Twilight for the energy conditions?

Matt Visser
Physics Department
Washington University
Saint Louis
USA

Collaborators:
Carlos Barceló (Wash U)
David Hochberg (LAEFF, Madrid)
Carmen Molina–París (CAB, Madrid)

Marcel Grossmann 9
Roma
July 2000

Abstract:

The energy conditions of general relativity permit one to deduce very powerful and general theorems.

However, the energy conditions are now realized to be a lot less secure than they once seemed:

— There are quantum effects that violate all of the energy conditions.

— There are relatively benign classical systems that violate all the energy conditions.

This opens up a Pandora's box of rather disquieting possibilities.

Is it twilight time for the energy conditions?

Overview:

Classical: In classical general relativity the energy conditions are used to prove lots of general theorems...

Quantum: Everyone who thought about it expected the energy conditions to eventually break down once one reaches the Planck slop...

Surprise 1: The energy conditions already fail miserably in **semi-classical quantum gravity**.

Failures occur at first order in $\hbar$, long before one reaches the Planck slop...

The quantum failures are widespread, albeit small...

Surprise 2: The energy conditions also fail miserably in quite reasonable **classical systems**.

Energy conditions:

Standard (pointwise) energy conditions:

TEC — trace energy condition
(now abandoned).

NEC — null energy condition.

WEC — weak energy condition.

SEC — strong energy condition.
(*aka* unphysical energy condition.)

DEC — dominant energy condition.

Standard averaged energy conditions:

ANEC — averaged null energy condition.

AWEC — averaged weak energy condition.

ASEC — averaged strong energy condition.

Standard propaganda:

Physically reasonable Lagrangians give classical theories satisfying the energy conditions...

The standard propaganda is wrong.

Definition:

In a Lorentzian spacetime:

The null energy condition [NEC] is said to hold at a point x, if for all null vectors k

$$T_{\mu\nu}\, k^\mu k^\nu \geq 0.$$

NEC Is the weakest pointwise energy condition in common use.

Notes:

DEC $\Rightarrow$ WEC $\Rightarrow$ NEC.

SEC $\Rightarrow$ NEC.

SEC does *not* imply WEC —
— the terminology is misleading.

Uses of the Energy Conditions:

- Penrose singularity theorem (WEC).

- Hawking–Penrose singularity theorem (SEC). [Relevant to the cosmological singularity.]

- Tipler's version of the Hawking–Penrose singularity theorem (WEC+ASEC).

- Schoen–Yau positive mass theorem (DEC).

- Witten's variant positive mass theorem (DEC).

Hypotheses are all stronger than the NEC.

Uses of the ANEC:

ANEC is the weakest averaged energy condition in common use.

The ANEC is used as input hypotheses in proving:

- Focussing theorems for null geodesics. [Borde]
- Generalized Penrose singularity theorem. [Roman]
- Topological censorship theorem. [Friedman–Schleich–Witt]
- Generalized positive mass theorem. [Penrose–Sorkin–Woolgar]

Quantum violations of the energy conditions:

- 2-particle Fock states. (NEC+)
- Casimir vacuum. (NEC+)
  [DeWitt, Einstein Centenary Survey]
- Hawking radiation. (NEC+)
- Squeezed vacuum. (WEC+DEC)
  [Morris–Thorne]
- Conformal anomaly (NEC+)
  [Visser, PLB 349 (1995) 443–447; gr-qc/9409043].
- Gravitational vacuum polarization (NEC+)
  [Visser, gr-qc/9604007; 9604008; 9604009; 9703001].
- Cosmological particle production. (SEC)

Classical violations of the energy conditions:

"Observational"

- Cosmological inflation. (SEC)
  [Minimally coupled massive scalar]

- Cosmological inflation. (NEC+)
  [Conformally coupled massive scalar]

- Galaxy formation: $0 < z < 10$. (SEC)
  [Visser, Science 276 (4 April 1997) 88]

- Accelerating universe. (SEC)

Classical violations of the energy conditions:

"Theoretical"

- Tolman wormholes (SEC)
  [Hochberg, Molina–París, Visser]

- Massless conformally coupled scalar (ANEC+)
  QFT: new improved energy-momentum
  [Barceló–Visser, gr-qc/9908029, PLB]
  [Barceló–Visser, gr-qc/0001099, Cosmo99]

- Non-minimally coupled scalar (ANEC+)
  massive/massless
  [Wald–Flanagan, gr-qc/9602052, PRD]
  [Barceló–Visser, gr-qc/0003025, CQG]

- String moduli fields (ANEC+)
  [Barceló–Visser, gr-qc/0001099, Cosmo99]

- Negative tension branes (ANEC+)
  [Barceló–Visser, hep-th/0004022, NPB]

Tolman wormhole:

Q: What are the *minimal* conditions for a "bounce"?

D: (Tolman wormhole $\equiv$ "bounce".)

A: Perform a model-independent analysis of the geometry near a bounce, along the lines of the Morris–Thorne analysis for traversable wormholes.

Details:
gr-qc/9810023, PLB455 (1999) 90-95
[Molina-París, Visser]
gr-qc/9810029, PRD59 (1999) 044011
[Hochberg, Molina-París, Visser]

Flare-out at the bounce $\Rightarrow$ SEC violated at or near the bounce.

Notes:

SEC violations are a *necessary* but not *sufficient* condition for a “bounce”.

You do not *need* to violate NEC, WEC, or DEC to get a “bounce”.

If you believe inflation you have already abandoned the SEC anyway.

Inflation will not *guarantee* a bounce, but it opens the door.

New improved stress tensor plus gravity:

Take a massless conformally coupled scalar field and add Einstein gravity.
(Static, spherically symmetric.)

Absurdly easy problem; surprising result.

— NEC and ANEC are often violated.

— There is a three-parameter class of exact solutions (total mass, scalar charge, scalar field at infinity).

Special cases:

(1) Schwarzschild/ anti-Schwarzschild.

(2) Naked singularities.

(3) Naked singularities hiding behind wormhole throats. (Not really "traversable".)

(4) Traversable wormholes with two asymptotically flat regions.

Details: gr-qc/9908029, Barceló–Visser, PLB.

Non-minimally coupled scalars plus gravity:

Take a generic non-minimally coupled scalar field and add Einstein gravity.
(Static, spherically symmetric.)

Absurdly easy problem; surprising result.

— NEC and ANEC are often violated.

— There is a four-parameter class of exact solutions (total mass, scalar charge, scalar field at infinity, and curvature coupling).

Special cases:

(1) Schwarzschild/ anti-Schwarzschild.
(2) Naked singularities.
(3) Naked singularities hiding behind wormhole throats. (Not really "traversable".)
(4) Traversable wormholes with two asymptotically flat regions.
For $\xi \leq 0$ no traversable wormholes.
For $\xi > 0$ get traversable wormholes.

Details: gr-qc/0003025, Barceló–Visser, CQG.

Negative tension branes plus gravity:

Take a negative tension brane and add Einstein gravity.
(Static, spherically symmetric, (3+1)-D.)

(Negative tension branes are now extremely common in brane cosmology, and variants of the Randall–Sundrum scenario. More than 50 papers as of July 2000.)

Absurdly easy problem, surprising result.

— NEC and ANEC are always violated.

— There is a four-parameter class of exact solutions (total mass, brane tension, bulk cosmological constant, electric charge).

Special cases:

Traversable wormholes with two asymptotically de Sitter regions.

Details: hep-th/0004022, Barceló–Visser, NPB.

Specific Implications:

Don't focus on the specific technical details. (Naked singularities, traversable wormholes). The main points are:

- Classical violations of the NEC arise in these very reasonable classical systems.

- Conformally coupled scalars are from a QFT perspective the preferred choice — corresponding to the new improved stress-energy tensor.

- String moduli are generic.

- Negative tension branes are ubiquitous.

- It's the fact that you get classical NEC violations in such simple physical systems that's worrying — the fact that these NEC violations are big enough to support traversable wormholes is a bonus.

General Implications:

- We do not currently have an acceptable positive mass theorem.
- We do not currently have an acceptable singularity theorem.
- We do not currently have an acceptable topological censorship theorem.
- Traversable wormholes almost begin to look physically reasonable.
- Tolman wormholes almost begin to look physically reasonable.
- This opens up a whole mess of weird possibilities...

Conclusions:

We need:

- Improved understanding of just what conditions can sensibly be put on the stress-energy tensor.
  (Quantum inequalities?)

- Improved energy conditions —
  you do not want a free-for-all.
  (Arbitrary stress-energy tensor $\Rightarrow$ arbitrarily weird physics.)

- Improved (positive-mass/ singularity/ censorship) theorems of all types.

— # # # —