Contiguity and Distributivity in the Enumerable Turing Degrees-Corrigendum

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1 Introduction

A computably enumerable Turing degree $\mathbf{a}$ is called *contiguous* iff it contains only a single computably enumerable weak truth table degree. (Ladner and Sasso [2]) In [1], the authors proved that a nonzero computably enumerable degree $\mathbf{a}$ is contiguous iff it is *locally distributive*: that is, for all $\mathbf{a_1}, \mathbf{a_2}, \mathbf{c}$ with $\mathbf{a}_1 \cup \mathbf{a}_2 = \mathbf{a}$ and $\mathbf{c} \leq \mathbf{a}$, there exist $\mathbf{c}_i \leq \mathbf{a}_i$ with $\mathbf{c}_1 \cup \mathbf{c}_2 = \mathbf{c}$.

In the proof it was observed that a certain set U of the proof needed only to be Δ^0_2. (Page 1222, paragraph 4) From this it was claimed that a consequence to the proof was that every contiguous computably enumerable degree was, in fact, *strongly contiguous*, in the sense that all (not necessarily computably enumerable) sets of the degree had the same weak truth table degree. From this some corollaries were proven, such as the corollary that no contiguous degree is m-topped. (That is, there is a computably enumerable set A in the degree such that for all computably enumerable sets $B \leq_T A$, $B \leq_m A$.)

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Andre Nies [3] observed that while the claim that the set U need not be computably enumerable *was* correct, the conclusion that the degree was strongly contiguous did *not* directly follow. All that followed was that

deg(A) *was contiguous and had the additional property that for all (not necessarily c.e.) sets* $B \leq_T A$*,* $B \leq_{wtt} A$*.*

While this weaker statement is enough for the corollaries that no m-topped or tt-topped degree is contiguous (same proofs) our claim that all contiguous degrees are strongly contiguous was now unproven, and with this note we wish to withdraw that claim.

we state the following as a conjecture:

Conjecture *Every contiguous computable enumerable degree is strongly contiguous.*

References

[1] Downey, R. and S. Lempp, Contiguity and distributivity in the enumerable degrees, *Journal of Symbolic Logic*, Vol. 62 (1997), 1215-1240.

[2] R. E. Ladner and L. P. Sasso, Jr., The weak truth table degrees of recursively enumerable sets, *Ann. Math. Logic*, **8** (1975), 429-448.

[3] Nies, A., personal communication, January, 2001.