A wormhole for photons

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"A non-gravitational wormhole", F. Baldovin, M. Novello, S. E. Perez Bergliaffa, and J. M. Salim Class. Quantum Grav. **17**, *3265 (2000)*

Effective geometry

M. Novello, V. De Lorenci, J. M. Salim, and R. Klippert PRD **61***, 045001 (2000), Phys.Lett.* **B482** *(2000), 134.*

Non-linear electrodynamics described by the Lagrangian $L = L(F, G)$
$F = F_{\mu\nu}F^{\mu\nu}$, $G = F^{\mu\nu}\,{}^*F_{\mu\nu}$.

The equations of motion are

$$\partial_\nu \left(L_F F^{\mu\nu} + L_G\,{}^*F^{\mu\nu}\right) = 0, \qquad (1)$$

$$\partial_\mu\,{}^*F^{\mu\nu} = 0. \qquad (2)$$

Light rays do not follow geodesics of the background spacetime metric $\eta^{\mu\nu}$ but of the effective metric

$$g^{\mu\nu}_{\pm} = L_F\,\eta^{\mu\nu} - 4\Big[\left(L_{FF} + \Omega_\pm L_{FG}\right)F^{\mu}_{\ \lambda}F^{\lambda\nu} + \left(L_{FG} + \Omega_\pm L_{GG}\right)F^{\mu}_{\ \lambda}{}^*F^{\lambda\mu}\Big] \qquad (3)$$

where

$$\Omega_{\pm} = \frac{-\Omega_2 \pm \sqrt{\Delta}}{2\Omega_1}, \tag{4}$$

$$\Delta = (\Omega_2)^2 - 4\Omega_1\Omega_3, \tag{5}$$

and

$$\Omega_1 = -L_F L_{FG} + 2F L_{FG} L_{GG} + G(L_{GG}^2 - L_{FG}^2), \tag{6}$$

$$\begin{aligned} \Omega_2 \;=\;& (L_F + 2G L_{FG})(L_{GG} - L_{FF}) + \\ & 2F(L_{FF} L_{GG} + L_{FG}^2), \end{aligned} \tag{7}$$

$$\Omega_3 = L_F L_{FG} + 2F L_{FF} L_{FG} + G(L_{FG}^2 - L_{FF}^2). \tag{8}$$

In concrete examples, all the derivatives of $L(F, G)$ in Eqn.(3) are evaluated in the background field configuration.

For $L = L(F)$, the nonlinear equations are

$$\partial_\nu \left(L_F F^{\mu\nu} \right) = 0, \qquad \partial^\nu F^*_{\mu\nu} = 0, \tag{9}$$

and the effective geometry reduces to

$$g^{\mu\nu} = L_F \eta^{\mu\nu} - 4 L_{FF} F^\mu_\alpha F^{\alpha\nu}. \tag{10}$$

Special case: Born-Infeld electrodynamics,

$$L = b^2 \left(1 - \sqrt{1 + \frac{F}{2b^2} - \frac{G^2}{16b^4}} \right). \tag{11}$$

Here, all the Ω_i's are zero, and the effective metric is given by

$$g^{\mu\nu} = \left(b^2 + \frac{F}{2} \right) \eta^{\mu\nu} + F^\mu_\alpha F^{\alpha\nu}. \tag{12}$$

For a static and spherically symmetric electric field we get from the equations of motion that

$$F^{tr}(r) = \frac{\alpha}{r^2 L_F}. \tag{13}$$

From this equation and Eqn.(11),

$$F(r) = -2b^2 \, \frac{16\alpha^2}{b^2 r^4 + 16\alpha^2} \tag{14}$$

The metric can be now computed from Eqn.(12), and is given by

$$ds^2 = dt^2 - dr^2 - \Delta(r) d\Omega^2, \tag{15}$$

where

$$\Delta(r) = r^2 + \frac{r_{\text{th}}^4}{r^2}, \tag{16}$$

and $r_{\text{th}} = 2\sqrt{|\alpha|/b}$.

Wormholes

Very restricted definition: spherically symmetric and static.

(M.S. Morris and K.S. Thorne, Am. J. Phys. **56**, 395 (1988)).

Metric:

$$ds^2 = e^{2\phi(l)}dt^2 - dl^2 + r^2(l)d\Omega^2 \qquad (17)$$

such that

- $\phi(l)$ is finite everywhere (absence of horizons),

- $r(l)$ has a minimum, r_{th} (radius of the throat),

- conditions at infinity .

- traversability.

For more general definitions see
D. Hochberg and M. Visser, PRD **56** (1997) 4745, and PRD **58** (1998) 044021.

The metric (15) satisfies this conditions, and consequently it describes a wormhole, with a throat at r_{th} and $\phi(r) = 0$.

The metric of the wormhole given by Eqn.(15) can be also written in the “standard” form if we make the coordinate change:

$$\rho^2 = r^2 + \frac{r_{\text{th}}^2}{r^2} \tag{18}$$

The result is

$$ds^2 = dt^2 - \frac{1}{1 - \frac{b_\pm(\rho_\pm)}{\rho_\pm}} d\rho_\pm^2 - \rho_\pm^2 d\Omega^2, \tag{19}$$

with

$$b_\pm(\rho_\pm) = \frac{2\rho_{\text{th}}^4 - \rho_\pm^4 \pm \rho_\pm^2 \sqrt{\rho_\pm^4 - \rho_{\text{th}}^4}}{\rho_\pm^3 \pm \rho_\pm \sqrt{\rho_\pm^4 - \rho_{\text{th}}^4}}, \tag{20}$$

and $\rho_{\text{th}} \leq \rho_{\pm} < \infty$.

- The "$+$" patch covers the region $r_{\text{th}} \leq r < \infty$, and the "$-$" patch, the region $0 < r \leq r_{\text{th}}$.

- The throat is located now at $\rho_{\text{th}} \equiv \sqrt{2}\, r_{\text{th}}$.

- Because two different shape functions $b_{+}(\rho_{+})$ and $b_{-}(\rho_{-})$ are needed, the wormhole is asymmetric under the interchange $\rho_{+} \leftrightarrow \rho_{-}$.

- Only the region covered by the "$+$" patch is asymptotically flat.

Effective potential

The metric for a symmetric wormhole with a shape function given by $b(\rho) = \rho_{\text{th}}^2/\rho$ is given by

$$ds^2 = dt^2 - \frac{d\rho^2}{1 - \frac{\rho_{\text{th}}^2}{\rho^2}} - \rho^2 \, d\Omega^2, \tag{21}$$

and the corresponding effective potential is (see Fig. 1)

$$V = \left(1 - \frac{\rho_{\text{th}}^2}{\rho^2}\right)\left(\frac{L^2}{\rho^2} - E^2\right) + E^2. \tag{22}$$

For the metric Eqn.(19), it is given by (see Figs. 2, 3, and 4)

$$V(\rho_\pm) = 2\left(\frac{L^2}{\rho_\pm^2} - E^2\right)\frac{\rho_\pm^4 - \rho_{\text{th}}^4}{\rho_\pm^2(\rho_\pm^2 \mp \sqrt{\rho_\pm^4 - \rho_{\text{th}}^4})} + E^2. \tag{23}$$

The dielectric analogy

The electromagnetic field inside a medium can be represented by two antisymmetric tensors: $F_{\mu\nu}$, and the polarization $P_{\mu\nu}$. They obey the equations

$$\partial^\nu P_{\mu\nu} = 0, \qquad \partial^\nu\, {}^*F_{\mu\nu} = 0. \tag{24}$$

Comparing with

$$\partial_\mu (L_F F^{\mu\nu}) = 0 \tag{25}$$

we see that the nonlinear theory in vacuum is equivalent to Maxwell's theory in an isotropic medium characterized by an electric susceptibility ϵ and a magnetic permeability μ given by

$$\epsilon = \mathcal{L}_F, \qquad \mu = \frac{1}{\mathcal{L}_F} \tag{26}$$

We can then replace the nonlinear EM theory in vacuum with the linear theory in the presence of a medium.

In the case under consideration,

$$\epsilon(r) = -\frac{1}{4}\left(1 + \frac{r_{\text{th}}^4}{r^4}\right)^{1/2}, \tag{27}$$

or, in terms of the electric field,

$$\epsilon(E) = -\frac{1}{4}\left(1 - \frac{E^2}{b^2}\right)^{-1/2}. \tag{28}$$

Conclusions

- Non-linear electrodynamics can be used to generate gravitational-like structures for the photons.

- The wormhole described here does not require the violation of the energy conditions (the effective metric is generated by the nonlinear dynamics, and not by Einstein's equations).

- The dielectric analogy can be useful to build gravitational analogs in the laboratory.