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Δ_2^0 DEGREES AND TRANSFER THEOREMS

BY

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1. The main goal of this paper is to demonstrate how weak truth table/Turing degree "transfer" techniques may be used to obtain information about the Δ_2^0 (Turing) degrees. Such techniques have previously been applied by Ladner-Sasso [13], Stob [18] and others to obtain information about **R**, the r.e. *T*-degrees. The best known example of this phenomenon is Ladner and Sasso's [13] use of contiguous degrees to show that every nonzero r.e. degree has a predecessor with the anticupping property.

Let **D** denote the degrees, **W** the r.e. weak truth table (*W*-)degrees and $\mathbf{D}_W$ the weak truth table degrees. Modifying the Ladner-Sasso analysis to Δ_2^0 degrees, we shall give a new and relatively easy proof of a result independently proved by Cooper [5] and Slaman and Steel [16] about structural interactions of **R** and **D**:

THEOREM A. $\exists \mathbf{a}, \mathbf{b} \in \mathbf{R}(\mathbf{0} < \mathbf{b} < \mathbf{a}$ *and* $\forall \mathbf{c} \in \mathbf{D}(\mathbf{c} \cup \mathbf{b} = \mathbf{a} \rightarrow \mathbf{c} = \mathbf{a}))$

Such a degree **a** is said to have the *strong anticupping property with witness* **b**. Actually, we get a slight improvement by constructing **a** with witnesses that are "downward dense" in **R**. To prove Theorem A, we first analyse how **D** and **W** interact and then prove some results about $\mathbf{D}_W$ and **W**. In particular, one result we shall establish is that every nonzero r.e. weak truth table degree has the *global anticupping property*, that is:

THEOREM B.

$$\forall \mathbf{a} \in \mathbf{W}(\mathbf{a} \neq \mathbf{0} \rightarrow \exists \mathbf{b} \in \mathbf{W}(\mathbf{0} < \mathbf{b} < \mathbf{a} \text{ and } \forall \mathbf{c} \in \mathbf{D}_W(\mathbf{a} \leq \mathbf{c} \cup \mathbf{b} \rightarrow \mathbf{a} \leq \mathbf{c}))).$$

Theorem B also implies that the elementary theory of (for example) the weak truth table degrees below $\mathbf{0}'_W$ and the Δ_2^0 degrees are different (since Posner and Robinson [15] have shown that the nonzero *T*-degrees below $\mathbf{0}'$ all cup to $\mathbf{0}'$).

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Finally, we shall give a couple of other examples to indicate some further applications of Δ_2^0 transfer theorems. For example, we show that there exist nonzero r.e. degrees **a** that split in a very strong way over all lesser Δ_2^0 degrees; namely, if $\mathbf{b} < \mathbf{a}$ and $\mathbf{b} \in \mathbf{D}$ then there exists an r.e. splitting $\mathbf{a}_1 \cup \mathbf{a}_2 = \mathbf{a}$ of **a** with $\mathbf{b} \cup \mathbf{a}_1, \mathbf{b} \cup \mathbf{a}_2 < \mathbf{a}$.

Our notation is standard and follows Soare [17]. T-reductions will be denoted by $(\Phi, \Gamma, \ldots)$ and those with "hats" $(\hat{\Phi}, \hat{\Gamma}, \ldots)$ will denote W-reductions. The recursive use corresponding to the latter will be the corresponding lower case greek letter (e.g., use $(\hat{\Phi}) = \varphi$, use $(\hat{\Gamma}) = \gamma, \ldots$). Unless stated otherwise, we denote T-degrees by lower case boldface letters $(\mathbf{a}, \mathbf{b}, \ldots)$. Finally all computations, etc., are bounded by s at stage s.

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2. We shall first construct an r.e. degree with the strong anticupping property. To do this we modify the transfer analysis of Ladner and Sasso [13] which gave a new proof of Lachlan's result [10] that there is an r.e. degree with the anticupping property. The Ladner-Sasso analysis is summarized by the combination of (2.1) and (2.2) below.

(2.1) There exists a nonzero contiguous r.e. degree, namely a nonzero r.e. degree **a** consisting of a single *r.e.* W-degree; meaning that if A and B are r.e. and of degree **a**, then $A \equiv_W B$.

(2.2) $\forall \mathbf{a} \in \mathbf{W}(\mathbf{a} \neq \mathbf{0} \to \exists \mathbf{b} \in \mathbf{W}(\mathbf{0} < \mathbf{b} < \mathbf{a} \text{ and } \forall \mathbf{c} \in \mathbf{W}(\mathbf{c} \cup \mathbf{b} \geq \mathbf{a} \to \mathbf{c} \geq \mathbf{a})))$.

We shall replace (2.1) and (2.2) by:

(2.1)′ There exists an r.e. degree $\mathbf{a} \neq \mathbf{0}$ such that all (not necessarily r.e.) sets A, B of degree **a** are of the same W-degree. We call such a degree *strongly contiguous*.

(2.2)′ (Theorem B) Every nonzero r.e. W-degree has the global anticupping property.

Then we see that—in the same way as [13]—(2.1)′ and (2.2)′ imply Theorem A.

We now turn to the proof of (2.1)′, namely the construction of a strongly contiguous degree. For convenience, we modify the presentation of Ambos-Spies [1]. We satisfy the following requirements.

P_e: $\bar{A} \neq W_e$.

N_e: $\Phi_e(A)$ total {and (0,1}-valued, by convention) and $\Gamma_e(\Phi_e(A)) = A$ implies $A \leq_W \Phi_e(A)$.

$\hat{N}_e$: $\Phi_e(A)$ total and $\Gamma_e(\Phi_e(A)) = A$ implies $\Phi_e(A) \leq_W A$.

Here (Φ_e, Γ_e) denotes a standard enumeration of all pairs of T-reductions. Both N_e and $\hat{N}_e$ are met by similar (completely compatible) techniques.

Due to the similarity of our method of satisfying N_e (and $\hat{N}_e$) with the case where $\Phi_e(A)$ is r.e., it will suffice (in each case) to discuss the strategy for a single requirement, and then to leave the details of coordination of the requirements to the reader.

Let $l(e,s) = \max\{x: \forall y < x(\Gamma_{e,s}(\Phi_{e,s}(A_s);\ y) = A_s(y))\}$. We meet N_e (and $\hat{N}_e$) by essentially the same cancellation procedure as for the case $\Phi_e(A)$ r.e. in a contiguous degree construction. The only difficulty is to see that it also works for $\Phi_e(A)$ only Δ_2^0. Specifically each follower x of P_j for $j > e$ is equipped with a guess as to whether or not $l(e,s) \to \infty$. If a follower x is guessing that $l(e,s) \not\to \infty$ then if

$$l(e,s) > ml(e,s) =_{\text{df}} \max\{l(e,t): t < s\}$$

we shall cancel x. The other key *follower rules* are:

(2.4) If x is appointed at stage s then $x = s$, and if $l(e,s) > ml(e,s)$ we give x a guess that $l(e,s) \to \infty$; otherwise x guesses $l(e,s) \not\to \infty$.

(2.5) If $x < y$ and x and y are followers and if x enters A_s, then x cancels y.

(2.6) If x and y are followers and $y > x$ (so that, by (2.4) y is appointed after x) and x is uncancelled at stage y, then y has lower priority than x.

The basic idea for N_e is this. For each follower x following some P_j for $j > e$ guessing $l(e,s) \to \infty$, we wait for the *first stage* when $l(e,s) > x$. At this stage (with x least) we declare x as *e-confirmed* and cancel all followers y for $y > x$. This gives the situation in Figure 1.

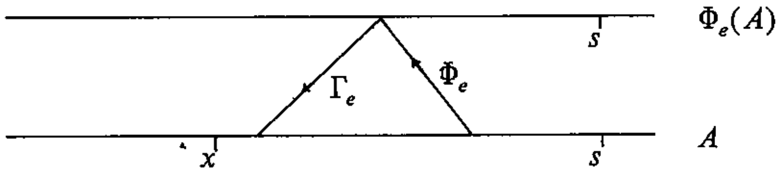

FIG. 1

Now the crucial points are that for this situation to occur x must be guessing that $l(e,s) \to \infty$, and there are no followers left alive between x and s. We claim that this insures that $A \leq_W \Phi_e(A)$ as follows: Let $u = \max\{u(\Phi_{e,s}(A_s;\ y)): y \leq x\}$. To determine whether $x \in A$ compute the least stage $t > s$ with $l(e,t) > ml(e,t)$ and

$$\Phi_{e,t}(A_t)[u] = \Phi_e(A)[u].$$

(Notice here we are not asking that $\forall t' > t(\Phi_{e,t'}(A_{t'})[u] = \Phi_{e,t}(A_t)[u])$ as would occur in the r.e. case). We claim that $x \in A$ iff $x \in A_t$. There are two cases to consider.

Case 1. $\Phi_{e,s}(A_s)[u] = \Phi_{e,t}(A_t)[u]$. In this case the situation of Figure 1 is unchanged and because u measures a use function it must be that $A[x] = A_s[x] = A_t[x]$.

Case 2. Otherwise. Since there were no numbers z alive at stage s with $x \le z < s$, by (2.4) the only way this can occur is if some follower $y \le x$ enters $A - A_s$. By (2.5) such a follower either cancels x or equals x. In either case $x \in A$ iff $x \in A_t$.

As with the case where $\Phi_e(A)$ is r.e., the cancellation/confirmation procedure implemented for N_e also meets $\hat{N}_e$. To see this, we must show that the cancellation of numbers between x and s in Figure 1 also allows A to w-compute $\Phi_e(A)$. Let z be given. To compute whether $z \in \Phi_e(A)$ first find the least stage s_1 where $l(e, s_1) > z$ and $l(e, s_1) > ml(e, s_1)$. Now A can only change (allowing $\Phi_{e,s_1}(A_{s_1})(z)$ to change) due to the entry of followers. At stage s_1 the only such followers g left alive must be guessing that $l(e, s) \to \infty$. By the way we appoint followers (2.4), if no follower $< s_1$ enters A after stage s_1 it must be that

$$\Phi_{e,s_1}(A_{s_1};\, z) = \Phi_e(A;\, z).$$

If $\Phi_{e,s_1}(A_{s_1};\, z)$ is to change, it follows that some follower g alive at stage s_1 must enter $A - A_{s_1}$. Suppose g_1 is the first such, and g_1 enters at stage t. Let s_2 be the least stage $> t$ with $l(e, s_2) > ml(e, s_2)$. Let $\hat{g}_1$ be the least follower that enters at any stage t' with $t \le t' < s_2$. Then $\hat{g}_1 \le g_1$ and $\hat{g}_1$ was present at stage s_1 (by (2.4)).

The crucial observation is:

(2.8) There are no followers x left alive with $\hat{g}_1 \le x < s_2$ at stage s_2.

To see this first observe that by (2.5), when $\hat{g}_1$ enters A—say at stage $\hat{t}$—it must cancel all followers p with $\hat{g}_1 \le p \le \hat{t}$. By choice of s_2 as the least stage $> t$ with $l(e, s_2) > ml(e, s_2)$, any follower q appointed after stage $\hat{t}$ but before stage s_2 must be guessing that $l(e, s) \nrightarrow \infty$ (by (2.4)). But then we automatically cancel such q at stage s_2. These observations give (2.8).

Now, we see that after stage s_2 either $A_{s_2}[\hat{g}_1] = A[\hat{g}_1]$ and so by (2.8), $A_{s_2}[s_2 - 1] = A[s_2 - 1]$ implying that $\Phi_{e,s_2}(A_{s_2};\, z) = \Phi_e(A;\, z)$ or some number $\le \hat{g}_1$ must enter A after stage s_2.

In the latter case, repeating the above process, we eventually arrive at a $\hat{g}_2$ and s_3 (say) etc. Combining all the above ideas, we get to our desired w-reduction: To compute $\Phi_e(A;\, z)$, find the least stage $\hat{s} > s$, with

$$l(e, \hat{s}) > ml(e, \hat{s}) \quad \text{and} \quad A_{\hat{s}}[s_1] = A[s_1].$$

Then it must be that $\Phi_{e,\hat{s}}(A_{\hat{s}};\ z) = \Phi_e(A;\ z)$ since the only followers below $u(\Phi_{e,\hat{s}}(A_{\hat{s}};\ z))$ alive at stage $\hat{s}$ were already present at stage s_1.

The remaining details of the full construction are to organize the above strategies with the usual π_2-guessing tree. Should the reader be unfamiliar with this, we refer him to [1] for further details. ∎

We now turn to the proof of Theorem B.

(2.3)′ *Every nonzero r.e. W-degree has the global anticupping property.*

Proof. Let A be a given r.e. nonrecursive set. We construct a coinfinite r.e. set $B = \bigcup_s B_s$ in stages to satisfy the following.

P_e: $|W_e| = \infty$ implies $W_e \cap B \neq \emptyset$

N_e: If C is any set and $\hat{\Gamma}_e(B \oplus C) = A$ then $A \leq_W C$.

We remind the reader that here $\hat{\Gamma}_e$ denotes the e-th W-reduction with use γ_e. This particular result gives a nice demonstration of the way some results for $\mathbf{D}_W$ can be obtained using techniques not applicable in the r.e. case. The reader should note that in this construction we cannot know C since there may be $2^{\aleph_0}$ possibilities. The key point, though, is that no matter which C pertains the use γ_e is the same. There are several ways to satisfy condition N_e above, but it seems easiest to use a construction similar to one of Ladner and Sasso [13]. We shall use an "almost monotone" restraint $r(e, s)$ which only drops when the "A-side" changes. To do this, we define a marking function $\alpha(e, s)$ as follows: Let $\sigma, \tau, \ldots$ denote strings. Define $\alpha(e, 0) = 0$. Set $\alpha(e, s+1)$ as the least x such that one of the following holds:

(i) $x < \alpha(e, s)$ and $x \in A_{s+1} - A_s$;

(ii) $x > \alpha(e, s)$ and $l(e, s) = x + 1$ where

$$l(e, s) = \max\{y\colon \exists\sigma\forall z < y(\hat{\Gamma}_{e,s}(B_s \oplus \sigma;\ z) = A_s(z))\};$$

(iii) (ii) does not apply and $x = \alpha(e, s)$.

We shall then define

$$r(e, s) = 1 + \max\{\gamma_{e,s}(z)\colon z \leq \alpha(e, s)\}$$

and

$$R(e, s) = \max\{r(j, s)\colon j \leq e\}.$$

There are two crucial observations regarding the relationship of α, r and A.

(2.7) If $l(e, s) > x$, $t_1, t_2 > s$ and $\alpha(e, t_1) = \alpha(e, t_2) = x$ then $r(e, t_1) = r(e, t_2)$. That is, once we see $l(e, s) > x$ we always know what $r(e, t)$ "will be", should x be the least number to occur in $A_{t+1} - A_t$ for $t > s$. We denote this by $m(e, x)$, that is, we define $m(e, x) = r(e, t) = 1 + \max\{\gamma_e(z)\colon t \leq x\}$

(2.8) Note that $y = \alpha(e, s+1) \le \alpha(e, s)$ iff $y = \mu z(z \in A_{s+1} - A_s) < \alpha(e, s)$. In particular, we ignore the B-side when it comes to dropping α.

Construction, stage $s + 1$. If $W_{e,s+1} \cap B_s = \varnothing$ then put $x \in B_{s+1} - B_s$ if $x > 2e$, $x > R(e, s+1)$, $A_{s+1}[x] \ne A_s[x]$ and $x \in W_{e,s}$ and x is least with these properties.

Verification. We only sketch some points due to their similarities with [13]. The reader should note that (2.7) allows us to show that all the P_e are met, by a permitting argument: For suppose P_e fails to be met. Let $z \in \omega$ be given. Let s_1 be a stage such that

$$\forall t \ge s_1(\alpha(j, t) = \alpha(j, s_1)) \quad \text{for } j \le e \text{ with } l(j, s) \nrightarrow \infty.$$

To decide if $z \in A$ or not find a stage $s = s(z) > s_1$ such that $l(j, s) > z$ for all j with $j \le e$ and $l(j, s) \to \infty$ (so that $m(j, z)$ of (2.7) is defined) and such that $y \in W_{e,s}$ with $y > \max\{2e, s_1, m(j, z)\colon j \le e\}$. By the observation (2.7) should ever $z \in A_t - A_s$ the restraints all drop so that we will be free to add y to A meeting P_e. Hence $A_s[z] = A[z]$ and so A is recursive, a contradiction.

Finally we verify N_e. Suppose C is any set with $\hat{\Gamma}_e(B \oplus C) = A$. Notice that appropriate σ exist to satisfy (ii) of the definition of $\alpha(e, s)$ and so $l(e, s) \to \infty$. Let z be given. Let $\sigma(z)$ denote $C[\gamma_e(z)]$. To C-recursively compute $A(z)$ find the least stage $s = s(z)$ such that

(i) all the P_j for $j < e$ cease activity, and

(ii) $\alpha(e, s) > z$ and $\forall y \le z(\hat{\Gamma}_{e,s}(B_s \oplus \sigma(z);\, y) = A_s(y))$.

We claim that $A_s[z] = A[z]$: For suppose otherwise. Let $\hat{z} \le z$ be the least number with $\hat{z} \in A - A_s$. By (2.8) we see that for all $t \ge s$, $\alpha(e, t) \ge \hat{z}$, and furthermore by (2.7), $r(e, t) \ge m(e, \hat{z})$. In particular, $B_s[\gamma_e(\hat{z})] = B[\gamma_e(\hat{z})]$. But now we see that $\hat{z}$'s entry into A causes the (preserved) disagreement

$$\hat{\Gamma}_e(B \oplus C;\, \hat{z}) = 0 \ne 1 = A(\hat{z}). \qquad \blacksquare$$

We get the following slightly strengthened form of Theorem A:

THEOREM A′. *There exists an r.e. degree* $\mathbf{a} \ne \mathbf{0}$ *such that for all r.e. degrees* $\mathbf{b} \ne \mathbf{0}$ *with* $\mathbf{b} < \mathbf{a}$ *there exists* $\mathbf{c} \le \mathbf{b}$ *with* $\mathbf{c}$ *a strong anticupping witness for* $\mathbf{a}$. *That is, strong anticupping witnesses are downward dense below* $\mathbf{a}$.

Proof. Let A be r.e. and of strongly contiguous degree. Let B be r.e. with $\varnothing <_T B <_T A$. By contiguity, $B \le_W A$. Now by (2.3)′ let C be an r.e. set $\le_W B$ such that for all sets D if $C \oplus D \ge_W B$, $D \ge_W B$. Now suppose for some set E we have $E \oplus C \equiv_T A$. By strong contiguity, $E \oplus D \equiv_W A\ {}_W\!\ge B$. Hence by choice of C, $E \ge_W B$ and so $E \equiv_W A$. $\blacksquare$

There are various other applications of the above approach. One must decide whether or not the permitting-type reductions built in the appropriate r.e. W-degree constructions may be replaced by Δ_2^0 permitting. Obviously, not all results on $\mathbf{W}$ may be changed in this way. For example Lachlan [La2] has shown that not every degree in $\mathbf{W}$ bounds a minimal pair (in $\mathbf{W}$) (strictly speaking this is a T-degree result that also must work in W), yet well known cone-avoidance full approximation arguments show that

$$\forall \mathbf{a}, \mathbf{b} \in \mathbf{W}\big(\mathbf{0} < \mathbf{a} < \mathbf{b} \to \exists \mathbf{c} \in \mathbf{D}_W(\mathbf{c} \cap \mathbf{a} = \mathbf{0} \quad \text{and} \quad \mathbf{0} < \mathbf{c} < \mathbf{b})\big).$$

In fact we may choose $\mathbf{c}$ of minimal T-degree. We refer the reader to [12] and [9]. One nice corollary of (2.3)′ is:

(2.9) THEOREM. *Suppose* $\mathbf{a}$ *and* $\mathbf{b}$ *are* W*-degrees with* $\mathbf{a} \geq \mathbf{b} > \mathbf{0}$ *and* $\mathbf{b}$ *r.e. Suppose that* $\mathbf{c}$ *is a* T*-degree with* $\mathbf{c} \geq \mathbf{0}'$. *Then the elementary theories of the upper semilattices* $[\mathbf{0}, \mathbf{a}]$ *and* $[\mathbf{0}, \mathbf{c}]$ *are different. In the language* $L(\leq, \vee, 0, 1)$ *the difference occurs by the two quantifier level.*

Proof. By Posner and Robinson [15, Theorem 3] the following sentence γ is not satisfiable in $[\mathbf{0}, \mathbf{c}]$:

$$\gamma =_{\text{df}} \exists x\big(x \neq 0 \quad \text{and} \quad \forall y(y \vee x \geq 1 \to y \geq 1)\big).$$

However, by (2.3)′, γ is satisfiable in $[\mathbf{0}, \mathbf{a}]$. ∎

To close this paper, we shall briefly point out a couple of further applications of Δ_2^0 transfer techniques. One example—transferring "backwards"–concerns the structure of W-degrees in a given degree. An r.e. degree $\mathbf{a}$ is *strongly* W*-bottomed* if there is an r.e. set A of degree $\mathbf{a}$ such that for all sets B of degree $\mathbf{a}$, $A \leq_W B$. It is unknown whether there is a nice characterization of such degrees. It is conjectured that they all must be low$_2$, since all contiguous degrees are low$_2$ (Cohen [4]). We prove a weaker result.

(2.10) THEOREM. *No high degree is strongly* W*-bottomed.*

Proof. Let A of degree $\mathbf{a}$ be the r.e. strong W-bottom. Let $B <_W A$ be a global anticupping witness for A given by (2.3)′. Notice $B <_T A$. Now by Epstein's theorem [9] there is an Δ_2^0 set C such that $C <_T A$ and $C \oplus B \equiv_T A$. Now $A \leq_W C \oplus B$ by choice of A. But then $A \leq_W C$ by choice of B, a contradiction. ∎

As our last example we again modify a construction from [13]

(2.11) THEOREM. *Every strongly contiguous r.e. degree* $\mathbf{a}$ *strongly splits over all lesser* Δ_2^0 *degrees in the sense that if* $\mathbf{b}$ *is a* Δ_2^0 *degree* $< \mathbf{a}$ *then there exist r.e. degrees* $\mathbf{a}_1, \mathbf{a}_2$ *with* $\mathbf{a}_1 \cup \mathbf{a}_2 = \mathbf{a}$ *and* $\mathbf{b} \cup \mathbf{a}_1, \mathbf{b} \cup \mathbf{a}_2 < \mathbf{a}$.

This result follows from:

(2.12) LEMMA. *All r.e. W-degrees strongly split as above over all lesser Δ_2^0 W-degrees.*

Proof. We briefly indicate how to modify the proof from [13] using a marking function $\alpha(e, i, s)$ as in (2.3)'. Let $A = \bigcup_s A_s$ and $B = \lim_s B_s$ be given recursive enumerations with $B <_W A$. We need to construct a r.e. splitting $A = A_0 \cup A_1$ satisfying

$$R_{e,i}\colon \hat{\Phi}_e(B \oplus A_i) \neq A_{i-1}.$$

Now we define $\alpha(e, i, s)$. Let $\alpha(e, i, s) = 0$ and let $\alpha(e, i, s+1)$ be the least y such that one of the following holds:

(i) $y < \alpha(e, i, s)$ and $y \in A_{i-1,s+1} - A_{i-1,s}$.

(ii) $y \geq \alpha(e, i, s)$ and $l(e, i, s) = y$ where

$$l(e, i, s) = \max\{z\colon \forall z < y(\hat{\Phi}_{e,s+1}(B_{s+1} \oplus A_{i,s+1};\ y) = A_{i-1,s+1}(y))\}.$$

(iii) (ii) does not pertain and $y = \alpha(e, i, s)$.

Let $r(e, i, s)$ be $1 + \sum_{z \leq \alpha(e,i,s)} \gamma_{e,s}(z)$.

Now one performs the usual Sacks splitting construction, but with $r(e, i, s)$ in place of the usual Sacks restraints. Then a permitting argument ensures that all the $R_{e,i}$ above are eventually met by a finite restraint (or else $A \leq_W B$). We refer the reader to [13] for further details. ■

The famous nonsplitting result of Lachlan [11] shows that (2.11) fails for arbitrary r.e. **a** (even for **b** r.e.). We do not know if theorem (2.11) is valid if we replace "**a** strongly contiguous" by "**a** low$_2$". The relevant result here is Bickford and Mills' [3] and Harrington's (unpublished) result that all r.e. low$_2$ degrees split over all lesser ones.

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