness: WPS vs UPS(4), UPS(5).

V. NUMERICAL RESULTS

In this section, we compare the performance of WPS against UPS(4) and UPS(5) in terms of the localization time, total number of transmissions until successful localization and probability of unique localization via simulations conducted using the Qualnet simulator [12].

We assume that the reference nodes are deployed according to Figure 2. In each simulation run, node s is deployed randomly within the space S. In addition, the processing delay at each node is fixed at δ=0.01, the beacon size is 256 bytes and the link rate is 5kbps. We vary the channel quality by considering p to be in the range [0.05:0.3]. For each parameter setting, we obtain the mean and 95% confidence interval over 1000 simulation runs. The results are plotted in Fig. 5.

As expected, the localization time and total number of transmissions obtained with WPS are bounded by the corresponding performance with UPS(4) and UPS(5). WPS achieves between 10 to 20% performance gain compared with UPS(5). Although its performance is significantly worse than UPS(4), it guarantees unique localization for the given deployment, while UPS(4) achieves unique localization between 10 to 12% of the time.

VI. CONCLUSIONS AND FUTURE WORK

In this paper, we consider the problem of underwater localization. We generalize the recently proposed range-based Underwater Positioning System with an infrastructure of N reference nodes (UPS(N)), which relies on minimal transmission from the node to be localized and more importantly, is not based on the premise of synchronized clocks. While the original scheme (UPS(4)) requires minimal infrastructure for 3-D localization, it does not guarantee unique localization, i.e., there exist an infeasible region. We illustrate the extent of this infeasible region and quantify the conditions for unique localization. We show that, by introducing a fifth reference node, unique localization can be guaranteed with high probability using UPS(5).

Accordingly, we propose a Wide Coverage Positioning System (WPS) that (i) relies on an infrastructure of 5 reference nodes but (ii) only utilizes beaconing from the fifth reference node when required, so as to minimize the localization time. We show, via simulations, that WPS achieves better localization speed with lower communication costs than UPS(5). Although it performs worse than UPS(4), it guarantees unique localization, while the latter does so only 10-12% of the time.

For future work, we plan to combine WPS (that offers unique localization with high probability) with E-UPS [3], which improves the robustness of UPS(4) in harsh underwater channel conditions. In addition, we plan to consider a more realistic underwater acoustic channel model in future performance evaluations.

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