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NONINERTIAL ZERO-POINT FLUCTUATIONS: A LABORATORY-ORIENTED INTRODUCTION

INVITED TALK

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Motto

Analogies are helpful in transferring information gained in one research area to another area. They are particularly useful in (quasi)exact sciences and could be a way to progress. The history of physics shows that this indeed has been the case in many circumstances. In general, one should know in a profound way the physics of the fields related by analogy, because any analogy means much similarity but also key differences. Thus, employing analogies is not in general an easy task.

ABSTRACT. In an introductory and nonrigorous manner, the write-up of the talk extends and updates several sections of my review gr-qc/9406012, last updated in 1997. I present the recently introduced glass analogy for black holes but in order to have a more detailed picture I collected from the literature some useful material related to the violations of the fluctuation-dissipation theorem in glass physics and stationary driven systems. Nonequilibrium effective temperatures from glass irreversible thermodynamics are considered as useful and quite general concepts for noninertial quantum fluctuations, though the analogy is not fully disentangled. Next, I discuss the stationary and nonstationary scalar vacuum noises in some detail and I emphasize the radiometric nature of the Frenet invariants of the stationary worldlines, rather than sticking to the thermal interpretation of the vacuum excitations as apparent for the uniformly accelerated quantum detector. I go on with the Hacyan-Sarmiento approach for calculating electromagnetic vacuum physical quantities. The application to the circular case led Mane to propose the identification of the Hacyan-Sarmiento zero-point radiation with the ordinary synchrotron radiation, but the issue remains still open. I also briefly discuss the spin flip synchrotron radiation in the context of Bell and Leinaas proposal. Jackson's approach showing why this proposal is implausible is included.

Finally, I end up with a short random walk in the literature on the fluctuation-dissipation relationship.

SECTION HEADINGS: 1. Nonequilibrium thermometry in glass physics. 2. The glass analogy for black holes. 3. Stationary and nonstationary scalar vacuum field noises. 4. The Hacyan-Sarmiento approach: The circular electromagnetic vacuum noise. 5. Synchrotron radiation in storage rings. 6. Spin motion at storage rings and circular vacuum noise. 7. The fluctuation-dissipation theorem.

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1. NONEQUILIBRIUM THERMOMETRY IN GLASS PHYSICS

Forward - An interesting analogy between black hole thermodynamics and the nonequilibrium thermodynamics of glasses has been put forth by Th. Nieuwenhuizen in 1998. To understand this analogy and also the recently introduced modified fluctuation-dissipation theorem, I start with a rather detailed presentation of nonequilibrium thermometry, in which interesting model thermometers and associated thermometric principles can be found in the literature. Hopefully, one can adapt these results to quantum field theory of vacuum fluctuations and high-energy radiation patterns. This was the main motivation underlying the writing of this lecture, although there are more themes included.

Let's start by a what-type question.
What does non-stationary dynamics (relaxation) mean in glass physics?

This can be understood by the following example:

1.1 Magnetization relaxation of the "thermo-remanent magnetization" [TRM]

The system (sample) is cooled in a small field from above T_g down to some $T_0 < T_g$. It "waits" in the field at T_0 for a time t_w, after which the field is cut, and the subsequent decrease of the TRM from the field-cooled value is recorded as a function of t. Following an immediate fall-off of the magnetization, which depending on the sample and temperature is of the order of 50% to 90% , a slow log-like relaxation takes place; it is believed to go towards zero, although never reaching an end at laboratory time scales.

These 'endless' relaxations, and the existence of 'aging' phenomena (dependence on t_w) are the main salient features of spin-glass dynamics. For different values of the waiting time t_w different TRM-decay curves are obtained.

1.2 Facts of the decay dynamics

- depends on two independent time-scales:

1. t - observation time

2. t_w - waiting time

• it is nonstationary: the response at $t+t_w$ to an excitation at t_w depends on $t+t_w$ and t_w, and not only on t (i.e., there is breakdown of the time translation invariance (TTI) property).

1.3 In general, in glasses there are two relaxations corresponding to two classes of degrees of freedom:

• the short time or β relaxation (STR) for fast modes

• the structural long time or α relaxation (LTR) for slow modes.

In the LTR relaxation the glass falls out of local equilibrium $\rightarrow$ the fluctuation-dissipation (FD) theorem breaks down and can be replaced by the more general relation

$$R(t, t_w) = \frac{X(t, t_w)}{T} \frac{\partial C(t, t_w)}{\partial t_w} ,$$

where

$C(t, t_w)$ - a two-time correlation function

$R(t, t_w)$ with $t > t_w$ - associated response

$X(t, t_w)$ - a function measuring the departure from the FD theorem.

The ratio $T_{eff} = T/X$ is interpreted as an effective temperature. If $X = 1$, $T_{eff} = T$ and we get the usual form of the FD theorem.

1.4 Measuring principles for T_{eff} in nonequilibrium glass systems

• The system (S) and the thermometer (TM) - both described by Langevin equations (LE) and have different heat baths.

• By using TTI, LE is transformed to an algebraic equation in the Fourier space.

• During the measuring time S and TM are brought in ‘contact’.

• The exchanged energy between S and TM in the stationary regime is monitored.

1.5 The thermometric principle is the following:

The readings of TM correspond to the temperature of the TM bath for which the net energy flow between S and TM vanishes.

1.6 Experimentally, one measures the time-integrated susceptibility:

$$\xi(\tau) = \int_0^{\tau} d\tau^{'} R(\tau^{'}) \ .$$

At equilibrium, from the FD theorem one gets $\xi(\tau) = \frac{C(0)-C(\tau)}{T}$. Since for paramagnets $C(0) = 1$, a parametric plot of $\xi(\tau)$ vs $C(\tau)$ is a straight line with slope equal to $-1/T$. Violations of the FD theorem will manifest as deviations from this slope to a slope $-1/T_{eff}$.

Why-type question:
Why T_{eff} introduced in this way is indeed a temperature ?

Answers:

• It is the temperature associated to a given time scale and it is the one measured on the system by a TM whose reaction time is equal to the time scale.

• T_{eff} determines the direction of the heat flow within a particular time scale, which essentially is from slower to faster relaxing modes.

• It acts as a criterion for thermalization, i.e., it determines the average energy of an oscillator coupled to the nonequilibrium sample (generalized equipartition theorem).

1.7 Operational definition of T_{eff} as a function of the susceptibility at a given time scale

$$T_{eff}(time \quad scale) = -\frac{1}{\chi^{'}}(time \quad scale) \ .$$

1.8 Stationary driven systems

Some *stationary* nonequilibrium (driven) systems show up very similar features. In such cases, a parameter v which measures the intensity of the driving force is introduced. It plays a role similar to t_w: the smaller v the "older" the system. Although in driven systems TTI is satisfied, the FD theorem is not, even in the limit of vanishing v.

A time scale $\tau(q, v)$ is defined by means of the relation

$$C(\tau, v) = q \ .$$

If q is larger than a threshold value q_{EA} (called the Edwards-Anderson order parameter) one has

$$lim_{v \to 0} \tau < \infty \ .$$

This is equivalent to the following definition of q_{EA}:

$$q_{EA} = lim_{\tau \to \infty, v \to 0} C(\tau, v) \ .$$

On the other hand, if $q < q_{EA}$, the time $\tau(v)$ diverges as v goes to zero.

For a thermometric measurement performed on a characteristic time scale τ, T_{eff} is given by

$$T_{eff}(\tau) = -\frac{1}{\chi'(q)}|_{q=C(\tau,v)} \ .$$

• At the present time, there is an experiment of Grigera and Israeloff (Phys. Rev. Lett. 83 # 24, 1999) in which they were able to detect directly weak violations of the FD theorem in a glycerol sample. Their procedure is based on the detection of the Nyquist-Johnson noise power spectral density from a capacitor whose dielectric is the glycerol. They performed the fitting of the data with a modified Nyquist formula in which an effective temperature parameter has been used.

NOTES

NOTE 1: The thermometer in quantum field theory

In the paper *The Unruh effect in extended thermometers* by Bell, Hughes, Leinaas (1985) the generality of the Unruh effect is related, following Sewell, to the PCT symmetry. At the same time, they give there a definition of the quantum field thermometer.

In quantum field theory the TM is supposed to be a Lorentz invariant quantum system of finite spatial extent.

This finite extent can create an ambiguity in the temperature parameter similar to that of a body in thermal equilibrium in a gravitational field (Tolman, 1934).

Bell *et al* give the following two examples of quantum field thermometers:

• Unruh's academic TM: Relativistic quantum field confined in a box whose walls are, for it, impenetrable, but which are freely penetrated by the original quantum field whose temperature is to be taken. One can imagine such a box held together by some unspecified physical force and uniformly accelerated by some other.

• Bell-Leinaas TM: An electron confined and accelerated by electromagnetic fields. The spin of the electron can in principle be considered as the 'internal degree of freedom of the quantum detector'.

NOTE 2: semiclassical relativity

Within semiclassical relativity and using the influence action, Hu and Sukanya (1995) wrote down the back reaction equation of matter fields on the dynamics of geometry in the form of an Einstein-Langevin equation. It would be interesting to develop a TM concept similar to that in glass physics in this context.

NOTE 3: On the order parameter q_{EA}

The EA order parameter q_{EA} describes freezing of local spins S_i in random directions. It is worth noting that a plot of the response function versus

q for a paramagnetic TM coupled to the asymmetric spherical Sherrington-Kirkpatrick model (a quadratic Hamiltonian in Ising spin variables with random couplings) is similar to the plot of the polarization at storage rings versus the magnetic moment.

2. THE GLASS ANALOGY FOR BLACK HOLES

Forward - I report here on the paper of Th. Nieuwenhuizen (Phys. Rev. Lett. 81, 2201 (1998)) on the glass analogy for the black hole (BH) thermodynamics (TDM).

First and Second Laws of BH TDM

First law of BH TDM

$$dU = \frac{\kappa}{8\pi} dA + \Omega dJ + \Phi dq \ , \tag{1}$$

$U = Mc^2$ – BH energy
κ – surface (BH horizon) tension
$A = 4\pi R_s^2$ – horizon area
Ω – horizon's angular velocity
Φ – electrostatic potential at the horizon.

The last two terms correspond to work terms. On the other hand, a heat-like interpretation is given to the first term in the following way. If one introduces an entropy

$$S_{BH} = \frac{k_B A}{4L_P^2} \ , \tag{2}$$

where $L_P = \sqrt{\hbar G/c^3}$, and the Hawking temperature

$$T_H = \frac{\hbar G \kappa}{2\pi c^3 k_B} \tag{3}$$

that can also be written $T_H = \frac{\hbar c^3}{8\pi G M k_B}$ for nonrotating, neutral BHs, then one can write the first law in a truly TDM form

$$dU = T_H dS_{BH} + \mathrm{d}W \ . \tag{4}$$

Generalized second law of Bekenstein

$$dS_{BH} + dS_m \geq 0 \ , \tag{5}$$

where S_m is the entropy of the matter outside the BH.

Nieuwenhuizen's objections

1. *First objection: Defining the TDM system*

For Nieuwenhuizen the literature on BH TDM looks somewhat confusing because one should first clearly define the system for which a TDM description is to be given. One natural choice as a system could be the BH and a "gedanken" sphere around it, of say, 100 times the Schwarzschild radius. Alternatively, one can take the whole universe as an isolated container.

2. *Second objection: Formulation of the first TDM law for BHs is not correct*

Employing

$$dU = \mathrm{d}Q + \mathrm{d}W \ , \tag{6}$$

the heat term should be determined for the system under consideration. In general, the second TDM law is only an inequality. Only for the equilibrium case, it is an equality. The equality sign has been used in the second BH TDM law assuming $T = T_H$ and $S = S_{BH}$, i.e., assuming these are the equilibrium values. These values have been inserted in the first law. However, Bekenstein's specific heat $C_{Bek} = dU/dT_H$ being negative points to a *nonequilibrium-driven* system. Even assuming an initial equilibrium situation, the fluctuations drive the system away of it.

The main conclusion of Nieuwenhuizen regarding the first two objections is that Eqs. (4) and (5) should not be applied simultaneously, since they refer to different cases, i.e., "to different time scales" (?!).

3. *Final objection (considered severe by Nieuwenhuizen): What is the heat bath ?*

Is it that at T_H or that at $T_{cb} = 2.73$ K ? If it is the latter, then one deals with a system - the BH - whose dynamical laws work at a second temperature T_H. Thus, a two-temperature description is required.

Nieuwenhuizen's solution

The best way to think of nonequilibrium TDM methods is to use the effective temperature concept $T_{eff}(t)$ by which one can naturally divide the dynamical modes in two classes:

- fast - in thermal equilibrium at T
- slow - in quasiequilibrium at T_{eff}. For glasses $T_{eff} > T$.

By eliminating t, one may specify the cooling trajectory by a function $T_{eff}(T)$.

The entropies of fast and slow modes:

- The fast modes have the equilibrium entropy S_{eq}
- the slow modes have the *information entropy* or *complexity* $\mathcal{I}$.

Then, one can write the heat flow in the form

$$\mathrm{d}Q = TdS_{eq} + T_{eff}d\mathcal{I} \ , \tag{7}$$

which satisfies $\mathrm{d}Q \leq TdS$, since $T_{eff} > T$ and $d\mathcal{I} < 0$. For the latter inequality, one can invoke that in the course of time the system will go to lower, less degenerate modes.

Specific heat:

$$C \equiv \frac{\partial U}{\partial T}|_p = T\frac{\partial S_{eq}}{\partial T} + T_{eff}\frac{\partial \mathcal{I}}{\partial T} \ . \tag{8}$$

Since both entropies are functions of T and T_{eff}, the specific heat can be written in a form first given by Tool in 1946

$$C = C_1 + C_2\frac{\partial T_{eff}}{\partial T} \ . \tag{9}$$

Nieuwenhuizen used these ideas to claim that T_{BH} plays the role of an "internal temperature" of the black hole system while the cosmic background temperature T_{bgr} that of the bath temperature.

He also considered the combined effect of Hawking emission and matter accretion including background photon absorption. Regarding this issue, Sivaram (Phys. Rev. Lett. 84, 3209, 2000) pointed that for an evaporating BH of mass M to accrete (absorb) either ambient radiation or matter, its luminosity due to Hawking radiation ($L_H \approx \hbar c^6/G^2M^2$) must be less than the corresponding Eddington luminosity ($L_E \approx 10^{38}(M/M_{\odot})$) ergs/s at which radiation pressure from the hole drives away the ambient medium.

Using this, Sivaram shows that an estimate in the work of Nieuwenhuizen (the very last equation 19) is not correct.

I currently investigate the possibility to consider T_{BH} not as a one-time quantity as Nieuwenhuizen did, but as a two-time effective temperature.

3. STATIONARY AND NONSTATIONARY SCALAR VACUUM FIELD NOISES

Forward - If stationary, the spectrum of vacuum field noise (VFN) is an important ingredient to get information about the curvature invariants of classical worldlines (relativistic classical trajectories). For scalar quantum field vacua there are six stationary cases as shown by Letaw some time ago, these are reviewed here. However, the non-stationary vacuum noises are not out of reach and can be processed by a few mathematical methods which I briefly comment on. Since the information about the kinematical curvature invariants of the worldlines is of radiometric origin, it can have useful application to radiation and beam radiometric standards at relativistic energies.

3.1 THE DETECTOR METHOD IN QUANTUM FIELD THEORY

First, let's discuss briefly the Unruh-DeWitt detector procedure.
The interaction between the detector and the scalar field is described by

$$L_{int} = Q(\tau)\phi(\tau) \ ,$$

where $\phi(\tau) = \phi(x(\tau))$ is the field along the worldline of the detector.

How does the detector perform the measurement ?

At $\tau = -\infty$, the detector is in the ground-state $|E_0\rangle$ and the field is in the Minkowski vacuum $|0_M\rangle$. After the detector-field interaction is switched on, the detector would not remain in $|E_0\rangle$, but would make a transition to an excited state. It is said that the detector "detects" some particles. Then, the transition amplitude for the detector-field system to be found in $|E_1, \psi\rangle$ at $\tau = \infty$ is given by first order perturbation theory as

$$A = i\langle E_1, \psi| \int_{-\infty}^{\infty} d\tau Q(\tau)\phi(\tau)|E_0, 0_M\rangle \ .$$

To apply only the first order of the perturbation one has to assume that the matrix element of Q is sufficiently small.

On the other hand, from the time evolution of the operator Q in the Heisenberg picture

$$Q(\tau) = e^{iH_D\tau\hbar}Q(0)e^{-iH_D\tau\hbar} \ ,$$

where H_D is the detector Hamiltonian, which is assumed to have stationary eigenstates E_i, one immediately obtains

$$i\langle E_1|Q(0)|E_0\rangle \int_{-\infty}^{\infty} d\tau e^{i(E_1-E_0)}\tau\hbar\langle\psi|\phi(\tau)|0_M\rangle \ .$$

After summation over all final states of the field $|\psi\rangle$, the transition rate, i.e., the transition probability per unit proper time from E_0 to E_1 is

$$\frac{dP}{d\tau} = |\langle E_1|Q(0)|E_0\rangle|^2 F\left(\frac{E_1 - E_0}{\hbar}\right) \ ,$$

where

$$F(\omega) = \int_{-\infty}^{\infty} d(\tau - \tau^{'})e^{-i\omega(\tau-\tau^{'})}g(\tau - \tau^{'})$$

and

$$g(\tau - \tau^{'}) = \langle 0_M|\phi(\tau)\phi(\tau^{'})|0_M\rangle \ .$$

Thus, F looks like a response function (or power spectrum) and g as the "quantum noise" in the Minkowski vacuum along the worldline $x(\tau)$.

3.2 SIX TYPES OF STATIONARY SCALAR VFNs

In general, the scalar quantum field vacua do not possess stationary vacuum excitation spectra (abbreviated as SVES) for all types of classical relativistic trajectories on which the DW detector moves. Nevertheless, the linear uniform acceleration is *not* the only case with that property as was shown by Letaw who extended Unruh's considerations, obtaining six types of worldlines with SVES for DW monopole detectors (SVES-1 to SVES-6, see below). The line of arguments is the following.

The DW detector is effectively immersed in a scalar bath. Its rate of excitation is determined by the energy spectrum of the scalar bath that can be expressed as the density of states times a cosine Fourier transform of the Wightman correlation function (WCF) of the scalar field. Since the WCF is directly expressed in terms of the inverse of the geodetic interval what one needs to calculate is a Fourier transform of the inverse of the geodetic interval dx_{μ}^2.

Stationarity $\rightarrow$ The WCF depends only on the proper time interval.

As shown by Letaw, the stationary worldlines are solutions of some generalized Frenet equations on which the condition of constant curvature invariants is imposed, i.e., constant curvature κ, torsion τ, and hypertorsion ν, respectively. Notice that one can employ other frames such as the Newman-Penrose spinor formalism as recently did Unruh but the Frenet-Serret one is in overwhelming use throughout physics.

3.3 THE SIX STATIONARY CASES ONE BY ONE

1. $\kappa = \tau = \nu = 0$

[inertial, uncurved worldlines].

SVES-1 is a trivial cubic spectrum

$$S_1(E) = \frac{E^3}{4\pi^2} \ , \tag{1}$$

i.e., as given by a vacuum of zero point energy per mode $E/2$ and density of states $E^2/2\pi^2$.

2. $\kappa \neq 0$, $\tau = \nu = 0$

[hyperbolic worldlines].

SVES-2 is Planckian allowing the interpretation of $\kappa/2\pi$ as 'thermodynamic' temperature. In the dimensionless variable $\epsilon_\kappa = E/\kappa$ the vacuum spectrum reads

$$S_2(\epsilon_\kappa) = \frac{\epsilon_\kappa^3}{2\pi^2(e^{2\pi\epsilon_\kappa} - 1)} \ . \tag{2}$$

3. $|\kappa| < |\tau|$, $\nu = 0$, $\rho^2 = \tau^2 - \kappa^2$

[helical worldlines].

SVES-3 is an analytic function corresponding to the case 4 below only in the limit $\kappa \gg \rho$

$$S_3(\epsilon_\rho) \stackrel{\kappa/\rho \to \infty}{\longrightarrow} S_4(\epsilon_\kappa) \ . \tag{3}$$

Letaw plotted the numerical integral $S_3(\epsilon_\rho)$, where $\epsilon_\rho = E/\rho$ for various values of κ/ρ.

4. $\kappa = \tau$, $\nu = 0$

[the spatially projected worldlines are the semicubical parabolas $y = \frac{\sqrt{2}}{3}\kappa x^{3/2}$ containing a cusp where the direction of motion is reversed].

SVES-4 is analytic, and since there are two equal curvature invariants one can use the dimensionless energy variable ϵ_κ

$$S_4(\epsilon_\kappa) = \frac{\epsilon_\kappa^2}{8\pi^2\sqrt{3}} e^{-2\sqrt{3}\epsilon_\kappa} \ . \tag{4}$$

It is worth noting that S_4, being a monomial times an exponential, is rather close to the Wien-type spectrum $S_W \propto \epsilon^3 e^{-\mathrm{const.}\epsilon}$.

5. $|\kappa| > |\tau|$, $\nu = 0$, $\sigma^2 = \kappa^2 - \tau^2$

[the spatially projected worldlines are catenaries, i.e., curves of the type $x = \kappa \cosh(y/\tau)$].

In general, SVES-5 cannot be found analytically. It is an intermediate case, which for $\tau/\sigma \to 0$ tends to SVES-2, whereas for $\tau/\sigma \to \infty$ tends toward SVES-4

$$S_2(\epsilon_\kappa) \stackrel{0 \leftarrow \tau/\sigma}{\longleftarrow} S_5(\epsilon_\sigma) \stackrel{\tau/\sigma \to \infty}{\longrightarrow} S_4(\epsilon_\kappa) \ . \tag{5}$$

6. $\nu \neq 0$

[rotating worldlines uniformly accelerated normal to their plane of rotation].

According to Letaw, SVES-6 forms a two-parameter set of curves. These trajectories are a superposition of the constant linearly accelerated motion and uniform circular motion. The corresponding vacuum spectra have not been calculated by Letaw, not even numerically.

Thus, only the hyperbolic worldlines, having just one nonzero curvature invariant, allow for a Planckian SVES and for a strictly one-to-one mapping between the curvature invariant κ and the 'thermodynamic' temperature ($T_U = \kappa/2\pi$). The VFN of semicubical parabolas can be fitted by

Wien-type spectra, the radiometric parameter corresponding to both curvature and torsion. The other stationary cases, being nonanalytical, lead to approximate determination of the curvature invariants defining locally the classical worldline on which a relativistic quantum particle moves.

One very general and acceptable statement on the *universal* nature of the kinematical Frenet parameters occuring in a few important quantum field model problems can be formulated as follows

There exist accelerating classical trajectories (worldlines) on which moving ideal (two-level) quantum systems can detect the scalar vacuum environment as a stationary quantum field vacuum noise with a spectrum directly related to the curvature invariants of the worldline, thus allowing for a radiometric meaning of those invariants.

According to these results, the thermal interpretation is merely replaced by the more general radiometric interpretation of the Frenet invariants.

3.4 NONSTATIONARY VACUUM FIELD NOISES

The nonstationary VFNs have a time-dependent spectral content requiring joint time and frequency information, i.e., one needs generalizations of the power spectrum concept. One can think of:

- tomographical processing
- wavelet transform analyses

The so-called non-commutative tomography (NCT) transform $M(s, \mu, \nu)$ proposed by Manko and Vilela Mendes seems to be an attractive way of processing the analytic nonstationary signals and perhaps one can hope to get unambiguous information for nonanalytic signals as well. In the definition of M, s is just an arbitrary curve in the non-commutative time-frequency plane. The most simple examples have been given by Manko and Vilela Mendes by means of the axes $s = \mu t + \nu\omega$, where μ and ν are linear combination parameters. The NCT transform is related to the Wigner-Ville quasidistribution $W(t, \omega)$ by an invertible transformation of the form

$$M(s, \mu, \nu) = \int \exp[-ik(s - \mu t - \nu\omega)] W(t, \omega) \frac{dk d\omega dt}{(2\pi)^2} \ . \tag{6}$$

According to Man'ko and Vilela Mendes the well-known interpretation ambiguities of the Wigner-Ville transform can be avoided in the case of NCT transform, i.e., a rigorous probability interpretation can be assigned.

Other useful properties of M:

- $M(t,1,0) = |f(t)|^2$
- $M(\omega,0,1) = |f(\omega)|^2$

f is the analytic signal, which is simulated by M.

- The most interesting property of M is the possibility to detect the presence of signals in noises for small signal-to-noise ratios.

This last property may be very effective for detecting VFNs, which are very small 'signals' with respect to more common noises.

On the other hand, since in the quantum detector method the vacuum autocorrelation functions are the essential physical quantities, and since according to fluctuation-dissipation theorem(s) they are related to the linear (equilibrium) response functions to an initial condition/vacuum, the FD approach has been revealed and promoted by Sciama.

In principle, the generalization of the FD theorem for some classes of out of equilibrium relaxational systems looks also promising for the case of nonstationary vacuum noises. As we have already discussed in Section 1, using the two-time FD ratio $X(t,t^{'})$, one writes

$$T_{\rm eff}R(t,t^{'}) = X(t,t^{'})\frac{\partial C(t,t^{'})}{\partial t'}~.$$

The FD ratio is employed to perform the separation of scales. Moreover, $T_{\rm eff}$ are time-scale-dependent quantities, making them promising for relativistic VFNs, which correspond naturally to out of equilibrium conditions.

3.5 GETTING THE SVES FORMULAS

One can calculate all sorts of SVES by means of the following general formula

$$S_j(E) = |\frac{E^2}{4\pi^3}\int_{-\infty}^{\infty}\frac{e^{-iEs}ds}{([x_{\mu}(s)-x_{\mu}(0)][x^{\mu}(s)-x^{\mu}(0)])}| = \frac{E^2}{4\pi^3}|{\rm I}_j|~, \quad j=1...6~, \qquad (S1)$$

where $x^\mu(s)$ is an arbitrary point on the worldline and $x^\mu(0)$ is the initial point. The signature of the Minkowski metric is $\eta_{\mu\nu} = (1, -1, -1, -1)$. I confirm Letaw's results by sketching the calculation of the integrals I_j for the six stationary cases. Simple details that have been skipped by Letaw can be found here. In general, the VFN spectra present a fourth power of a Frenet-type invariant as a scaling parameter (κ, ρ and σ, respectively) that should be taken into account when performing the calibration of the spectra.

1. **The recta**.
$x^\mu(s) = (s, 0, 0, 0)$; $x^\mu(0) = (0, 0, 0, 0)$. The integral is

$$\mathrm{I}_1 = \int_{-\infty}^{+\infty} \frac{e^{-iEs}}{s^2} ds \ . \tag{S2}$$

It can be obtained by (i) Cauchy's residue theorem and (ii) $e^{-iEs} = (1 - iEs + ...)$. The value of the integral is $\pi i(-iE) = \pi E$, and therefore one gets the cubic spectrum. This inertial zero-point cubic spectrum will appear in all the other five stationary spectra as an additive background and therefore one may take into account only the non-cubic contributions as a measure of noninertial vacuum effects.

2. **The hyperbola**.
$x^\mu(s) = \kappa^{-1}(\sinh \kappa s, \cosh \kappa s, 0, 0)$ and $x^\mu(0) = \kappa^{-1}(0, 1, 0, 0)$. Now the integral is

$$\mathrm{I}_2 = \int_{-\infty}^{+\infty} \frac{e^{-i\epsilon_\kappa u}}{2(\cosh u - 1)} du \ . \tag{S3}$$

Writing $e^{-i\epsilon_\kappa u} = \cos \epsilon_\kappa u - i \sin \epsilon_\kappa u$, one makes use of formula 3.983.3 at page 505 in the fourth edition of the *Table* of Gradshteyn and Ryzhik (GR) to get

$$\int_0^{+\infty} \frac{\cos ax}{\cosh x - 1} dx = -(\pi a)\mathrm{cth}(\pi a) \ , \tag{S4}$$

and of the Cauchy theorem for the integral in the sinus function

$$-i \int_{-\infty}^{+\infty} \frac{\sin(\epsilon_\kappa x) dx}{\cosh x - 1} = \pi \epsilon_\kappa \ . \tag{S5}$$

Thus

$$\mathrm{I}_2 = -\pi\epsilon_\kappa \mathrm{cth}(\pi\epsilon_\kappa) + \frac{\pi\epsilon_\kappa}{2} = \pi\epsilon_\kappa[1 - \mathrm{cth}(\pi\epsilon_\kappa)] - \frac{\pi\epsilon_\kappa}{2} = -2\pi\epsilon_\kappa \frac{1}{e^{2\pi\epsilon_\kappa} - 1} - \frac{\pi\epsilon_\kappa}{2} \ , \tag{S6}$$

where the first term leads to the Planckian spectrum and the latter to the cubic zero-point contribution.

3. **The helix**.
$x^{\mu}(s) = \rho^{-2}(\tau\rho s, \kappa \cos\rho s, \kappa\sin\rho s, 0)$, $x^{\mu}(0) = \rho^{-2}(0, \kappa, 0, 0)$. The integral reads

$$\mathrm{I}_3 = \rho^3 \int_{-\infty}^{+\infty} \frac{e^{-i\epsilon_{\rho}u}}{2\kappa^2(\cos u - 1) + \tau^2 u^2} du \ . \qquad (S7)$$

According to Letaw this integral is non-analytic and indeed I was not able to find any helpful formula in the GR *Table*.

4. **The semicubical parabola**.
$x^{\mu}(s) = (s + \frac{1}{6}\kappa^2 s^3, \frac{1}{2}\kappa s^2, \frac{1}{6}\kappa^2 s^3, 0)$, $x^{\mu}(0) = (0,0,0,0)$. The integral reads

$$\mathrm{I}_4 = \kappa \int_{-\infty}^{+\infty} \frac{e^{-i\epsilon_{\kappa}u}du}{u^2(1 + \frac{1}{12}u^2)} = \kappa \mathrm{I}_1 - \kappa \int_{-\infty}^{+\infty} \frac{e^{-i\epsilon_{\kappa}u}du}{(12 + u^2)} \ . \qquad (S8)$$

Of interest is only the second integral that can be found in the GR *Table* at page 359

$$\int_{-\infty}^{+\infty} \frac{e^{-ipx}dx}{a^2 + x^2} = \frac{\pi}{|a|} e^{-|ap|} \ , \qquad (S9)$$

for $a > 0$ and p real. Thus, one gets

$$\int_{-\infty}^{+\infty} \frac{e^{-i\epsilon_{\kappa}u}du}{(12 + u^2)} = \frac{\pi}{\sqrt{12}} e^{-\sqrt{12}\epsilon_{\kappa}} \ . \qquad (S10)$$

The final result is

$$S_4 = \frac{\kappa^4 \epsilon_{\kappa}^2}{4\pi^2\sqrt{12}} e^{-\sqrt{12}\epsilon_{\kappa}} \ . \qquad (S11)$$

Interestingly, for a horizontal storage ring (guiding magnetic field in the vertical z direction) the orbit in the moving frame can be approximated for laboratory times such that $\gamma\omega_0|t| = O(1)$ by the following semicubical parabola

$$y^{'} \approx (\rho/2\gamma^2)(6\gamma^2|x^{'}|/\rho)^{2/3}$$

where ρ is the instantaneous radius of curvature of a particle's arbitrary trajectory (See Fig. 2 and Eq. 2 in J.D. Jackson, Rev. Mod. Phys. 48, 417 (1976)).

5. **The catenary**.
$x^{\mu}(s) = \sigma^{-2}(\kappa\mathrm{sinh}\sigma s, \kappa\mathrm{cosh}\sigma s, \tau\sigma s, 0)$ and $x^{\mu}(0) = \sigma^{-2}(0, \kappa, 0, 0)$. The integral is of the type

$$\mathrm{I}_5 = \int_{-\infty}^{+\infty} \frac{\sigma^3 e^{-i\epsilon_\sigma u} du}{2\kappa^2(\mathrm{cosh}u - 1) - \tau^2 u^2} \ . \tag{S12}$$

This integral turns into I_2 and I_4 in the limits mentioned in the text, respectively, but again there is no helpful formula in the GR *Table*, and thus I_5 appears to be non-analytic.

6. **The helicoid (helix of variable pitch)**.
$x^{\mu}(s) = (\frac{\Delta}{RR_+}\sinh(R_+ s), \frac{\Delta}{RR_+}\cosh(R_+ s), \frac{\kappa\tau}{R\Delta R_-}\cos(R_- s), \frac{\kappa\tau}{R\Delta R_-}\sin(R_- s))$ and $x^{\mu}(0) = (0, \frac{\Delta}{RR_+}, \frac{\kappa\tau}{R\Delta R_-}, 0)$, where:

$\Delta^2 = \frac{1}{2}(R^2 + \kappa^2 + \tau^2 + \nu^2)$
$R^4 = (\kappa^2 + \tau^2 + \nu^2)^2 - 4\kappa^2\tau^2$
$R_+^2 = \frac{1}{2}(R^2 + \kappa^2 - \tau^2 - \nu^2)$
$R_-^2 = \frac{1}{2}(R^2 - \kappa^2 + \tau^2 + \nu^2)$.

The following integral is obtained:

$$\mathrm{I}_6 = \frac{1}{2}\int_{-\infty}^{+\infty} \frac{e^{-iEs} ds}{(\frac{\Delta}{RR_+})^2[\cosh(R_+ s) - 1] + (\frac{\kappa\tau}{R\Delta R_-})^2[\cos(R_- s) - 1]} \ . \tag{S13}$$

This is the most complicated non-analytic stationary case, with no helpful formula in the GR *Table*.

4. THE HACYAN-SARMIENTO APPROACH: CIRCULAR ELECTROMAGNETIC VACUUM NOISE

Forward - In 1989, Hacyan and Sarmiento (HS) introduced a clear-cut method for calculating the main electromagnetic vacuum spectral quantities and they presented the basic cases of *linear acceleration* and *uniform rotation*. In the latter case, they obtained a nonzero energy flux in the direction of motion of the detector. This prompted Mane (1991) to suggest a connection with the synchrotron radiation.

4.1 HS APPROACH

In the following, a brief presentation of the Hacyan-Sarmiento procedure is given without referring to any difficult mathematical issue.

The electromagnetic energy-momentum tensor is

$$T_{\mu\nu} = \frac{1}{16\pi}\left(4F_{\mu\alpha}F^{\alpha}_{\nu} + \eta_{\mu\nu}F_{\lambda\beta}F^{\lambda\beta}\right) \ . \tag{1}$$

The two-point Wightman functions are defined as follows

$$D^{+}_{\mu\nu}(x,x^{'}) \equiv \frac{1}{4}\left(4F^{\alpha}_{(\mu}(x)F_{\nu)\alpha}(x^{'}) + \eta_{\mu\nu}F_{\lambda\beta}(x)F^{\lambda\beta}(x^{'})\right) \ . \tag{2}$$

$$D^{-}_{\mu\nu}(x,x^{'}) \equiv D^{+}_{\mu\nu}(x^{'},x) \ . \tag{3}$$

Because of the properties

$$\eta_{\mu\nu}D^{\pm}_{\mu\nu} = 0, \qquad D^{\pm}_{\mu\nu} = D^{\pm}_{\nu\mu}, \qquad \partial_{\nu}D^{\pm\nu}_{\mu} = 0, \tag{4}$$

the electromagnetic Wightman functions can be expressed in terms of the scalar Wightman functions as follows

$$D^{\pm}_{\mu\nu}(x,x^{'}) = c\partial^{2}_{\mu\nu}D^{\pm}(x,x^{'}) \ , \tag{5}$$

where c is a constant.

Using the Fourier transforms of the WFs (FTWF)

$$\tilde{D}^{\pm}(\omega,\tau) = \int_{-\infty}^{\infty} d\sigma e^{i\omega\sigma}D^{\pm} \ , \tag{6}$$

where ω is the frequency of the zero-point fields, the *particle number density of the vacuum seen by the moving detector* is given by

$$n(\omega,\tau)=\frac{1}{(2\pi)^2\omega}[\tilde{D}^{+}(\omega,\tau)-\tilde{D}^{-}(\omega,\tau)] \ . \tag{7}$$

On the other hand, the *energy density per mode* is given by

$$de=\frac{\omega^2}{\pi}[\tilde{D}^{+}(\omega,\tau)+\tilde{D}^{-}(\omega,\tau)]d\omega \ . \tag{8}$$

These are the main theoretical results obtained by Hacyan and Sarmiento.

An important application is a uniformly rotating 'observer' whose proper time is τ and angular speed is ω_0 follows the circular world line

$$x^{\alpha}=(\gamma\tau, R_0\cos(\omega_0\tau), R_0\sin(\omega_0\tau), \mathrm{const}) \ , \tag{9}$$

where R_0 is the rotation radius in the inertial frame and $\gamma=(1-v^2)^{-1/2}$.

In this important case there are two Killing vectors

$$k^{\alpha}=(1,0,0,0)$$

and

$$m^{\alpha}=(0,-R_0\sin(\omega_0\tau),R_0\cos(\omega_0\tau),0) \ .$$

Expressing the Wightman functions in terms of these two Killing vectors, Hacyan and Sarmiento calculated the following physically observable spectral quantities (i.e., those obtained after subtracting the inertial zero-point field contributions)

The energy density

$$\frac{de}{d\omega}=\frac{\gamma^3}{2\pi^3R_0^3}\frac{\omega^2+(\gamma v\omega_0)^2}{\omega^2}\frac{v^3w^2}{w^2+(2\gamma v)^2}h_{\gamma}(2\omega/\omega_0) \tag{10}$$

The flux density

$$\frac{dp}{d\omega}=\frac{\gamma^3}{2\pi^3R_0^3}\frac{\omega^2+(\gamma v\omega_0)^2}{\omega^2}4v^4k_{\gamma}(2\omega/\omega_0) \tag{11}$$

The stress density

$$\frac{ds}{d\omega} = \frac{\gamma^3}{2\pi^3 R_0^3}\frac{\omega^2 + (\gamma v \omega_0)^2}{\omega^2}\frac{v^3 w^2}{w^2 + (2\gamma v)^2} j_\gamma(2\omega/\omega_0) \ , \tag{12}$$

where

$$h_\gamma(w) \equiv \int_0^\infty [\frac{(3+v^2)x^2 + (v^2+3v^4)\sin^2 x - 8v^2 x \sin x}{\gamma^2[x^2 - v^2\sin^2 x]^3} - \frac{3}{x^4} + \frac{2\gamma^2 v^2}{x^2}] \cos(wx) dx \tag{13}$$

$$k_\gamma(w) \equiv -\int_0^\infty \left(\frac{x^2 + v^2\sin^2 x - (1+v^2) x \sin x}{\gamma^2[x^2 - v^2\sin^2 x]^3} - \frac{3}{x^4} - \frac{\gamma^2}{6x^2} \right) \cos(wx) dx \tag{14}$$

$$j_\gamma(w) \equiv \int_0^\infty \left(\frac{1}{\gamma^4[x^2 - v^2\sin^2 x]^2} - \frac{1}{x^4} + \frac{2\gamma^2 v^2}{3x^2} \right) \cos(wx) dx \ , \tag{15}$$

where $w = \frac{2\omega}{\omega_0}$ and $x = \frac{\sigma\omega_0}{2}$.

In addition, they noticed that the quantities $H_\gamma = \frac{v^3 w^2}{w^2 + (2\gamma v)^2} h_\gamma(2\omega/\omega_0)$, $K_\gamma = 4v^4 k_\gamma(w)$ and $J_\gamma = \frac{v^3 w^2}{w^2+(2\gamma v)^2} j_\gamma(2\omega/\omega_0)$ have in the ultrarelativistic limit $\gamma \gg 1$ the following scaling property

$$X_{k\gamma}(kw) = k^3 X_\gamma(w) \ , \tag{16}$$

where k is an arbitrary constant, and $X = H, K, J$. This is the same scaling property as of a Planckian distribution with a temperature proportional to γ.

4.2 NONRELATIVISTIC LIMIT OF THE HS APPROACH

Kim, Soh, and Yee (1987) used the HS approach to calculate the nonrelativistic limit of zero-point field of parameters v and ω_0, and not acceleration and speed as used by Letaw and Pfautsch (1980).

They calculated the *particle number density of the vacuum as seen by the moving detector* according to the Hacyan-Sarmiento formula (7) and also the *energy density per mode* (8). For the inertial, uniformly accelerated, and the circular cases one always gets for the difference of the Fourier-transformed WFs (FTWFs) the same result $\frac{\omega}{2\pi}$ and therefore the particle number density of the vacuum is the same. What differs is the energy density per mode.

For the circular case, the FTWFs are

$$\tilde{D}^{\pm}(\omega,\tau)=\frac{-1}{4\pi^2}\frac{\omega_0}{2\gamma}\int_{-\infty}^{\infty}ds\frac{\exp(2iWs)}{(s\mp i\epsilon)^2-v^2\sin^2(s)}\ , \tag{17}$$

where $s=\gamma\omega_0\sigma/2$ and $W\equiv\omega/\gamma\omega_0$. It is easy to see that the nontrivial effects in $de/d\omega$ are given by the function $\tilde{D}^{-}(\omega)$. It can be expanded in power series

$$\tilde{D}^{-}(\omega)=\frac{-1}{4\pi^2}\frac{\omega_0}{2\gamma}\int_{-\infty}^{\infty}ds\exp(2iWs)\sum_{n=0}^{\infty}\frac{v^{2n}\sin^{2n}(s)}{(s+i\epsilon)^{2n+2}}\ . \tag{18}$$

Integrating term by term leads to

$$\tilde{D}^{-}(\omega)=\frac{\omega_0}{2\pi\gamma}\sum_{n=0}^{\infty}\frac{v^{2n}}{2n+1}\sum_{k=0}^{n}(-1)^k\frac{(n-k-W)^{2n+1}}{k!(2n-k)!}\cdot\Theta(n-k-W)\ , \tag{19}$$

where Θ is the usual step function.

This formula has been used by J. Rogers [Phys. Rev. Lett. **61**, 2113 (1988)] in his geonium proposal to detect the circular electromagnetic vacuum noise. See also H. Rosu, Nuovo Cimento B 111, 507 (1996) [quant-ph/9509017].

4.3 MANE'S ARGUMENT

Mane (1991) used the HS formula for the energy flux to argue that its time component is related to the synchrotron radiation.

The HS Poynting flux is directed along the Lorentz boost from the laboratory frame to the rest frame of the observer, which is taken as the y axis. It can be written

$$p_y=\frac{1}{1440\pi^2}\frac{\hbar\gamma^8\omega_0^4 v}{c^4}(50-47\gamma^{-2})\ . \tag{20}$$

p_y is proportional to $\hbar$ and therefore becomes zero in the classical limit. However, for particles, such as the electron, which couples to the flux by the fine structure constant $\alpha=e^2/\hbar c$, the radiation effect looks totally classical. The recoil induced by the flux of the vacuum fluctuations on the four-momentum of the particle per unit proper time is

$$\alpha A p_y \propto \frac{e^2\gamma^4\omega_0^2 v}{c^3} \ , \tag{21}$$

where $A \approx R_0^2 c^2/(v^2\gamma^4)$ is the transverse interaction area between the electron and the electromagnetic field. In the laboratory frame, the energy loss of the particle per unit laboratory time is given by the Larmor formula

$$I = \frac{2}{3}\frac{e^2}{c^3}(\gamma^2\omega_0 v)^2 \ . \tag{22}$$

This is related to the damping force F in the form $I = \vec{F}\cdot\vec{v}$ and therefore the recoil induced by synchrotron radiation on the four-momentum of the particle per unit proper time is again $\propto \frac{e^2\gamma^4\omega_0^2 v}{c^3}$. Thus, the order of recoil of the particle induced by this interaction is equal to that derived by the Larmor formula in the ultrarelativistic limit.

NOTES

NOTE 1: As quoted by D.P. Barber, another interesting interpretation of the synchrotron radiation is as an inverse Compton scattering of equivalent photons in a generalization of the Fermi-Weizsäcker-Williams method. This interpretation has been put forth by Lieu and Axford.

R. Lieu, W. Ian Axford, "Synchrotron radiation: An inverse Compton effect", Astrophys. J. 416, 700 (1993); "Quantum-limited synchrotron radiation in inhomogeneous magnetic fields", Astrophys. J. 447, 302 (1995).

NOTE 2: By examining the shape of the HS spectra, one can infer that the vacuum fluctuations could be supplemented by radiation reaction effects in order to fully explain the synchrotron radiation. This could be similar to the Heisenberg picture of quantum electrodynamics where this type of explanation has been given a long time ago by Senitzky (Phys. Rev. Lett. 31, 955 (1973)) and Milonni, Ackerhalt, Smith (Phys. Rev. Lett. 31, 958 (1973)).

Radiation reaction effects on the synchrotron radiation have been considered for example by R.W. Nelson & I. Wasserman, "Synchrotron radiation with radiation reaction", Astrophys. J. 371, 265 (1991).

5. SYNCHROTRON RADIATION IN STORAGE RINGS

Forward - If one goes as far as accepting the idea that synchrotron radiation is due to noninertial electromagnetic vacuum fluctuations one should reproduce in this approach many basic features of the synchrotron radiation that are known from measurements at storage rings. We review here these experimental facts.

Schwinger (1949) gave the equation for the synchrotron power radiated by a relativistic electron in an arbitrary orbit. The spectral intensity of the magnetobremsstrahlung in the synchrotron regime is proportional to the so-called shape function

$$W_{\omega} \propto F\Big(\frac{\omega}{\omega_m}\Big) ~, \tag{1}$$

where ω_m is given in terms of the cyclotron radian frequency ω_c as $\omega_m = \omega_c\,\gamma^3$, and F is given by $F(\xi) = \frac{9\sqrt{3}}{8\pi}\xi\int_{\xi}^{\infty}K_{5/3}(z)dz$, where K is the MacDonald (modified Bessel) function of the quoted fractional order. The small and large asymptotic limits of the synchrotron shape function are as follows

$$F(\xi \ll 1) \approx 1.33\;\xi^{1/3} \tag{2}$$

and

$$F(\xi \gg 1) \approx 0.78\;\xi^{1/2}e^{-\xi} ~, \tag{3}$$

with a maximum (amount of radiation) to be found at the frequency $\frac{1}{3}\omega_m$.

However, accelerator people use a somewhat different formula giving the power radiated per amper (amp) of current (I) per radian of horizontal orbit (Θ) per unit bandwidth ($d\lambda$) for electrons confined in a storage ring

$$\frac{d^3P(\lambda)}{dId\Theta d\lambda} = \frac{3^{5/2}}{160\pi^2}\frac{ec\gamma^7}{\rho^2}\left(\frac{\lambda_c}{\lambda}\right)^3\int_y^{\infty}K_{5/3}(\eta)d\eta ~. \tag{4}$$

This formula is in cgs units. $y = \lambda_c/\lambda$, where

$$\lambda_c = \frac{4\pi\rho}{3\gamma^3} \tag{5}$$

is the famous critical wavelength of the synchrotron radiation. Half of the power is radiated above λ_c and half below. For electrons $\gamma = 1956.8E$ (GeV).

The number of photons in a resolution interval $\Delta\lambda$ defined by an instrument of resolving power R (i.e. $\Delta\lambda = \lambda/R$) is given by

$$N(\lambda)\Delta\lambda = 2.4567 \times 10^{13} \frac{Ei\theta}{R} \left(\frac{\lambda_c}{\lambda}\right) \int_y^\infty K_{5/3}(\eta) d\eta \tag{6}$$

in photons/sec. E is the energy in GeV, i is the current in milliamps, and θ is the amount of orbit seen in milliradians.

The total power radiated is

$$P(\mathrm{kW}) = 8840 \frac{E^4}{\rho} I(\mathrm{amps}) \ . \tag{7}$$

Another important formula is the power radiated as a function of vertical angle ψ

$$\frac{d^2 P}{d\psi d\lambda} = \frac{27ec}{320\pi^3} \frac{\gamma^8}{\rho^2} \left(\frac{\lambda_c}{\lambda}\right) I\Theta F(\lambda_c/2\lambda, \psi), \tag{8}$$

where $F(\lambda_c/2\lambda, \psi)$ is given by the sum of the parallel (to the orbit plane) and perpendicular polarizations, $F_{||}$ and $F_\perp$, respectively

$$F(\lambda_c/2\lambda, \psi) = F_{||} + F_\perp = (1+X)^2 K_{3/2}^2(\xi) + X(1+X) K_{1/3}^2(\xi) \ , \tag{9}$$

where $X = (\gamma\psi)^2$ and $\xi = \lambda_c/2\lambda(1+X)^{3/2}$.

The degree of linear polarization parallel to the plane of the electron orbit is given as a function of ψ by

$$J_{||}(\psi) = \frac{F_{||}}{F_{||} + F_\perp} \ . \tag{10}$$

In the plane of the ring, i.e., $\psi = 0$, the degree of polarization is 100% at all wavelengths because $F_\perp = 0$. As ψ is increased, the polarization state changes from linear to elliptical and finally circular with the phase difference between the parallel and perpendicular components always being $\pi/2$.

6. SPIN MOTION AT STORAGE RINGS AND CIRCULAR VACUUM NOISE

Forward - The electromagnetic circular vacuum noise is interesting not only because of the Hacyan-Sarmiento results but also due to the proposal of Bell and Leinaas (1983) to relate electron depolarization at storage rings to this type of vacuum noise. However, there are strong arguments against this interpretation. I give here a brief introduction to this problem.

6.1 Introduction

The main idea of Bell and Leinaas relies on the spin degree of freedom of the electron in an external magnetic field B_0 along the z axis that may be thought to have two (quasi)stationary states corresponding to $\sigma_z = \pm 1$, with an energy splitting $\Delta = 2|\mu||B_0|$. This is valid when a second term in the effective spin-field interaction Hamiltonian due to the so-called Thomas precession is not included. Thus, the electron looks like a Unruh-DeWitt detector. The transitions between the two spin states induced by the radiation field are then written in terms of first-order, time-dependent perturbation theory and the thermal ratio is obtained as if produced by the equilibrium ratio of populations of the upper and lower levels. The effect of the Thomas precession term in the effective Hamiltonian does not alter the shape of the polarization curve and only shifts it horizontally when plotted as a function of the magnetic moment.

However, there is a quantum electrodynamical explanation of the polarization effect at storage rings in terms of the so called spin-flip synchrotron radiation that has been proposed by Sokolov and Ternov in 1963. The spin-flip radiated power is very small with respect to the ordinary synchrotron radiation becoming of the same order only at $\gamma_{sf} = (mc\rho/\hbar)^{1/2}$, which for a common storage ring is around 6×10^6. This is more than two orders of magnitude higher than the actual $\gamma < 10^4$ of electrons in current storage rings leading to a spin-flip radiated power representing only 10^{-11} of the usual (non spin-flip) synchrotron emitted power.

6.2 Elementary description of radiative polarization (cf. Jackson, 1976)

The elementary (naive) description is based on the following ideas:

1. The effect involves spin-flip.
2. The rate is very slow since it is related to magnetic dipole transition between states with a small energy difference.
3. The electrons and positrons are polarized with their magnetic moments parallel to the magnetic field, corresponding to the state of lowest energy of an isolated spin system.

In the rest frame of the orbiting electron $K^{'}$, let us consider a simple $M1$ transition from the upper energy level to the lower.

Though we know that for relativistic particles the radiative process is due to a segment of trajectory of length $d \approx \rho/\gamma$ corresponding to an angular deflection $\Delta\theta \approx 1/\gamma$, we consider a particle of charge e and mass m moving at constant speed $v = c\beta$ in a circular orbit of radius ρ in a uniform static magnetic field B.

Suppose that the spin degree of freedom can be treated nonrelativistically in this frame. With magnetic moment

$$\vec{\mu} = \frac{g}{2} \cdot \frac{e\hbar}{2mc}\vec{\sigma} \ ,$$

the spin degree of freedom has two energy levels in $K^{'}$ with frequency difference

$$\omega_{12}^{'} = |\frac{g}{2}| \cdot \frac{eB^{'}}{mc} = |\frac{g}{2}|\gamma^2\omega_0 \ .$$

The transition probability per unit time for a spontaneous magnetic dipole transition from the upper state to the lower is

$$w^{'} = \frac{4}{3\hbar}\left(\frac{\omega_{12}^{'}}{c}\right)|\langle 2|\vec{\mu}|1\rangle)|^2 \ .$$

The laboratory transition rate is reduced by one power of γ due to time dilation effects in $K^{'}$. With $\omega_0 = c/\rho$ for a relativistic particle, one gets a characteristic time

$$\tau_{naive} = \left[\frac{2}{3}|\frac{g}{2}|^5\frac{e^2\hbar\gamma^5}{m^2c^2\rho^3}\right]^{-1} \ .$$

The difference with respect to the Sokolov-Ternov formula is in the numerical factors, $\frac{2}{3}|\frac{g}{2}|^5$ for τ_{naive} with respect to $\frac{5\sqrt{3}}{8}$ for the Sokolov-Ternov time τ_{ST}. Moreover, for $|g| = 2$, the difference is of the order unity. However, spontaneous emission from the upper to the lower energy level leads trivially to 100 % polarization with the correct senses for electrons and positrons.

Referring to previous Russian works, Jackson argued that the above elementary argument, which is also basic for the vacuum fluctuation interpretation, is exact only for neutral particles possessing magnetic moments. For charged particles, the simple argument is reliable only for large $|g|$, whereas for small values of $|g|$ the magnetic moment does not precess rapidly enough to discard important spin-orbit effects. Moreover, the actual degree and sense of the polarization depends sensitively on the value of g. For the range $0 < g < 1.2$, there is a sort of laser effect: the upper energy level is populated preferentially over the lower one.

Jackson showed that the spacing between orbital levels is very small compared to the M1 transition energy and therefore the M1 transition will involve some changes in the orbital quantum number. According to Jackson, these exchanges of energy between spin and orbital degrees of freedom provide little significance to the two-level concept for the spin system alone. In 1973, Derbenev and Kondratenko obtained a formula for the equilibrium polarization where spin-orbit effects are included through a spin-orbit coupling function. Their formula is considered as the standard result for the transverse polarization at storage rings. In 1987, Bell and Leinaas published a more detailed analysis of their proposal, in which still assuming a thermal spectrum of the spin excitations, they took into account the fluctuations in the orbital motion. They obtained a polarization formula rather close to the standard one with some differences only close to a narrow depolarizing resonance. Thus, the confirmation of their calculation and of a thermal spin distribution means precise experimental measurements (still to be performed).

There was some recent interest in the Bell-Leinaas proposal due to the Monterey conference on quantum aspects of beam physics organized by Pisin Chen in 1998. Leinaas, Unruh, and Barber reviewed the physics implied by this proposal. Their works can be found in the Los Alamos archive.

7. THE FLUCTUATION-DISSIPATION RELATIONSHIP

Forward - I make here a random walk through the literature on the FD theorem that I think to be relevant for the topics presented herein.

7.1 Nyquist (1928): discovery of FD theorem

The equilibrium TDM quantities, which are either extensive or intensive state parameters,undergo spontaneous fluctuations. In 1928, H. Nyquist published the derivation of an equation relating the fluctuations in voltage in linear electrical systems, usually referred to as Nyquist or Johnson noise (J.B. Johnson performed the first measurements at that time), to the electrical resistance, which is a parameter describing an irreversible process. Such relationships are therefore very helpful in virtually all fields of physics and are known as FD theorems. Nyquist's proof of the FD theorem is based on a combined usage of the second law of TDM and a direct calculation of the fluctuations in an ideal transmission line.

7.2 Callen & Welton and Callen & Greene (1951, 1952): General form of the FD theorem

Callen and Welton (1951) and Callen and Greene (1952) gave a sufficiently general form of Nyquist's result, as a relation between the 'impedance' in a general linear disipative system (power dissipation is quadratic in the magnitude of the perturbation) and the fluctuations of appropriate generalized TDM forces.

For the proof, one works towards comparing the expressions for $R(\omega)/|Z(\omega)|^2$ and of $\langle V^2\rangle$. This leads to

$$\langle V^2\rangle = (2/\pi)\int_0^\infty R(\omega)E(\omega,T)d\omega \ ,$$

where

$$E(\omega,T) = \frac{1}{2}\hbar\omega + \hbar\omega[\exp(\hbar\omega/kT) - 1]^{-1} \ .$$

At high temperatures, $E(\omega,T)\approx kT$ and one gets the Nyquist form

$$\langle V^2 \rangle \approx (2/\pi)kT \int R(\omega)d\omega \ .$$

7.3 Candelas & Sciama (1977): First application of FD theorem to BHs

In 1977, Candelas and Sciama showed that the BHs obey the principles of irreversible TDM in the form of a FD theorem for their zero-point quantum fluctuations. In Rindler coordinates, they considered the coupling of the particle to the scalar field through the interaction Lagrangian

$$L_{int} = \epsilon \cdot m(x)\phi(x) \ ,$$

where $m(x)$ is a monopole charge and ϵ is a small coupling constant. The rate at which the detector makes transitions from an energy eigenstate corresponding to a frequency ν_1 to another eigenstate corresponding to another frequency $\nu_2 = \nu_1 + \nu$ is

$$R(\nu;\nu_1) = \epsilon^2 M^2 \int_{-\infty}^{\infty} d\tau e^{i\nu\tau} W(\tau) \ ,$$

where $M = \langle \nu_1 + \nu | m(0) | \nu_1 \rangle$ and $W(\tau) = \langle 0|\phi(\tau)\phi(0)|0\rangle$ is the expectation value of the two-point Wightman function. As remarked by Candelas and Sciama this relationship can be considered as a FD theorem since R determines not only the *equilibrium* internal state of the charge but also the *rate* at which the equilibrium is approached, or equivalently the *dissipation rate of any correlations* that might initially be present.

R is essentially determined by the Fourier transform of the autocorrelation function of the scalar field, which is just the power spectrum of the noise evaluated along the worldline of the particle (this is called Wiener-Khinchin theorem in Mathematics).

This type of reasoning has been applied by Candelas and Sciama to a detector constrained to corotate near the (outer) horizon of a Kerr BH and weakly coupled to the fluctuations of the gravitational shear field σ of the null congruence generating the future horizon. They obtained the following asymptotic result at the outer horizon, $r \to r_+$, for the Fourier transform of the autocorrelation function of σ in the Unruh vacuum (outgoing flux of radiation at large radii)

$$\int_{-\infty}^{\infty} d\tau e^{i\omega\tau} \langle U|\sigma^*(\tau)\sigma(0)|U\rangle \sim \frac{4}{5}\frac{1}{r-r_+}\frac{\omega(\omega^2+4\kappa^2)}{e^{2\pi\omega/\kappa}-1}$$

They interpreted this relationship as a FD theorem, with $\omega^2 + 4\kappa^2$ playing the simultaneous role of the impedance function and the density-of-states factor. For a critique of this result, see B.L. Hu, A. Raval, S. Sinha, gr-qc/9901010, contribution to Vishveshwara's festschrift (Kluwer, 1999) pp. 103-120.

7.4 Mottola (1986): Application of FD theorem to General Relativity

In 1986, Mottola discussed a quantum form of the FD theorem for General Relativity. The basic points are the following.

• The analytic property of periodicity in imaginary time of the Green's functions generates a simple relationship between the symmetric and antisymmetric parts of the polarization tensor, which is one form of the FD theorem.

• The existence of the Killing field $\partial/\partial t$ is necessary in order that some concept of time independence and equilibrium remains intact in the full general relativistic setting. The thermal Green's functions are uniquely defined by their regularity on the Euclidean section, $t \to it$ and the boundary conditions at spacelike infinity (vanishing if the space is not compact). Corresponding to this thermal Green's function is a state of the quantum fields, or, more precisely, a thermal density matrix.

• Suppose one asks how the expectation value of a physical quantity in this state, such as the energy-momentum tensor T_{ab}, changes as the background metric is changed. Then, this is the linear response of $\langle T_{ab}(x)\rangle$ to an external perturbation $\delta g_{cd}(x^{'})$, to first order in the perturbation.

• In nonrelativistic statistical mechanics the dissipative effects are described through the antisymmetric function. To show that this is the case in curved space requires to relate the rate of increase of matter entropy due to the particle production process to this commutator. This is most easily accomplished by considering the linear response equation with time-asymmetric boundary conditions. The FD theorem in this context is related to the so called "vacuum viscosity" concept used by some authors in the literature.

7.5 Hu & Sinha (1995): Application of FD theorem to semiclassical cosmology

In 1995, Hu and Sinha used the concept of open systems, where a classical geometry is treated as the system and quantum matter fields as the environment, to derive FD theorems for semiclassical cosmology. This is only one paper from a long list belonging to Hu and collaborators spreaded over the last two decades. Similar results have been obtained by Verdaguer's group.

7.6 Terashima (1999): FD Theorem and the Rindler noise

• A calculation of the Rindler noise by means of the FD theorem has been recently provided by Terashima. He used the formulation of Callen and Welton, and the well-known fact that the "response function" (the Fourier transform of the autocorrelation function) depends only on the field and the worldline of the detector but not on the internal structure of the detector.

Terashima explicitly calculated the dissipative coefficient from the equations of motion of the detector and the field. Then, using the FD theorem, he immediately obtained the vacuum Rindler noise.

On the other hand, Takagi showed in 1986 that the Rindler noise is thermal in even number of dimensions but displays an inversion of statistics (becoming Fermi-Dirac) in odd dimensions.

Terashima proved that the statistics inversion is due to the "temperature" dependence of the dissipative coefficient.

7.7 Eyink (2000): Fluctuation-response relations for multitime correlations

Using a variational approach, Eyink derives a general fluctuation-response relation (he prefers to call the FD theorem in this way) for arbitrary multitime correlations.

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