The multiple deaths of Palatini $f(R)$ gravity

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University of Canterbury, Feb 13th 2009
Motivation

Theoretical: Clash between GR and QFT

* intrinsic limits, conceptual clash, ...
* Quantum corrections, string theory...
* higher order theories, Lorentz or EP violations...

Observational: Inability to explain cosmological/astrophysical riddles without dark matter/energy

* 4% baryons, 20% dark matter, 76% dark energy!
* acceleration, deceleration, then acceleration
* cosmological constant and coincidence problems
Alternative theory of gravity which:

- comes as a low energy limit of a more fundamental theory
- includes ultraviolet/infrared corrections with respect to General Relativity
- can account for some or all the unexplained observations
But we don't know the fundamental theory!

* we could blame it on others...

* cook up toy theories and maybe even give them feedback - sounds much better!

Typical Example: f(R) gravity

\[ S = \frac{1}{16\pi G} \int d^4x \sqrt{-g}R \quad \rightarrow \quad S = \frac{1}{16\pi G} \int d^4x \sqrt{-g}f(R) \]

Three versions of f(R) gravity

1. **Metric**: variation only wrt the metric

2. **Palatini**: variation wrt metric and connection
   - the connection is an independent variable but does not enter the matter action!

3. **Metric-affine**: variation wrt metric and connection
   - the connection is an independent variable and enters the matter action

Metric and Palatini variations both lead to GR for Einstein-Hilbert action (textbook)
Classification

f(R) gravity

Metric-Affine f(R) gravity

Palatini f(R) gravity

Metric f(R) gravity

General Relativity

\[ \Gamma^{\lambda}_{\mu\nu} \neq \{^{\lambda}_{\mu\nu}\} \]

\[ \Gamma^{\lambda}_{\mu\nu} = \{^{\lambda}_{\mu\nu}\} \]

\[ S_M = S_M(g_{\mu\nu}, \psi) \]

\[ f(R) = R \]
Field equations:

\[ f'(R)R_{\mu\nu} - \frac{1}{2} f(R)g_{\mu\nu} = 8\pi G T_{\mu\nu} \]

\[ \bar{\nabla}_\lambda \left( \sqrt{-g} f'(R) g^{\mu\nu} \right) = 0 \]

Trace of 1st field eq.:

\[ f'(R)R - 2f(R) = 8\pi G T \]

Solving for the connection:

\[ \Gamma^\lambda_{\mu\nu} = \frac{g^{\lambda\sigma}}{f'(R)} \left[ \partial_\mu (f'(R)g_{\nu\sigma}) + \partial_\nu (f'(R)g_{\mu\sigma}) - \partial_\sigma (f'(R)g_{\mu\nu}) \right] \]
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Palatini $f(R)$ gravity

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Starting with

\[ S = \frac{1}{16\pi G} \int d^4 x \sqrt{-g} f(R) \]

introducing an auxiliary scalar field yields

\[ S = \frac{1}{16\pi G} \int d^4 x \sqrt{-g} \left( f(\chi) + f'(\chi)(R - \chi) \right) \]

where variation gives

\[ f''(\chi)(R - \chi) = 0 \]

I.e. Dynamically equivalent actions
Introducing the variables

\[ \phi = f'(\chi), \quad V(\phi) = \chi(\phi)\phi - f(\chi(\phi)) \]

the action takes the form

\[ S = \frac{1}{16\pi G} \int d^4 x \sqrt{-g}(\phi\mathcal{R} - V(\phi)) \]

Using the field equations

\[ \mathcal{R} = R + \frac{3}{2(f'(\mathcal{R}))^2} (\nabla_\mu f'(\mathcal{R})) (\nabla^\mu f'(\mathcal{R})) + \frac{3}{f'(\mathcal{R})} \Box f'(\mathcal{R}) \]
Introducing the variables

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the action takes the form

\[ S = \frac{1}{16\pi G} \int d^4x \sqrt{-g}(\phi R - V(\phi)) \]

Using the field equations

\[ R = R + \frac{3}{2(f'(R))^2} (\nabla_\mu f'(R)) (\nabla^\mu f'(R)) + \frac{3}{f'(R)} \Box f'(R) \]
Equivalence with Brans-Dicke Theory

Outcome:

\[ S = \frac{1}{16\pi G} \int d^4 x \sqrt{-g} \left( \phi R + \frac{3}{2\phi} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right) \]

\( \checkmark \) Palatini \( f(R) \) gravity is equivalent to \( \omega_0 = -3/2 \) Brans-Dicke Theory!

Field equation for the scalar:

\[ (2\omega_0 + 3) + 2V - \phi V' = 8\pi GT \]

The scalar field is not dynamical!
PPN limit and metric

Remarkable result*: Whether the theory has the correct Newtonian limit depends on the density!

PPN metric:

\[ h_{00}(t, x) = 2G_{\text{eff}} \frac{M_\odot}{r} + \frac{V_0}{6\phi_0} r^2 + \Omega(\rho) \]

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Non-perturbative corrections and strong coupling in the matter sector at low energies! *

consider some matter field, e.g. the Higgs
the connection is an auxiliary field
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Conflict with Particle Physics

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Singularities on stellar surfaces

Surface singularities in spherically symmetric polytropes for $\frac{3}{2} < \Gamma < 2$! EoS: $p = k \rho^{\Gamma}$

- Unique exterior solution
- Matching with any interior leads to singularity

Polytropes are restricted but...

- No physically meaningful solution for isentropic gas or degenerate non-relativistic gas, $\Gamma = 5/3$
- The problem is not restricted to polytropes

Non-cumulativity: the root of all evil

Field equations after eliminating the connection:

\[
G_{\mu\nu} = \frac{8\pi G}{f'} T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \left( \mathcal{R} - \frac{f}{f'} \right) + \frac{1}{f'} \left( \nabla_{\mu} \nabla_{\nu} - g_{\mu\nu} \Box \right) f' \\
- \frac{3}{2} \frac{1}{f'^2} \left[ \left( \nabla_{\mu} f' \right) \left( \nabla_{\nu} f' \right) - \frac{1}{2} g_{\mu\nu} \left( \nabla f' \right)^2 \right]
\]

- second order in the metric - higher order in the matter fields!
- matter enters the gravitational action through the back door leading to aforementioned issues
Conclusions and Outlook

Astrophysics, Cosmology, Theoretical Physics all provide constraints for gravity theories

...but also...

Toy theories of gravity can teach us a lot about the gravitational interaction

Several difficulties and subtleties in modified gravity:

- Many constraints to satisfy
- Many directions to take
- A long way to go...