Sonoluminescence — a QED vacuum effect?

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UMSL/U Maryland

November 1999.

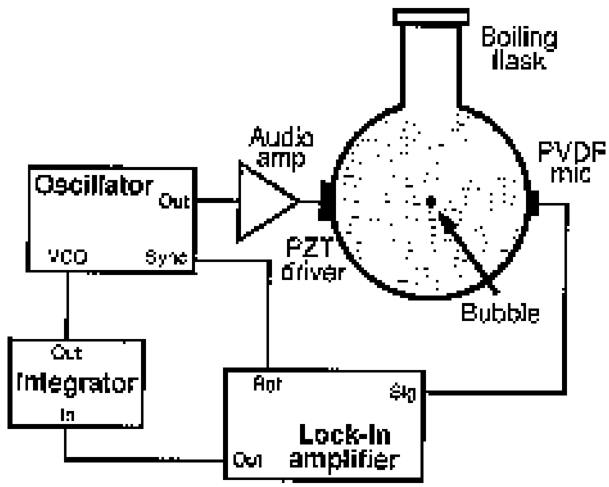
Abstract:

Sonoluminescence occurs when a bubble (typically an air bubble in water) is driven to expand and collapse by imposing an ultrasonic sound wave (about 40 kHz). In stable single-bubble sonoluminescence, every time the bubble goes through the collapse phase there is a brief flash of visible light (frequencies about 10^{15} Hz). While everyone agrees that the effect is real the mechanism is highly controversial.

In my colloquium I will describe and extend one of the candidate theories: Julian Schwinger's suggestion that collapse of the bubble causes significant changes in the QED vacuum — this is sometimes called the dynamical Casimir effect. I will explore the consequences of taking this QED vacuum effect seriously, and try to indicate some make-or-break issues that could experimentally settle this matter.

The Basics — I:

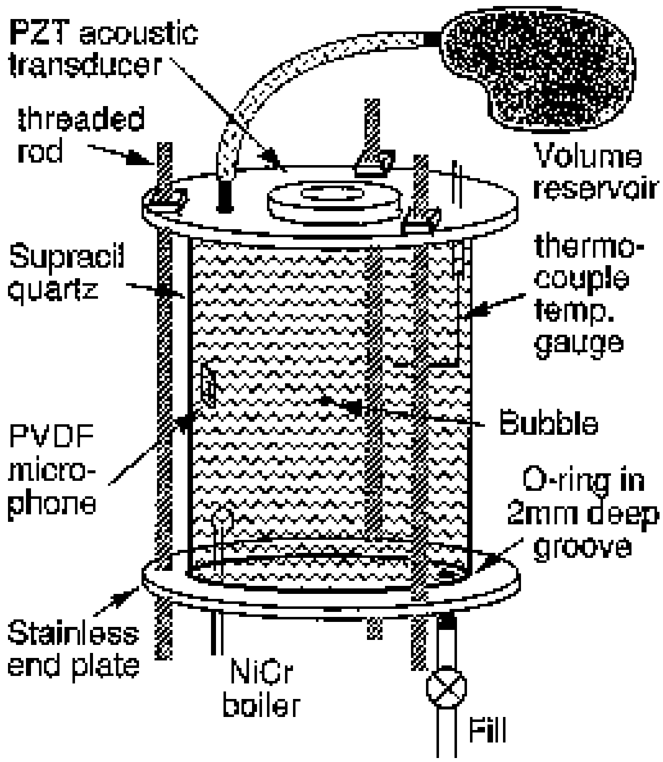
You need a few litres of water, a single bubble of air, and a powerful loudspeaker.



Turn on the loudspeaker, at about 40 kHz.

(There is nothing magic about 40 kHz, it's just safely out of human hearing range — the loud-speaker has to be loud, and having to explain deaf graduate students to OSHA is not a good plan...)

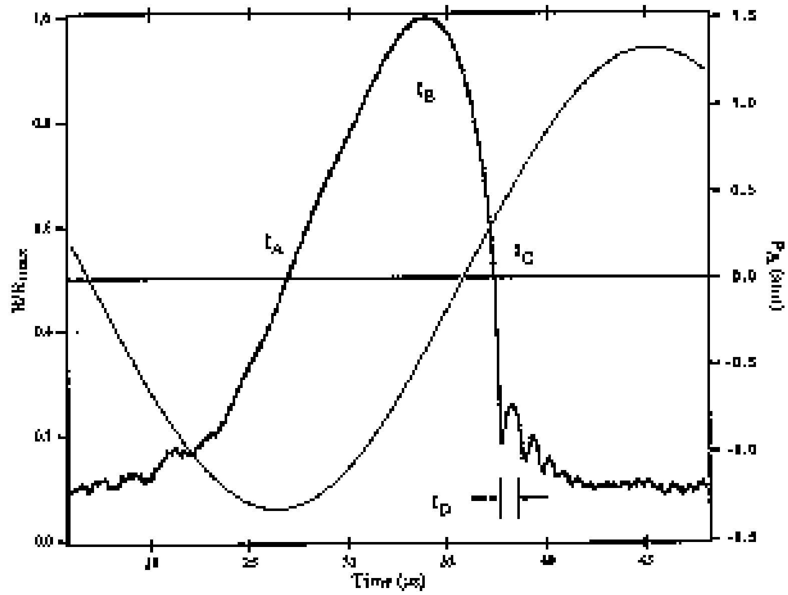
The Basics — II:



Sketch of a cylindrical resonator for obtaining sonoluminescence from a sealed system. This is important for controlling the composition and the partial pressure of the gas content in the resonator. The NiCr (toaster) wire is used to seed a bubble by boiling the liquid locally. The vaporous cavity fills with whatever gas is dissolved in the liquid, and is at the same time yanked to the velocity node of the sound field where it emits light at a sufficiently high acoustic drive.

The Basics — III:

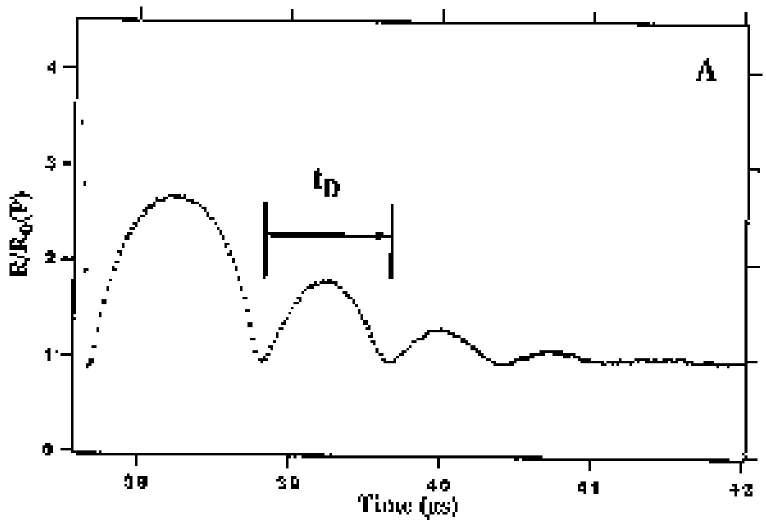
The bubble will undergo expansion, collapse, and rebound on each acoustic cycle.



The radius $R(t)$ scaled to the maximum radius R_{max} during one acoustic cycle. The experimental data comes from an air bubble in water. Comparison with the Rayleigh–Plesset equation of bubble dynamics indicates that for these data $R_{ambient} = 4$ microns, and the amplitude of the acoustic overpressure is 1.35 atmospheres.

The Basics — IV:

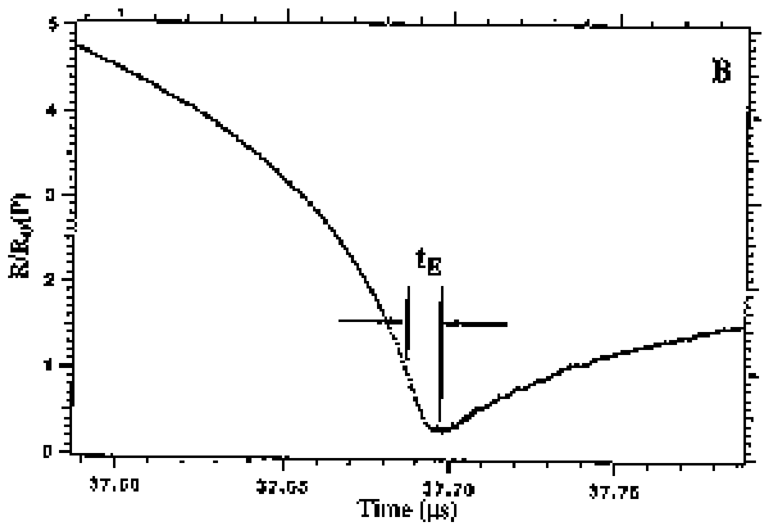
Bubble radius:
4.5 microns → 45 microns → 0.5 microns.



Detail of the bubble motion $R(t)$ near the minimum radius R_{min}, taken with continuous laser illumination. The observation that $R_{min}/R_{ambient} < 1/8$ indicates that the van der Waals hard core of the gas plays a role in limiting collapse of the bubble. The breathing period is t_D, and the time to go from the ambient radius to the collapse radius is t_E.

The Basics — V:

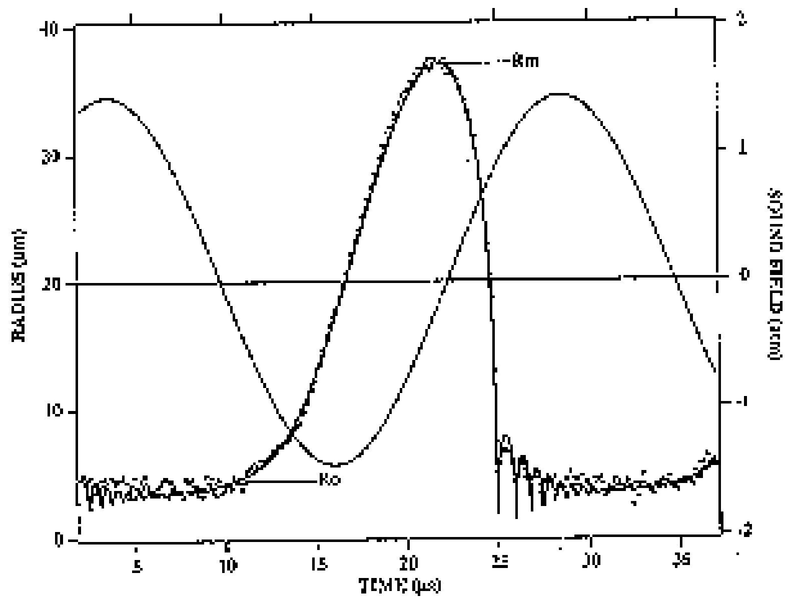
Bubble radius:
4.5 microns $\rightarrow$ 45 microns $\rightarrow$ 0.5 microns.



Detail of the bubble motion $R(t)$ near the minimum radius R_{min}, taken with continuous laser illumination. The observation that $R_{min}/R_{ambient} < 1/8$ indicates that the van der Waals hard core of the gas plays a role in limiting collapse of the bubble. The breathing period is t_D, and the time to go from the ambient radius to the collapse radius is t_E.

The Basics — VI:

Bubble radius:
4.5 microns → 45 microns → 0.5 microns.

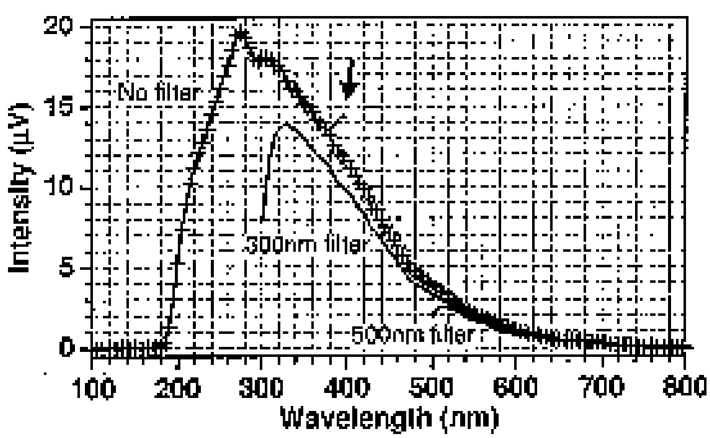


Radius of a 3 mm xenon bubble as a function of time over one acoustic cycle. When the acoustic pressure is negative the bubble expands. This is followed by collapse and ringing before the bubble sits dead in the water waiting for the next cycle. Calibrating the data by the Rayleigh–Plesset equation (solid line) the ambient radius is 4.3 microns and the acoustic overpressure is 1.45 atmospheres.

The Basics — VII:

On each acoustic cycle, there is a very brief flash, during which about one million photons of visible light are emitted.

Spectrum: Broad-band, (no obvious resonances, approximately power law at low frequencies). Up to $f \leq 3 \times 10^{15}$ Hz; $\lambda \geq 100$ nm.



Raw data showing the uncalibrated spectrum of light emitted by a sonoluminescing bubble of helium in water. Note that spectral response turns on at about 185 nm (6.5 eV).

The Basics — VIII:

Effective temperature:
40,000 K; $\rightarrow$ 70,000 K; $\rightarrow$ 100,000 K.

Q: Is this a real physical temperature?

Pulse width: $\tau < 10$ ps? $\tau < 350$ ps?

Pulse width independent of frequency?

Size of SL region: small... (0.5 microns?)

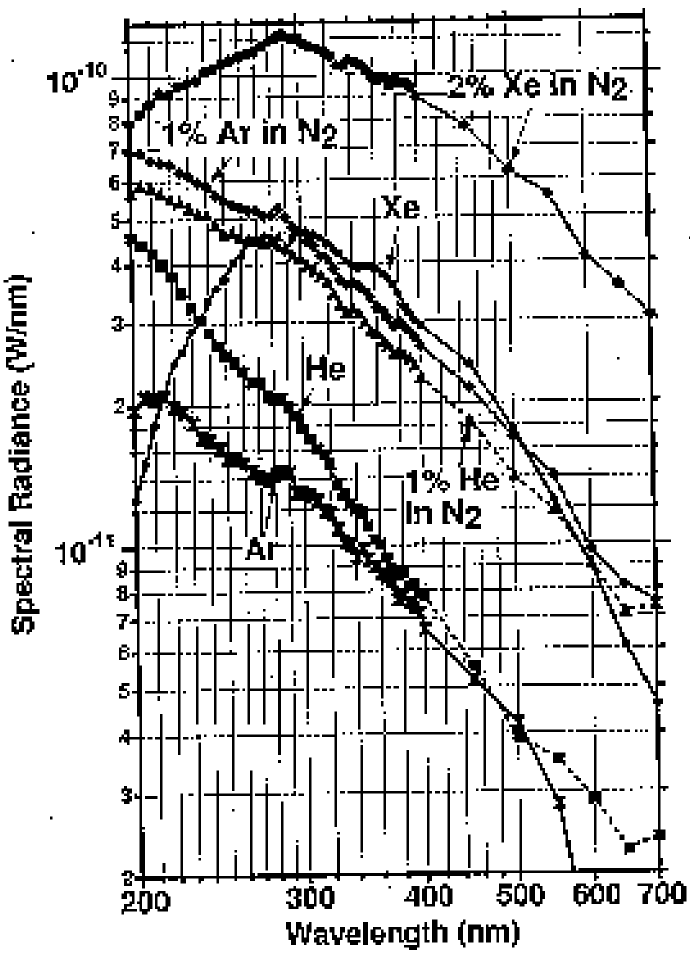
Time averaged power: 30 mW $\rightarrow$ 100 mW.

Best liquid: water.

Best gas: air with one percent noble gas.

The Basics — IX:

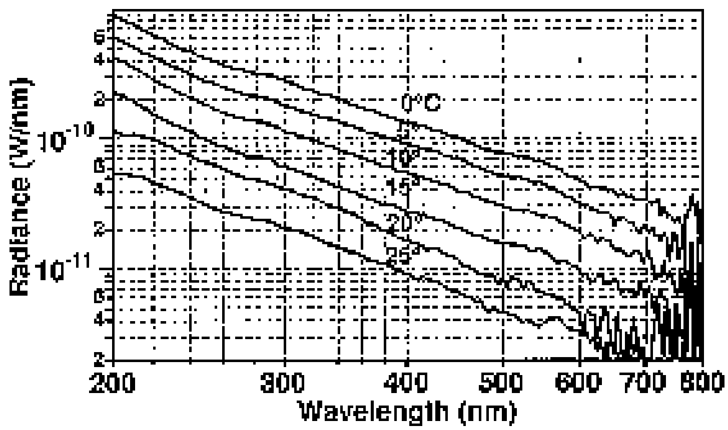
Sensitive to noble gas admixture
(Pure nitrogen or pure oxygen —
almost no effect).



Spectrum of SL for various gas mixtures dissolved in water at T = 24 Celsius. (All for $R_{ambient}$ = 150 mm.) Note that although helium is dimmer than xenon it has higher spectral density in the ultraviolet.

The Basics X:

Sensitive to ambient temperature
(T up $\Rightarrow$ SL decreases).

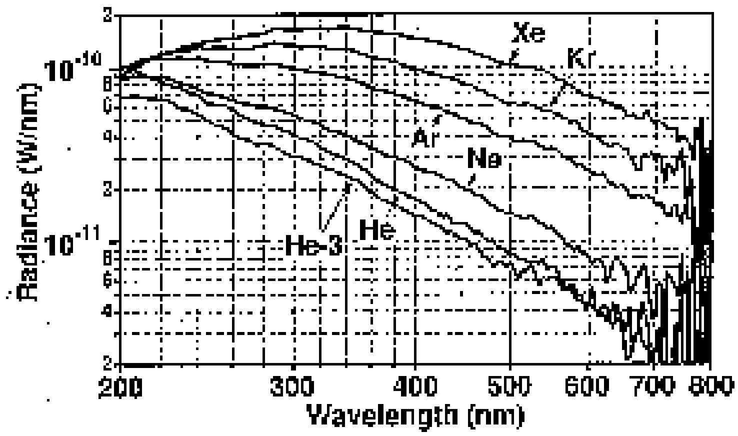


Corrected spectra for a 150 mm bubble of helium in water at various temperatures.

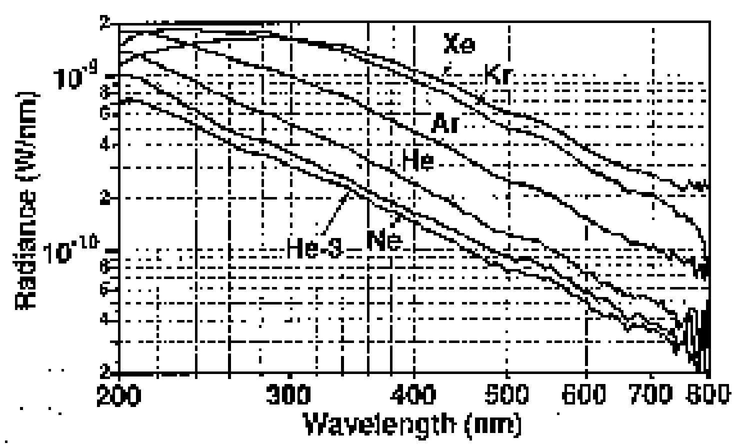
Sensitive to ambient magnetic fields
(B up $\Rightarrow$ SL decreases).

The Basics XI:

Sensitive to noble gas admixture (Pure nitrogen or pure oxygen — almost no effect).



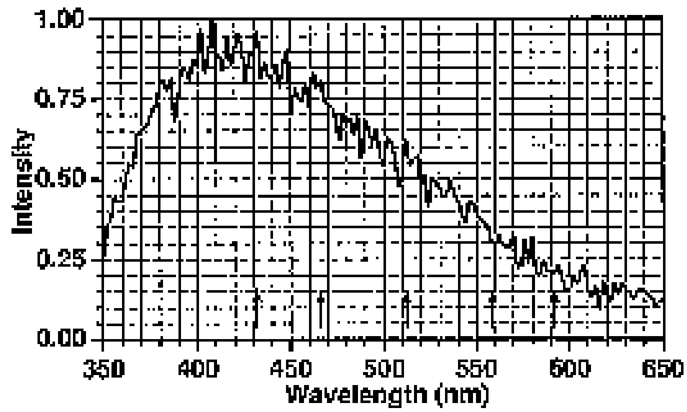
Room temperature spectra of various noble gasses in water.



Freezing point spectra of various noble gasses in water.

<u>The Basics XII:</u>

There is a marked absence of Swan lines:

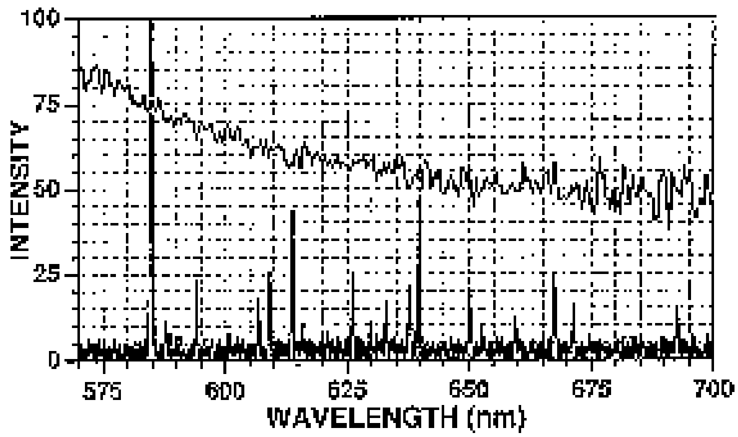


Spectrum of xenon in dodecane. Obtained at a resolution of 1 nm, this spectrum shows no evidence of the Swan lines which are emitted by excited carbon molecules. Swan lines *have* been observed in experiments designed to measure transient SL.

B.P. Barber et al./Physics Reports 281 (1997) 65–143.

The Basics XIII:

There is a marked absence of normal gas discharge lines:



Comparison of the spectrum of SL from a 3 Torr neon bubble and the spectrum of gas discharge through a 75 Torr neon atmosphere. Note that the dramatic peaks which give neon its characteristic orange-red colour are absent for SL.

B.P. Barber et al./Physics Reports 281 (1997) 65–143.

Key Questions:

What is the physics of the frequency conversion?
(kilo-Hertz acoustic energy $\rightarrow$
peta-Hertz electromagnetic energy!)

That's a factor of 10^{12} in frequency!

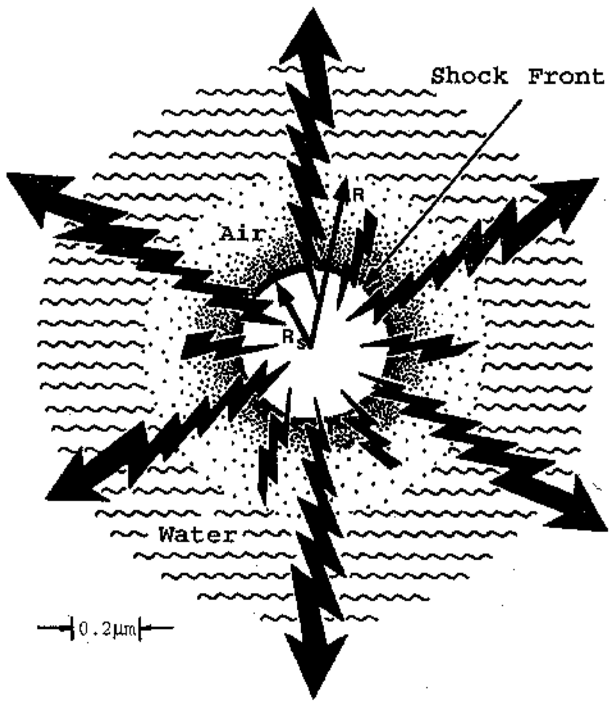
What is the physics of the cutoff?
Thermal? — adiabatic heating?
Transparency? — dissociation?
Refractive index? — plasma frequency?
Adiabatic response? — response times?

Is the bubble thermalized?
(Be suspicious, be very suspicious...)

Tentative models:

- Shock heating
- Adiabatic heating
- Bremsstrahlung
- Sudden ionization
- Dynamical Casimir effect

Tentative model — Shock heating:



Sketch of an imploding shock wave model of SL. The shock is launched by the supersonic motion of the bubble wall. The radius of the gas water interface is R and the radius of the shock is R_s. The shock first implodes to a focus and then explodes. This figure depicts the state reached about 100 ps after focussing.

B.P. Barber et al./Physics Reports 281 (1997) 65–143.

Tentative model — dynamical Casimir effect:

At least three variants of this model —

- Schwinger's original

  [quasi-static back-of-envelope estimate for energy budget; no real estimate of conversion efficiency]

  [highly contentious: serious disagreements regarding the definition of the static Casimir energy for dielectrics.]

- Eberlein's variant

  [estimates conversion efficiency using modified adiabatic approximation]

  [trying to fit the observed luminosity data drives the model out of the adiabatic regime]

- Our variant

  [analytic estimates for conversion efficiency in two regimes]

Schwinger's original model — I:

(Schwinger's calculation was static, not dynamical, despite his choice of terminology.)

Compare the static zero-point energy of a dielectric with the static zero point energy of ordinary vacuum

$$E = V \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \, \tfrac{1}{2}\hbar \left\{ \omega(\vec{k}) - c||\vec{k}|| \right\}.$$

In terms of the refractive index

$$E = V \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \, \tfrac{1}{2}\hbar c||\vec{k}|| \, \left\{ \frac{1}{n(\vec{k})} - 1 \right\} < 0.$$

In real life all materials have $n \to 1$ at high enough frequency.

Schwinger's original model — II:

Approximate the refractive index of water by 1.3 at low frequencies, and 1.0 at frequencies above the extreme ultraviolet.

Approximate the refractive index of air by 1.0 at all frequencies.

The static zero point energy of an air bubble in water, with respect to pure water is

$$E = V \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \, \tfrac{1}{2} \, \hbar c ||\vec{k}|| \, \left\{ 1 - \frac{1}{n(\vec{k})} \right\} > 0.$$

Assume that as the bubble collapses all this ZPE is converted to real photons.

Insert numbers.

Schwinger's original model — III:

Good: The energy budget is correct.

Good: There is a natural cutoff in the extreme UV — because $n \to 1$ there.

Good: No need for thermalization.

Bad: There is no realistic way of estimating conversion efficiency.

Side issue: Continued bickering over how to define the static Casimir energy.

References:
hep-th/9609105 $\equiv$ PLB 395 (1997) 76-82;
hep-th/9702007 $\equiv$ PRD 56 (1997) 1262-1280;
hep-th/9707073 $\equiv$ PRD 56 (1997) 6629-6639.

(Collaborators: Carl Carlson [William+Mary],
Carmen Molina–Parıs [Los Alamos],
Juan Pérez–Mercader [LAEFF, Madrid])

Eberlein's variant — I:

Eberlein's calculation is technically correct, but not applicable to sonoluminescence.

Eberlein assumes adiabatic behaviour of the electromagnetic field, and develops a non-standard but consistent and correct formalism for calculation photon production due to adiabatic changes in the placement of dielectrics. (That is, adiabatic changes in the dielectric constant.)

But photon production due to adiabatic effects is always small, and trying to fit the observed luminosity using this formalism forces the bubble wall to move rapidly, undermining the physical justification for using the adiabatic approximation.

(Naive estimates obtained by forcing the observational data into the Eberlein framework imply superluminal motion for the bubble wall.)

Eberlein's variant — II:

Good: Clear evidence that changing the dielectric constant leads to photon production.

Good: Reasonable shape for the spectrum.

Bad: Clear evidence that SL cannot be an adiabatic process.

Our variant:

The observation of peta-Hertz photons tells us that something significant must be changing on approximately femtosecond timescales.

Q: Is there any way to get approximately femtosecond changes in the dielectric properties?

A1: Bubble collapse won't do, that's far too slow (nanoseconds at best).

A2: The van der Waals hard core bounce is much more promising — at minimum radius the gas bubble contents reach the absolute maximum density implicit in the van der Waals equation of state. The speed of sound (formally) goes to infinity.

References:

quant-ph/9805023 $\equiv$ PRL 83 (1999) 678-681;
quant-ph/9805031; quant-ph/9904008;
quant-ph/9904018; quant-ph/9905031.

Homogeneous dielectric approximation — I:

For a homogeneous time-varying dielectric

$$\epsilon(t)\frac{\partial^2 E}{\partial t^2} - c^2 \nabla^2 E = 0.$$

The time-varying dielectric can be thought of as a external driving term that excites the electromagnetic field.

Second-quantize the EM field in this background.

If you interpret this process in terms of a Feynman diagram it's obvious you always get pair production.

If $\epsilon(t \to -\infty) = \epsilon_{in}$ and $\epsilon(t \to +\infty) = \epsilon_{out}$ then you have well-defined "in" and "out" vacuum states and can describe photon production in terms of Bogolubov coefficients.

Homogeneous dielectric approximation — II:

To solve the model, introduce a "pseudo-time"

$$\tau(t) = \int \frac{dt}{\epsilon(t)}.$$

Then

$$\frac{\partial^2 E}{\partial \tau^2} - c^2 \epsilon(\tau) \nabla^2 E = 0.$$

Pick a particular profile

$$\epsilon(\tau) = \tfrac{1}{2}(n_{\mathrm{in}}^2 + n_{\mathrm{out}}^2) + \tfrac{1}{2}(n_{\mathrm{out}}^2 - n_{\mathrm{in}}^2)\ \tanh(\tau/\tau_0).$$

(τ_0 represents the [pseudo-time] timescale for the change of the refractive index).

This profile picked for analytic tractability.

Homogeneous dielectric approximation — III:

Compute the Bogolubov coefficient.
Convert back to physical time.
Define $\langle n^2 \rangle = (n_{\text{in}}^2 + n_{\text{out}}^2)/2$:

$$\left|\beta(\vec{k}^{\text{ in}}, \vec{k}^{\text{ out}})\right|^2 = \frac{V}{(2\pi)^3}\,\delta^3(\vec{k}^{\text{ in}} + \vec{k}^{\text{ out}})$$
$$\times \frac{\sinh^2\left(\pi |n_{\text{in}}^2 \omega_{\text{in}} - n_{\text{out}}^2 \omega_{\text{out}}|\; t_0/(2\langle n^2\rangle)\right)}{\sinh\left(\pi\; n_{\text{in}}^2\; \omega_{\text{in}} t_0/\langle n^2\rangle\right)\; \sinh\left(\pi\; n_{\text{out}}^2\; \omega_{\text{out}} t_0/\langle n^2\rangle\right)}.$$

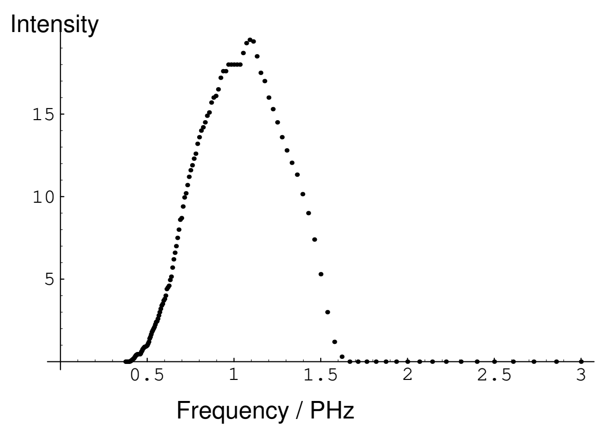
Here t_0 is now the physical timescale of the change in the refractive index.

For our temporal profile

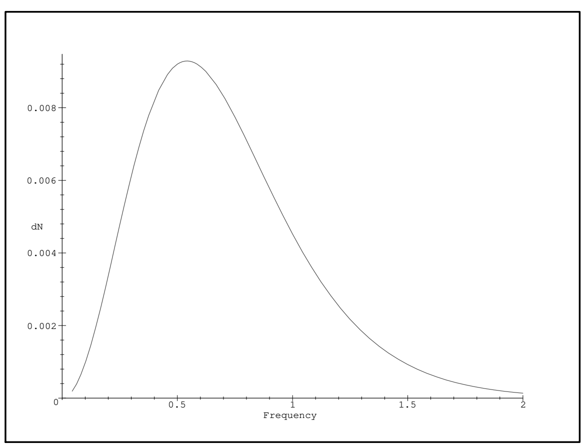
$$t_0 = \tfrac{1}{2}\tau_0 \left(n_{\text{in}}^2 + n_{\text{out}}^2\right).$$

From the Bogolubov coefficient you can estimate the spectrum.

Homogeneous dielectric approximation — IV:

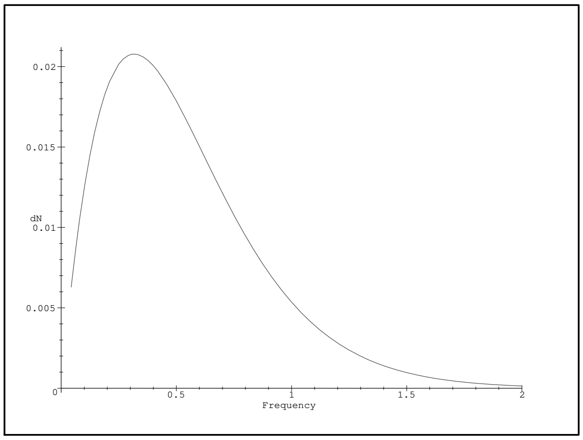


Typical experimental spectrum as a function of frequency.



Example of a theoretical spectrum estimated from the homogeneous dielectric model as a function of frequency.

Homogeneous dielectric approximation — V:



Planckian spectrum as a function of frequency.

Getting a million photons is not too difficult. (For example:
$n_{\text{in}} \approx 1$, $n_{\text{out}} \approx 12$;
$n_{\text{in}} \approx 70$, $n_{\text{out}} \approx 25$;
$n_{\text{in}} \approx 2 \times 10^4$, $n_{\text{out}} \approx 1$.)

Getting a picosecond timescale is not too difficult.

(The refractive index helps relax the physical timescale; giving more wriggle room.)

Finite-volume effects — I:

For finite bubbles you have to decompose the EM field in terms of spherical harmonics and Bessel functions.

Make the sudden approximation.

The technical details are a real mess, see quant-ph/9904013; quant-ph/9905034.

Introduce $\Delta n \equiv n_{\text{gas}}^{\text{in}} - n_{\text{gas}}^{\text{out}}$. The spectrum is

$$\frac{dN}{d\omega_{\text{out}}} = \tfrac{1}{4} R^2 (\Delta n)^2 \sum_{l=1}^{\infty} (2l+1)$$
$$\times \int d\omega_{\text{in}} \left\{ \frac{n_{\text{gas}}^{\text{out}} \omega_{\text{out}}^2 + n_{\text{gas}}^{\text{in}} \omega_{\text{in}}^2}{\omega_{\text{out}} + \omega_{\text{in}}} \right\}^2 \left| A_\nu^{\text{in}} \right|^2 \left| A_\nu^{\text{out}} \right|^2$$
$$\times \left[\frac{W[J_\nu(n_{\text{gas}}^{\text{out}}\, \omega_{\text{out}} r/c), J_\nu(n_{\text{gas}}^{\text{in}}\, \omega_{\text{in}} r/c)]_R}{(n_{\text{gas}}^{\text{out}}\, \omega_{\text{out}})^2 - (n_{\text{gas}}^{\text{in}}\, \omega_{\text{in}})^2} \right]^2 .$$

$W(f,g)$ is the Wronskian, and the A_ν^{in} are calculable coefficients depending on the frequency, refractive index of the bubble, and that of the ambient medium.

Finite-volume effects — II:

The above is a general result applicable to any dielectric sphere that undergoes sudden change in refractive index.

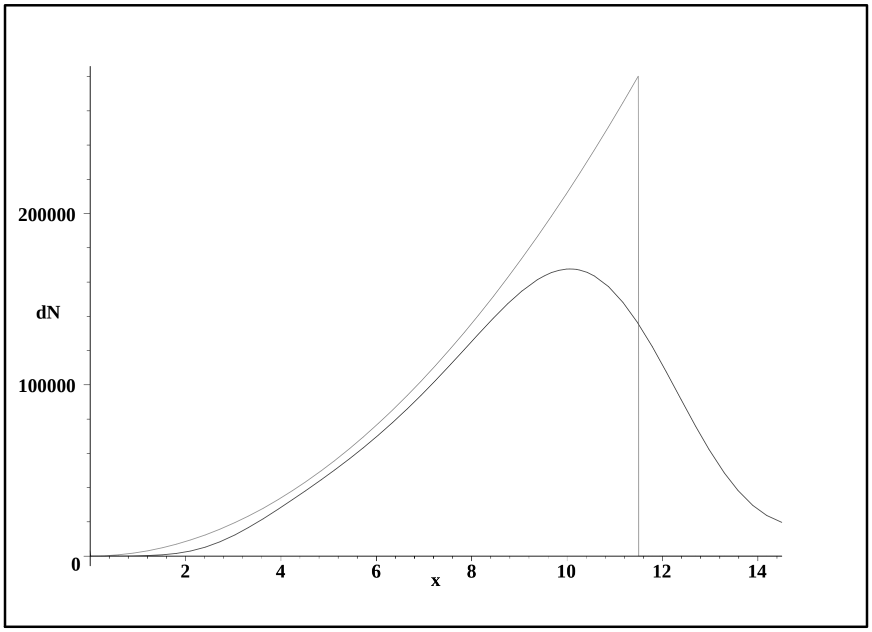
This expression is far too complex to allow a practical analytical resolution of the general case.

For the specific case of sonoluminescence, we have developed suitable numerical approximations.

$n_{\text{gas}}^{\text{in}}$	$n_{\text{gas}}^{\text{out}}$	Number of photons	$\langle E \rangle / \hbar\omega_{\text{max}}$
2×10^4	1	1.06×10^6	0.803
71	25	1.00×10^6	0.750
68	34	1.06×10^6	0.751
9	25	0.955×10^6	0.750
1	12	0.98×10^6	0.765

Table I: Some typical cases.

Finite-volume effects — III:



Spectrum dN/dx obtained by integrating the approximate Bogolubov coefficient.

The curve with the sharp cutoff is the infinite-volume sudden approximation.

Finite-volume effects tend to smear out the sharp discontinuity, but do not greatly affect the total number of photons emitted.

Ionization $\equiv$ dynamical Casimir effect?

Our calculation applies to any sudden change in refractive index, however generated.

In particular, sudden ionization can drive a sudden change in refractive index.

In this sense, ionization-based models for SL are a sub-class of those based on the dynamical Casimir effect.

Details: quant-ph/9904018.

Experimental tests:

Two-photon correlations are important.

Dynamical Casimir effect $\rightarrow$
vacuum squeezing $\rightarrow$
approximately back-to-back two-photon final states.

Still get quasi-thermal single-photon statistics if only one photon is detected.

Timing information is important.

Pulse width, pulse location,
temporal fine structure?

Details: quant-ph/9904013; quant-ph/9904018.

Conclusions:

We need:

- More experiments —
  Theory has probably gone as far as it can...

- Improved data on timing —
  Resolve the timespan on which the refractive index changes.

- Data on two-photon correlations —
  Squeezed states are generic to QED vacuum effects.

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