Energy conditions and their implications

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Abstract:

The energy conditions of general relativity permit one to deduce very powerful and general theorems about the behaviour of strong gravitational fields and cosmological geometries.

However, the energy conditions are beginning to look a lot less secure than they once seemed:

— There are quantum effects that violate all of the energy conditions.

— There are even relatively benign looking classical systems that violate all the energy conditions.

This opens up a Pandora's box of rather disquieting possibilities.

Overview:

Classical: In classical general relativity the various energy conditions are used to prove lots of lovely general theorems...

Quantum: Everyone who thought about it expected the various energy conditions to eventually break down once one reaches the Planck slop...

Surprise 1: The various energy conditions already fail miserably in **semi-classical quantum gravity**.

The failures occur at first order in $\hbar$, long before one reaches the Planck slop...

The failures are widespread, albeit small...

Surprise 2: The various energy conditions also fail miserably in some quite reasonable looking **classical systems**.

Energy conditions:

Standard (pointwise) energy conditions:

TEC — trace energy condition
(now abandoned).

NEC — null energy condition.

WEC — weak energy condition.

SEC — strong energy condition
(*aka* unphysical energy condition)
(now almost completely abandoned).

DEC — dominant energy condition.

Standard averaged energy conditions:

ANEC — averaged null energy condition.

AWEC — averaged weak energy condition.

ASEC — averaged strong energy condition.

Standard propaganda:

All reasonable Lagrangians give theories satisfying the energy conditions...

The standard propaganda is wrong.

Definition:

In a Lorentzian spacetime:

The null energy condition [NEC] is said to hold at a point x, if for all null vectors k

$$T_{\mu\nu}\, k^{\mu} k^{\nu} \geq 0.$$

Notes:

DEC $\Rightarrow$ WEC $\Rightarrow$ NEC.

SEC $\Rightarrow$ NEC.

SEC does *not* imply WEC —
— the terminology is misleading.

NEC Is the weakest pointwise energy condition in common use.

Uses of the Energy Conditions:

- Penrose singularity theorem (WEC).
  [Relevant to black hole formation.]

- Hawking–Penrose singularity theorem (SEC).
  [Relevant to the cosmological singularity.]

- Tipler's version of the Hawking–Penrose singularity theorem (WEC+ASEC).

- Schoen–Yau positive mass theorem (DEC).

- Witten's variant positive mass theorem (DEC).

Hypotheses are all stronger than the NEC.

Uses of the ANEC:

The ANEC is used as input hypotheses in proving:

- Focussing theorems for null geodesics. [Borde]
- Generalized Penrose singularity theorem. [Roman]
- Topological censorship theorem. [Friedman–Schleich–Witt]
- Generalized positive mass theorem. [Penrose–Sorkin–Woolgar]

ANEC is the weakest averaged energy condition in common use.

Quantum violations of the energy conditions:

- 2-particle Fock states. (NEC+)

- Casimir vacuum. (NEC+)
  [DeWitt; Einstein Centenary Survey]

- Hawking radiation. (NEC+)

- Squeezed vacuum. (WEC+DEC)
  [Morris–Thorne]

- Conformal anomaly (NEC+)
  [Visser, PLB 349 (1995) 443–447; gr-qc/9409043].

- Gravitational vacuum polarization (NEC+)
  [Visser, gr-qc/9604007; 9604008; 9604009; 9703001].

Classical violations of the energy conditions:

- Cosmological inflation. (SEC)
  [Minimally coupled massive scalar]
  [Theory only]

- Galaxy formation: $0 < z < 10$. (SEC)
  [Visser, Science 276 (4 April 1997) 88]
  [Observation: accelerating universe]

- Non-minimally coupled scalar (NEC+)
  massive/massless
  [Wald–Flanagan, gr-qc/9602052, PRD].

- Massless conformally coupled scalar (NEC+)
  QFT: new improved energy-momentum
  [Barceló–Visser, gr-qc/9908029, PLB]

Tolman wormhole:

Q: What are the *minimal* conditions for a "bounce"?

Definition: Tolman wormhole $\equiv$ "bounce".

A: Perform a model-independent analysis of the geometry near a bounce, along the lines of the Morris–Thorne analysis for traversable wormholes.

Details:
gr-qc/9810023, PLB455 (1999) 90-95
[Molina-París, Visser]
gr-qc/9810029, PRD59 (1999) 044011
[Hochberg, Molina-París, Visser]

Flare-out at the bounce $\Rightarrow$ SEC violated at or near the bounce.

Notes:

SEC violations are a *necessary* but not *sufficient* condition for a "bounce".

You do not *need* to violate NEC, WEC, or DEC to get a "bounce".

If you believe inflation you have already abandoned the SEC anyway.

Inflation will not *guarantee* a bounce, but it opens the door.

New improved stress tensor plus gravity:

Q: What happens if you take a massless conformally coupled scalar field and add Einstein gravity? (Static, spherically symmetric.)

This looks an absurdly easy problem, but the result is somewhat of a surprise.

A: There is a three-parameter class of exact solutions (total mass, scalar charge, scalar field at infinity).

Special cases:

(1) Schwarzschild/ anti-Schwarzschild.
(2) Naked singularities.
(3) Naked singularities hiding behind wormhole throats. (Not really "traversable".)
(4) Traversable wormholes with two asymptotically flat regions.

Details: gr-qc/9908029, Barceló–Visser, PLB.

Stress-energy tensor:

$$\begin{aligned} T_{\mu\nu} &= \nabla_\mu\phi_c\nabla_\nu\phi_c - \frac{1}{2}g_{\mu\nu}(\nabla\phi_c)^2 \\ &\quad +\frac{1}{6}\Big[G_{\mu\nu}\,\phi_c^2 - 2\,\nabla_\mu(\phi_c\nabla_\nu\phi_c) \\ &\qquad +2\,g_{\mu\nu}\,\nabla^\lambda(\phi_c\nabla_\lambda\phi_c)\Big]. \end{aligned}$$

Equation of motion for scalar:

$$(\nabla^2 - \frac{1}{6}R)\phi_c = 0.$$

Einstein equation:

$$\kappa\, G_{\mu\nu} = T_{\mu\nu}.$$

Aside (flat space):

$$T_{\mu\nu}\,k^\mu k^\nu = \left(\frac{d\phi_c}{d\lambda}\right)^2 - \frac{1}{6}\,\frac{d^2(\phi_c^2)}{d\lambda^2}.$$

Even in Minkowski space —
local minima of ϕ_c^2 provide NEC violations.

Eliminate the Einstein equations:

Use

$$T_{\mu\nu}\; g^{\mu\nu} = 0.$$

The equations reduce to

$$R_{\mu\nu} = \frac{\left[\frac{2}{3}\,\nabla_\mu\phi_c\,\nabla_\nu\phi_c - \frac{1}{6}\;\; g_{\mu\nu}(\nabla\phi_c)^2 - \frac{1}{3}\,\phi_c\nabla_\mu\nabla_\nu\phi_c\right]}{\left[\kappa - \frac{1}{6}\phi_c^2\right]}$$

$$\nabla^2\phi_c = 0.$$

Aside (curved space):

NEC violations (unusual sign for the null components of Ricci) can now arise — *either* from local minima of ϕ_c^2, *or* from regions where $\phi_c^2 > 6\kappa$.

$$T_{\mu\nu}\; k^\mu\; k^\nu \;\; = \;\; \frac{\kappa\;\left[\left(\frac{d\phi_c}{d\lambda}\right)^2 - \frac{1}{6}\;\frac{d^2(\phi_c^2)}{d\lambda^2}\right]}{\kappa - \frac{1}{6}\phi_c^2}.$$

ANEC violations:

After integrating by parts:

$$\oint T_{\mu\nu}\, k^{\mu}\, k^{\nu}\, d\lambda \;=\; \oint \frac{\kappa\,\left[\kappa - \frac{1}{18}\phi_c^2\right]}{\left[\kappa - \frac{1}{6}\phi_c^2\right]^2} \left(\frac{d\phi_c}{d\lambda}\right)^2 d\lambda.$$

ANEC violations are more difficult to get than NEC violations.

They require

$$\phi_c > \sqrt{18\kappa}.$$

Effective Newton constant:

$$\frac{1}{8\pi\, G_{\text{eff}}} = \kappa_{\text{eff}} = \kappa - \frac{1}{6}\phi_c^2.$$

Janis–Newman–Winicour–Wyman solution:

Coupled Einstein plus *minimal* scalar field

$$
\begin{aligned}
R_{\mu\nu} = &\ \kappa^{-1}\left(\nabla_\mu \phi_m \, \nabla_\nu \phi_m\right), \\
\nabla^2 \phi_m = &\ 0.
\end{aligned}
$$

JNWW solution —

$$
\begin{aligned}
ds_m^2 \;=\;& -\left(1-\frac{2\eta}{r}\right)^{\cos\chi} dt^2 \\
&+\left(1-\frac{2\eta}{r}\right)^{-\cos\chi} dr^2 \\
&+\left(1-\frac{2\eta}{r}\right)^{1-\cos\chi} r^2(d\theta^2+\sin^2\theta \, d\Phi^2),
\end{aligned}
$$

$$
\phi_m \;=\; \sqrt{\frac{\kappa}{2}}\,\sin\chi\,\ln\left(1-\frac{2\eta}{r}\right).
$$

Ansatz for conformal scalar field:

$$
(g_c)_{\mu\nu} = \Omega(\phi_m)^2 \,(g_m)_{\mu\nu}.
$$

$$
\phi_c = \phi_c(\phi_m).
$$

General solution:

$$\Omega = \alpha_+ \exp\left(+\frac{\phi_m}{\sqrt{6\kappa}}\right) + \alpha_- \exp\left(-\frac{\phi_m}{\sqrt{6\kappa}}\right),$$

$$\phi_c = \pm\sqrt{6\kappa}\times \left[\frac{\alpha_+ \exp\left(+\frac{\phi_m}{\sqrt{6\kappa}}\right) - \alpha_- \exp\left(-\frac{\phi_m}{\sqrt{6\kappa}}\right)}{\alpha_+ \exp\left(+\frac{\phi_m}{\sqrt{6\kappa}}\right) + \alpha_- \exp\left(-\frac{\phi_m}{\sqrt{6\kappa}}\right)}\right].$$

Parameters η and χ inherited from JNWW.

New parameter

$$\tan\Delta = \frac{(\alpha_+ - \alpha_-)}{(\alpha_+ + \alpha_-)}.$$

Generalizes Froyland, and Agnese–La-Camera.

Geometry:

As a function of r:

$$\Omega(r) = \alpha_+ \left(1 - \frac{2\eta}{r}\right)^{\frac{\sin\chi}{2\sqrt{3}}} + \alpha_- \left(1 - \frac{2\eta}{r}\right)^{-\frac{\sin\chi}{2\sqrt{3}}}.$$

Schwarzschild radial coordinate $\mathcal{R}$

$$\mathcal{R}(r) = r\left(1 - \frac{2\eta}{r}\right)^{\frac{1-\cos\chi}{2}} \Omega(r).$$

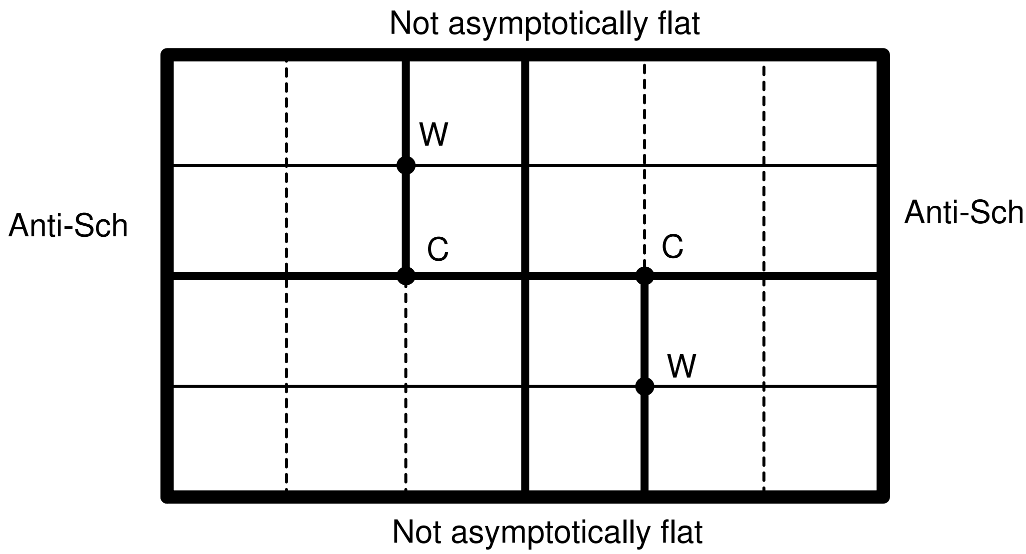
If $\chi \neq 0, \pm\pi$, and $\alpha_- \neq 0$, then $\Omega \to \infty$ as $r \to 2\eta$.

If $\chi \in (0, \pi/3)$, and $\alpha_- \neq 0$, then $\mathcal{R} \to \infty$ as $r \to 2\eta$.

This signals the presence of a wormhole throat (traversable in principle).

The rest of the calculation is an exercise in exploring various special cases.

Parameter space:



Sketch of the parameter space of our class of solutions to Einstein gravity coupled to the "new improved stress-energy tensor".

Using the symmetries discussed in the paper we have taken $\eta \geq 0$.

We sketch the range $\chi \in [-\pi, +\pi]$ and $\Delta \in [-\pi, +\pi]$.

Parameter space is an orbifold because of the remaining symmetry under $\{\eta, \chi, \Delta\}$ to $\{\eta, -\chi, -\Delta\}$.

Specific Implications:

Don't focus on the specific technical details. (Naked singularities, traversable wormholes). The main points are these:

- Classical arbitrarily large violations of the NEC arise in this otherwise very reasonable classical system.

- Conformally coupled scalars are from a QFT perspective the preferred choice — corresponding to the new improved stress-energy tensor.
  (Conformal coupling is a IR fixed point of the RG.)

- It's the fact that you get significant NEC violation in such a simple physical system that's worrying — the fact that this NEC violation is big enough to support traversable wormholes is a bonus.

General Implications:

- We do not currently have a physically acceptable positive mass theorem.
- We do not currently have a physically acceptable singularity theorem.
- We do not currently have a physically acceptable topological censorship theorem.
- Traversable wormholes almost begin to look physically reasonable.
- Tolman wormholes almost begin to look physically reasonable.
- This opens up a whole mess of weird possibilities...

Conclusions:

We need:

- Improved understanding of just what conditions can sensibly be put on the stress-energy tensor.

- Improved energy conditions —
  you do not want a free-for-all.
  (Arbitrary stress-energy tensor $\Rightarrow$
  arbitrarily weird physics.)

- Improved (positive-mass/ singularity/ censorship) theorems of all types.

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