

# Our Mathematical Age

Rod Downey  
Victoria University  
Wellington  
New Zealand

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## My apologies in advance

- ▶ I teach only university students, some school students for enrichment classes (particularly in Singapore), and Scottish Country dancing for which I actually have a teaching certificate!
- ▶ Indeed having taught SCD to a group of around 18, 9-10 years olds in Johnsonville School with a teacher I gained immense respect for her.
- ▶ However, I do have a bit of insight into modern mathematics so I hope the following might be of some use.

## My own educational experience

- ▶ Primary School 6 (**SIX**) schools in 7 years, moving from Toowoomba to parts of Brisbane and the Sunshine Coast.
- ▶ The final little school (Rainworth) had a clever old headmaster/teacher who tried to inspire with maths (and the cane!).
- ▶ Then Brisbane State High School, insisting to my parents that I would stay there even though we again moved to somewhere which gave 2 hours travel each way.
- ▶ Typical reports: “Could try harder”, “Has a bad attitude”, “Can be disruptive” (but the occasional “has a distinct aptitude for the subject” and “results speak for themselves”)
- ▶ I think I was bloody minded; but they did allow me to be the only student who did logic with the science students.
- ▶ University; I got all the Gough Whitlam support for poor people... These things really do work.

## And then

- ▶ My parents both working class; first Downey to ever go past 10th grade.
- ▶ Dad a storeman and packer, mum made people pay debts.
- ▶ Really the family had no idea why someone would even go to University; one of my main motivations being told not to do things.
- ▶ Actually, I had no idea why someone would go to University unless you wanted to be a doctor or a much-hated lawyer.
- ▶ Parents only really happy when I got paid.
- ▶ My wife from a professional family.
- ▶ After a wee bit abroad came to New Zealand for “a couple of years” ...
- ▶ 2 Boys, one a designer at Xero and one works in Google's Waymo.
- ▶ Newlands College....

# Extension

- ▶ Notwithstanding some conflicts here and there, boys thrived in Newlands.
- ▶ Great vertical support structures; good science and math teachers and good leadership.
- ▶ Carlton, the oldest was given extension mathematics.
- ▶ With the best of intentions, this consisted of being moved into the next year of maths.
- ▶ This was not good because stuff was left out and also for social development.
- ▶ We refused such things for the other boy.
- ▶ What might be good extension? What resources might be good for lateral extension?

# What's math anyway

- ▶ I think mathematicians build symbolic models of the world.
- ▶ This might be the physical world, but could be e.g. thought, logic.
- ▶ Then manipulating them allows them to understand/predict/explore.
- ▶ The Egyptians/Babylonians/Greeks/Chinese/Incas invented **geometry** to help building and the motions of the cosmos etc.
- ▶ They and others invented methods of calculating interest rates etc to make money.
- ▶ Later from physics we invented **differential equations** which can be used to describe rates of change.
- ▶ Logicians built models of **reasoning** and this led to computers and are the basis of modern society.
- ▶ It is the **reasoning** and **modelling** which is the core of maths, not the **procedures**.

- ▶ In my own experience I stay at this college in Singapore and give little lectures to arts student.  
<http://homepages.ecs.vuw.ac.nz/~downey/>
- ▶ I try to show them a wee bit of math/history and point at forefronts of modern mathematics.
- ▶ Each of the presentations has a lot of references I am sure a keen student could follow up on.

## Some names

- ▶ Graph Colouring as a Mathematical Minature (graph theory and combinatorics, leading to complexity theory)
- ▶ Codebreaking at Bletchley (about Turing etc, study in statistics) also a public lecture in my **talks**
- ▶ The life of  $\Pi$  (development of analysis and calculus-also Fascination of Science)
- ▶ I will look at these not because I think they are especially good, but they have good pointers for projects in them, and I know them.

- ▶ Can find these talks to download at  
<http://homepages.ecs.vuw.ac.nz/~downey/talks.html>
- ▶ Also an advanced class at NTU: (Order, Chaos and Algorithms-  
Unpublished Notes for Singapore Lectures at Nanyang for Gifted  
Undergraduates and Junior College Students) in  
<http://homepages.ecs.vuw.ac.nz/~downey/publications/>

## Some ideas 1

- ▶ My own background is in **discrete mathematics**.
- ▶ Theoretical computer science, algorithms, computation.
- ▶ **Graph theory and algorithmics** is an excellent vehicle.
- ▶ New Zealand has some excellent material in **Computer Science Unplugged** <https://csunplugged.org/en/> and <https://csfieldguide.org.nz/en/>
- ▶ **How not to be Wrong** (about \$NZ17) has a wealth of material about particularly statistics, and mathematical reasoning for the layman. Stresses **The Mathematical Way of Thought**.
- ▶ I am sure you are aware of videos like those on **Mathsreach**. <https://mathsreach.org/Welcome>
- ▶ Kid Crypto: Type this into Google with Fellows after it.

## For example

- ▶ **Graph theory and algorithmics** is an excellent vehicle.
- ▶ Euler's Formula: For  $E \geq 1$ , and  $G$  connected.  $V + E - F = 2$  "The Raging Sea Proof"
- ▶ Then the 5 Colour Theorem.

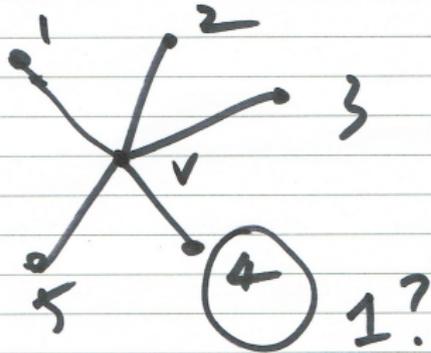
# The Raging Sea

- ▶ Your plane graph is sitting inside a raging sea, breaking on the structure from the outside.
- ▶ You remove an outer edge and the sea floods in.  $F := F - 1$ ,  $E := E - 1$ ,  $V := V$ .
- ▶ Continue till only a jetty remains. Then remove outer edges one at a time:  $E := E - 1$ ,  $V := V - 1$ ,  $F = 1$  still.
- ▶ At the end  $V + E - F$  has not changed, but you have a single edge with 2 vertices, and one face. so  $V + E - F = 2 + 1 - 1 = 2$ .
- ▶ **Then** show this means there must be a vertex of degree 5. (not too hard)

# The 5 Colour Theorem

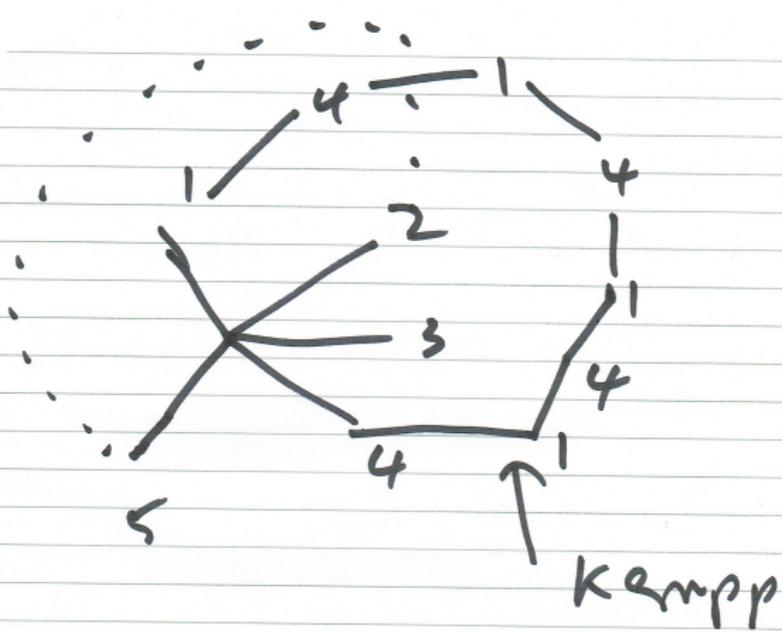
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- ▶ Heawood, 1890's-can talk about Kempp's proof and point them at the 4CT.
- ▶ I begin with the smallest planar graph that is not 5-colourable.
- ▶ If I **remove** a vertex  $v$  of degree  $\leq 5$  I can 5-colour the rest.
- ▶ So do this and 5-colour the smaller graph.
- ▶ Can I put  $v$  back in?



can I recolor  
as  $1 \leftrightarrow 4$

Smallest counterexample



IF NO !

Kempner chan

# Today

- ▶ I don't think standards have been falling. Certainly not since Euclid.
- ▶ There is a gap left by the loss of Euclidean Geometry, the only achievement standard which addresses this is 2.6 Apply algebraic methods in solving problems.
- ▶ I think that the students are a wee bit different now. (With no evidence except observations)
- ▶ It is hard to convince them about effort to get to deep knowledge.
- ▶ Adept at finding stuff out on the Internet. (“research”)

- ▶ But I believe that are a bit more easily demoralized by harder things that will come with effort but are not instantly understandable. For me, for example, proofs by induction.
- ▶ The current breadth in the school curriculum and choice of topics available, does make it more difficult for the student entering University.
- ▶ We find many students make bad choices, and need remedial work. How can kids know what they want so young and then make subject choices accordingly?
- ▶ We have had to upgrade the requirements for entry into core maths courses at VUW, such as 161, to ask for NCEA merits.
- ▶ I lament at the loss of geometry, because the heart and soul of maths is deductive reasoning. We need some kind of replacement in the new curriculum, discrete maths would do.

# Summary

- ▶ I hope this gives you some pointers as to what might be useful materials for your classes.
- ▶ It hope this gives you a wee bit of insight into things to say, why maths?
- ▶ Thanks for Listening!