Robust Cooperative Relay Beamforming For Cognitive Radio Networks

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***Abstract*—This paper considers a cognitive radio (CR) relay network which consists of a cognitive source, a cognitive destination and a number of cognitive relay nodes that share spectrum with a primary transmitter and receiver. The cognitive source is unable to communicate directly with the cognitive destination and hence uses cognitive relay nodes to create a link to the cognitive destination. Under the assumption of partial and imperfect channel state information (CSI), we propose a new robust cooperative cognitive relay beamformer that maximises the signal-to-interference-and-noise ratio (SINR) at the cognitive destination subject to a primary receiver outage probability constraint. We show that the robust beamforming problem can be stated as convex semidefinite program (SDP).**

I. Introduction

Cognitive radios (CR) can operate either using the interweave [1] or the underlay [2] schemes. The underlay scheme allows a secondary user (SU) to transmit concurrently with the primary user (PU) provided that the interference at the PU receiver can be maintained below some acceptable level. This is achieved by imposing either an average/peak interference constraint [2, 3], or a minimum signal-to-interference-and-noise ratio (SINR) constraint [4]. The advantage of using the SINR-based scheme is that it allows the SU to optimise its transmissions based on the quality of the primary user transmitter ($\mathrm{PU_{Tx}}$) to the primary user receiver ($\mathrm{PU_{Rx}}$) link.

The use of multiple antennas can result in significant improvements to the performance of underlay CR systems. These performance improvements can also be realised by system employing multiple single antenna relay nodes through a technique known as cooperative relaying [5–8]. Cooperation among geographically distributed relay nodes can be utilised to form a virtual antenna array and provide increased gains in capacity through distributed beamforming. Cooperative beamforming designs in the form of convex semidefinite programs (SDP) were formulated in [6].

The concept of cooperative beamforming has been applied to CR systems in [9, 10]. A CR system typically deploys relay nodes to allow a SU transmitter ($\mathrm{SU_{Tx}}$) to communicate with a distant SU receiver ($\mathrm{SU_{Rx}}$) when the link between the $\mathrm{SU_{Tx}}$ and $\mathrm{SU_{Rx}}$ is poor. While improving the SU performance through beamforming, cooperative beamforming at the relays also enables more control over the interference generated at the $\mathrm{PU_{Rx}}$. The design of cooperative beamformers under the assumption of perfect/full channel state information (CSI) have been studied in [9, 10]. In practical communication systems, this assumption may be over idealistic as perfect CSI for all links is rarely available. These imperfections arise due to many factors, some of which include channel estimation errors, limited CSI feedback and outdated channel estimates. The design of worst-case robust cooperative beamformers that are less susceptible to these imperfections has been investigated in [7]. Unfortunately, solutions obtained through the worst-case approach can be overly conservative because the true probability of worst-case errors may be extremely low [11].

In a CR relay network, the CSI of the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$, $\mathrm{SU_{Tx}}$ to $\mathrm{PU_{Rx}}$ and SU relays ($\mathrm{SU_{Rls}}$) to $\mathrm{PU_{Rx}}$ is generally the most difficult to acquire and some level of cooperation with the PU system may be required. The level of cooperation determines the quality of the CSI that is available to the SU. In this paper, we consider a CR relay network where i) only partial CSI is available for the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$ and $\mathrm{SU_{Tx}}$ to $\mathrm{PU_{Rx}}$ links; ii) the CSI of the $\mathrm{SU_{Rls}}$ to $\mathrm{PU_{Rx}}$ links is imperfect; and iii) perfect CSI is available for all other links. Under the assumption of partial and imperfect CSI, we propose a new robust CR cooperative relay beamformer that maximises the cognitive destination SINR subject to a $\mathrm{PU_{Rx}}$ outage probability constraint. We show that beamforming problem can be transformed into a convex SDP. In [12], the interference generated by $\mathrm{SU_{Tx}}$ at the $\mathrm{PU_{Rx}}$ was ignored. This paper extends the work of [12] by including this interference into the beamformer design problems.

The performance resulting from the optimisation problems outlined above is demonstrated by means of capacity cumulative distribution functions (CDFs) in flat Rayleigh channels.

In this paper, we assume both i) the proposed optimisation problems are solved by a central SU processing unit; and ii) a dedicated link, such as that in a distributed antenna system [13], between this central SU processing unit and each relay node exists.

Notation: Upper (lower) bold face letters are used for matrices (vectors); $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, $\mathbb{E}\{\cdot\}$ and $\|\cdot\|$ denote complex conjugate, transpose, Hermitian transpose, expectation and Euclidean norm, respectively. $|\cdot|^2$ denotes the magnitude squared operator for scalars and element-wise magnitude squared for vectors. $\mathrm{tr}(\cdot)$ and $\odot$ denote the matrix trace operator and element-wise product between vectors, respectively. $\mathbf{W} \succeq 0$ denotes that $\mathbf{W}$ is a positive semidefinite matrix. $\mathbf{x} \sim \mathcal{N}_\mathcal{C}(\mathbf{m}, \mathbf{\Sigma})$ states that $\mathbf{x}$ contains entries of complex Gaussian random variables, with mean $\mathbf{m}$ and covariance $\mathbf{\Sigma}$.

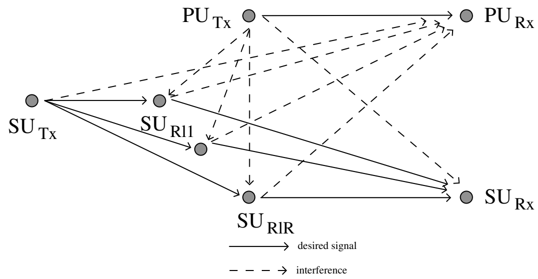

Fig. 1. System Model

II. System Model

Consider a underlay CR relay network which consists of a $\mathrm{SU_{Tx}}$, a $\mathrm{SU_{Rx}}$, R secondary relay ($\mathrm{SU_{Rl}}$) nodes and a $\mathrm{PU_{Tx}}$ and $\mathrm{PU_{Rx}}$ pair, as shown in Fig. 1. We assume that due to poor channel conditions between the $\mathrm{SU_{Tx}}$ and $\mathrm{SU_{Rx}}$, there is no reliable link between them. Hence, the $\mathrm{SU_{Tx}}$ employs the $\mathrm{SU_{Rls}}$ to communicate with the $\mathrm{SU_{Rx}}$. Since the PU and SU systems transmit concurrently, the $\mathrm{PU_{Rx}}$ experiences interference from the $\mathrm{SU_{Tx}}$ and the $\mathrm{SU_{Rl}}$ transmissions, and both $\mathrm{SU_{Rl}}$ and $\mathrm{SU_{Rx}}$ experience interference from the $\mathrm{PU_{Tx}}$ transmissions.

All links in the network are assumed to be independent, point-to-point, flat Rayleigh fading channels. The channel coefficients of the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$, $\mathrm{PU_{Tx}}$ to $\mathrm{SU_{Rl}}$ i, $\mathrm{PU_{Tx}}$ to $\mathrm{SU_{Rx}}$, $\mathrm{SU_{Tx}}$ to $\mathrm{SU_{Rl}}$ i, $\mathrm{SU_{Tx}}$ to $\mathrm{PU_{Rx}}$, $\mathrm{SU_{Rl}}$ i to $\mathrm{SU_{Rx}}$ and $\mathrm{SU_{Rl}}$ i to $\mathrm{PU_{Rx}}$ links are denoted by h_{pp}, $h_{\mathrm{pr}}^{(i)}$, h_{ps}, $h_{\mathrm{sr}}^{(i)}$, h_{sp}, $h_{\mathrm{rs}}^{(i)}$ and $h_{\mathrm{rp}}^{(i)}$, respectively. The instantaneous channel powers of these links are represented by $g_{\mathrm{pp}} = |h_{\mathrm{pp}}|^2$, $g_{\mathrm{pr}}^{(i)} = |h_{\mathrm{pr}}^{(i)}|^2$, $g_{\mathrm{ps}} = |h_{\mathrm{ps}}|^2$, $g_{\mathrm{sr}}^{(i)} = |h_{\mathrm{sr}}^{(i)}|^2$, $g_{\mathrm{sp}} = |h_{\mathrm{sp}}|^2$, $g_{\mathrm{rs}}^{(i)} = |h_{\mathrm{rs}}^{(i)}|^2$ and $g_{\mathrm{rp}}^{(i)} = |h_{\mathrm{rp}}^{(i)}|^2$ and have the means: $\Omega_{\mathrm{pp}} = \mathbb{E}\{g_{\mathrm{pp}}\}$, $\Omega_{\mathrm{pr}}^{(i)} = \mathbb{E}\{g_{\mathrm{pr}}^{(i)}\}$, $\Omega_{\mathrm{ps}} = \mathbb{E}\{g_{\mathrm{ps}}\}$, $\Omega_{\mathrm{sr}}^{(i)} = \mathbb{E}\{g_{\mathrm{sr}}^{(i)}\}$, $\Omega_{\mathrm{sp}} = \mathbb{E}\{g_{\mathrm{sp}}\}$, $\Omega_{\mathrm{rs}}^{(i)} = \mathbb{E}\{g_{\mathrm{rs}}^{(i)}\}$ and $\Omega_{\mathrm{rp}}^{(i)} = \mathbb{E}\{g_{\mathrm{rp}}^{(i)}\}$.

A two-step amplify-and-forward (AF) protocol is adopted in this work. During the first step, the $\mathrm{SU_{Tx}}$ broadcasts the signal $\sqrt{P_{\mathrm{s}}}s_{\mathrm{s}}$ to the relays, where P_{s} is the $\mathrm{SU_{Tx}}$ transmit power and s_{s} the information symbol. Simultaneously, the $\mathrm{PU_{Tx}}$ transmits the signal $\sqrt{P_{\mathrm{p}}}s_{\mathrm{p}}^{(1)}$, where P_{p} is the $\mathrm{PU_{Tx}}$ transmit power and $s_{\mathrm{p}}^{(1)}$ the information symbol. We assume that $\mathbb{E}\{|s_{\mathrm{s}}|^2\} = \mathbb{E}\{|s_{\mathrm{p}}^{(1)}|^2\} = 1$. The signal received at the ith relay is given by

$$x_i = \underbrace{\sqrt{P_{\mathrm{s}}}s_{\mathrm{s}}h_{\mathrm{sr}}^{*(i)}}_{\text{wanted signal}} + \underbrace{\sqrt{P_{\mathrm{p}}}s_{\mathrm{p}}^{(1)}h_{\mathrm{pr}}^{*(i)} + n_{\mathrm{r}}^{(i)}}_{\text{interference + noise}}, \tag{1}$$

and that at the $\mathrm{PU_{Rx}}$ by

$$z_p^{(1)} = \underbrace{\sqrt{P_{\mathrm{p}}}s_{\mathrm{p}}^{(1)}h_{\mathrm{pp}}^{(1)}}_{\text{wanted signal}} + \underbrace{\sqrt{P_{\mathrm{s}}}s_{\mathrm{s}}h_{\mathrm{sp}} + n_{\mathrm{p}}}_{\text{interference + noise}}, \tag{2}$$

where $h_{\mathrm{pp}}^{(1)}$ is the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$ channel coefficient in the first transmission step and $n_{\mathrm{r}}^{(i)}$ and n_{p} are the additive white Gaussian noise (AWGN) with powers σ_{r}^2 and σ_{p}^2 at the ith relay and the $\mathrm{PU_{Rx}}$, respectively.

During the second step, the ith relay transmits the signal

$$\begin{aligned} y_i &= x_i w_i \\ &= \sqrt{P_{\mathrm{s}}}s_{\mathrm{s}}h_{\mathrm{sr}}^{*(i)}w_i + \sqrt{P_{\mathrm{p}}}s_{\mathrm{p}}^{(1)}h_{\mathrm{pr}}^{*(i)}w_i + n_{\mathrm{r}}^{(i)}w_i, \end{aligned} \tag{3}$$

where w_i is the complex beamforming weight applied by the ith relay. During this time, the $\mathrm{PU_{Tx}}$ transmits the signal $\sqrt{P_{\mathrm{p}}}s_{\mathrm{p}}^{(2)}$, where $s_{\mathrm{p}}^{(2)}$ is the information symbol and is assumed to be different to that transmitted in the first step. We assume that $\mathbb{E}\{|s_{\mathrm{p}}^{(2)}|^2\} = 1$. At the $\mathrm{SU_{Rx}}$, the received signal can be expressed as

$$\begin{aligned} z_s &= \sum_{i=1}^{R} y_i h_{\mathrm{rs}}^{*(i)} + \sqrt{P_{\mathrm{p}}}s_{\mathrm{p}}^{(2)}h_{\mathrm{ps}}^* \\ &= \underbrace{\sqrt{P_{\mathrm{s}}}s_{\mathrm{s}}[\mathbf{h}_{\mathrm{sr}} \odot \mathbf{h}_{\mathrm{rs}}]^H\mathbf{w}}_{\text{wanted signal}} + \underbrace{[\mathbf{n}_{\mathrm{r}} \odot \mathbf{h}_{\mathrm{rs}}]^H\mathbf{w} + n_{\mathrm{s}}}_{\text{noise}} \\ &\quad + \underbrace{\sqrt{P_{\mathrm{p}}}s_{\mathrm{p}}^{(2)}h_{\mathrm{ps}}^* + \sqrt{P_{\mathrm{p}}}s_{\mathrm{p}}^{(1)}[\mathbf{h}_{\mathrm{pr}} \odot \mathbf{h}_{\mathrm{rs}}]^H\mathbf{w}}_{\text{interference}}, \end{aligned} \tag{4}$$

and that at the $\mathrm{PU_{Rx}}$ as

$$\begin{aligned} z_p^{(2)} &= \sqrt{P_{\mathrm{p}}}s_{\mathrm{p}}^{(2)}h_{\mathrm{pp}}^{(2)} + \sum_{i=1}^{R} y_i h_{\mathrm{rp}}^{(i)} \\ &= \underbrace{\sqrt{P_{\mathrm{p}}}s_{\mathrm{p}}^{(2)}h_{\mathrm{pp}}}_{\text{wanted signal}} + \underbrace{[\mathbf{n}_{\mathrm{r}} \odot \mathbf{h}_{\mathrm{rp}}]^H\mathbf{w} + n_{\mathrm{p}}}_{\text{noise}} \\ &\quad + \underbrace{\sqrt{P_{\mathrm{s}}}s_{\mathrm{s}}[\mathbf{h}_{\mathrm{sr}} \odot \mathbf{h}_{\mathrm{rp}}]^H\mathbf{w}}_{\text{SU interference}} + \underbrace{\sqrt{P_{\mathrm{p}}}s_{\mathrm{p}}^{(1)}[\mathbf{h}_{\mathrm{pr}} \odot \mathbf{h}_{\mathrm{rp}}]^H\mathbf{w}}_{\text{self interference}}, \end{aligned} \tag{5}$$

where $h_{\mathrm{pp}}^{(2)}$ is the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$ channel coefficient in the second transmission step, $\mathbf{h}_{\mathrm{sr}} \triangleq [h_{\mathrm{sr}}^{(1)}\ h_{\mathrm{sr}}^{(2)} \dots h_{\mathrm{sr}}^{(R)}]^T$, $\mathbf{h}_{\mathrm{rs}} \triangleq [h_{\mathrm{rs}}^{(1)}\ h_{\mathrm{rs}}^{(2)} \dots h_{\mathrm{rs}}^{(R)}]^T$, $\mathbf{h}_{\mathrm{pr}} \triangleq [h_{\mathrm{pr}}^{(1)}\ h_{\mathrm{pr}}^{(2)} \dots h_{\mathrm{pr}}^{(R)}]^T$, $\mathbf{h}_{\mathrm{rp}} \triangleq [h_{\mathrm{rp}}^{(1)}\ h_{\mathrm{rp}}^{(2)} \dots h_{\mathrm{rp}}^{(R)}]^T$, $\mathbf{w} \triangleq [w_1\ w_2\ \dots\ w_R]^T$, $\mathbf{n}_{\mathrm{r}} \triangleq [n_r^{(1)}\ n_r^{(2)}\ \dots\ n_r^{(R)}]^T$ and n_{s} is AWGN with power σ_{s}^2 at the $\mathrm{SU_{Rx}}$. Note that the relays also retransmit the PU's signal, hence, the $\mathrm{PU_{Rx}}$ also receives the $\mathrm{PU_{Tx}}$ symbol from the first step, which is treated as self interference in our analysis.

By assuming that s_{s}, $s_{\mathrm{p}}^{(1)}$, $s_{\mathrm{p}}^{(2)}$, $n_r^{(i)}$ $\forall i$, n_{s} and n_{p} are all uncorrelated from each other and perfect CSI is available, and therefore considering the channel coefficients as deterministic constants, the ith relay's transmit power is given by

$$P_{Rl}^{(i)} = \mathbf{E}_{ii}|w_i|^2, \tag{6}$$

where $\mathbf{E} = P_{\mathrm{s}}\mathrm{diag}\left(|\mathbf{h}_{\mathrm{sr}}|^2\right) + P_{\mathrm{p}}\mathrm{diag}\left(|\mathbf{h}_{\mathrm{pr}}|^2\right) + \sigma_{\mathrm{r}}^2\mathrm{I}$. The SINR at the $\mathrm{SU_{Rx}}$ is expressed as

$$\begin{aligned} \gamma_{\mathrm{s}} &= \frac{P_{\mathrm{s}}\left|[\mathbf{h}_{\mathrm{sr}} \odot \mathbf{h}_{\mathrm{rs}}]^H\mathbf{w}\right|^2}{P_{\mathrm{p}}\left|h_{\mathrm{ps}}\right|^2 + P_{\mathrm{p}}\left|[\mathbf{h}_{\mathrm{pr}} \odot \mathbf{h}_{\mathrm{rs}}]^H\mathbf{w}\right|^2 + \sigma_{\mathrm{r}}^2\left\|\mathbf{h}_{\mathrm{rs}} \odot \mathbf{w}\right\|^2 + \sigma_{\mathrm{s}}^2} \\ &= \frac{\mathbf{w}^H\mathbf{Q}\mathbf{w}}{P_{\mathrm{p}}|h_{\mathrm{ps}}|^2 + \mathbf{w}^H(\mathbf{R} + \mathbf{V})\mathbf{w} + \sigma_{\mathrm{s}}^2}, \end{aligned} \tag{7}$$

where $\mathbf{Q} = P_{\mathrm{s}}[\mathbf{h}_{\mathrm{sr}} \odot \mathbf{h}_{\mathrm{rs}}][\mathbf{h}_{\mathrm{sr}} \odot \mathbf{h}_{\mathrm{rs}}]^H$, $\mathbf{R} = P_{\mathrm{p}}[\mathbf{h}_{\mathrm{pr}} \odot \mathbf{h}_{\mathrm{rs}}][\mathbf{h}_{\mathrm{pr}} \odot \mathbf{h}_{\mathrm{rs}}]^H$ and $\mathbf{V} = \sigma_{\mathrm{r}}^2\mathrm{diag}\left(|\mathbf{h}_{\mathrm{rs}}|^2\right)$.

The SINR at the $\mathrm{PU_{Rx}}$ in the first step can be expressed as

$$\gamma_{\mathrm{p}}^{(1)} = \frac{P_{\mathrm{p}}|h_{\mathrm{pp}}^{(1)}|^2}{P_{\mathrm{s}}|h_{\mathrm{sp}}|^2 + \sigma_{\mathrm{p}}^2}, \tag{8}$$

and using the following definition

$$\begin{aligned} I_p \triangleq\ & P_{\mathrm{s}}\left|[\mathbf{h}_{\mathrm{sr}} \odot \mathbf{h}_{\mathrm{rp}}]^H\mathbf{w}\right|^2 + P_{\mathrm{p}}\left|[\mathbf{h}_{\mathrm{pr}} \odot \mathbf{h}_{\mathrm{rp}}]^H\mathbf{w}\right|^2 \\ & + \sigma_{\mathrm{r}}^2\left\|\mathbf{h}_{\mathrm{rp}} \odot \mathbf{w}\right\|^2, \end{aligned}$$

the SINR in the second step can be expressed as

$$\gamma_{\mathrm{p}}^{(2)} = \frac{P_{\mathrm{p}}|h_{\mathrm{pp}}^{(2)}|^2}{I_p + \sigma_{\mathrm{p}}^2} = \frac{P_{\mathrm{p}}|h_{\mathrm{pp}}^{(2)}|^2}{\mathbf{w}^H(\mathbf{B}+\mathbf{C}+\mathbf{D})\mathbf{w} + \sigma_{\mathrm{p}}^2}, \tag{9}$$

where $\mathbf{B} = P_{\mathrm{s}}[\mathbf{h}_{\mathrm{sr}} \odot \mathbf{h}_{\mathrm{rp}}][\mathbf{h}_{\mathrm{sr}} \odot \mathbf{h}_{\mathrm{rp}}]^H$, $\mathbf{C} = P_{\mathrm{p}}[\mathbf{h}_{\mathrm{pr}} \odot \mathbf{h}_{\mathrm{rp}}][\mathbf{h}_{\mathrm{pr}} \odot \mathbf{h}_{\mathrm{rp}}]^H$ and $\mathbf{D} = \sigma_{\mathrm{r}}^2 \mathrm{diag}\left(|\mathbf{h}_{\mathrm{rp}}|^2\right)$.

To guarantee a certain level of quality-of-service (QoS) to the primary user, in our beamformer design formulations under the assumption of perfect CSI, we impose $\mathrm{PU_{Rx}}$ instantaneous SINR constraints in both transmission steps, i.e., $\gamma_{\mathrm{p}}^{(1)} \geq \gamma_{\mathrm{T}}$ and $\gamma_{\mathrm{p}}^{(2)} \geq \gamma_{\mathrm{T}}$. These constraints are transformed into a probability based constraint in Section IV.

III. Beamformer Optimisation Under Full CSI

In this section, under the assumption of the availability of full CSI for all links at the SU system, we find the optimum beamforming weight vector, $\mathbf{w}$, that maximises the $\mathrm{SU_{Rx}}$ SINR subject to the $\mathrm{PU_{Rx}}$ QoS constraint and an individual maximum transmit power constraint, $P_{\mathrm{Rl,max}}^{(i)}$, on each relay node. In practice, the relay power constraint may be due either to regulatory or hardware limitations.

We assume that we are unable to control the PU's transmit power and the PU transmits at a constant power of P_{p}. The assumption of perfect CSI for all links allows us to obtain fundamental limits on performance. However, in practice, the channel would need to be estimated, hence the performance results obtained in this section provide an upper bound. In Section IV, we consider the case when perfect CSI is not available.

Since the $\mathrm{PU_{Rx}}$'s QoS requirement needs to be guaranteed in both transmission steps, the beamforming problem stated above is preceded by a $\mathrm{SU_{Tx}}$ power control stage whereby the optimum $\mathrm{SU_{Tx}}$ transmit power, P_{s}, is chosen. Note that it is not possible to have a joint power control and beamformer design because we assume that the channels are not static from the first transmission step to the second. Using (8), it is easily shown that the optimum transmit power is given by

$$P_{\mathrm{s}} = \begin{cases} 0 & \frac{P_{\mathrm{p}}|h_{\mathrm{pp}}^{(1)}|^2}{\gamma_{\mathrm{T}}} < \sigma_{\mathrm{p}}^2, \\ \min\left(\frac{P_{\mathrm{p}}|h_{\mathrm{pp}}^{(1)}|^2}{\gamma_{\mathrm{T}}|h_{\mathrm{sp}}|^2} - \frac{\sigma_{\mathrm{p}}^2}{|h_{\mathrm{sp}}|^2}, P_{\mathrm{s,max}}\right) & \text{otherwise} \end{cases} \tag{10}$$

where $P_{\mathrm{s,max}}$ is the maximum allowed $\mathrm{SU_{Tx}}$ transmit power. This choice of P_{s} maximises the SINR at the relay nodes while ensuring that the $\mathrm{PU_{Rx}}$ SINR constraint is satisfied. P_{s} is then used in the cooperative beamformer design which is described next.

The $\mathrm{SU_{Rx}}$ SINR maximisation problem is expressed as

$$\max_{\mathbf{w}} \quad \frac{\mathbf{w}^H\mathbf{Q}\mathbf{w}}{\mathbf{w}^H\left(\mathbf{R}+\mathbf{V}\right)\mathbf{w} + P_{\mathrm{p}}|h_{\mathrm{ps}}|^2 + \sigma_{\mathrm{s}}^2} \tag{11a}$$

$$\text{s.t.} \quad \mathbf{E}_{ii}|w_i|^2 \leq P_{\mathrm{Rl,max}}^{(i)}, \quad i = 1 \ldots R \tag{11b}$$

$$\mathbf{w}^H\gamma_{\mathrm{T}}\left(\mathbf{B}+\mathbf{C}+\mathbf{D}\right)\mathbf{w} + \gamma_{\mathrm{T}}\sigma_{\mathrm{p}}^2 - P_{\mathrm{p}}|h_{\mathrm{pp}}^{(2)}|^2 \leq 0 \tag{11c}$$

Problem (11) is a nonconvex optimisation problem; however, it can be transformed into an optimisation problem which has the structure of a linear-fractional program [14]. Using the definition $\mathbf{W} \triangleq \mathbf{w}\mathbf{w}^H$, problem (11) can be restated as

$$\max_{\mathbf{W}} \quad \frac{\mathrm{tr}\left(\mathbf{Q}\mathbf{W}\right)}{\mathrm{tr}\left(\left(\mathbf{R}+\mathbf{V}\right)\mathbf{W}\right) + P_{\mathrm{p}}|h_{\mathrm{ps}}|^2 + \sigma_{\mathrm{s}}^2} \tag{12a}$$

$$\text{s.t.} \quad \mathbf{E}_{ii}\mathbf{W}_{ii} \leq P_{\mathrm{Rl,max}}^{(i)}, \quad i = 1 \ldots R \tag{12b}$$

$$\gamma_{\mathrm{T}}\,\mathrm{tr}\left(\left(\mathbf{B}+\mathbf{C}+\mathbf{D}\right)\mathbf{W}\right) + \gamma_{\mathrm{T}}\sigma_{\mathrm{p}}^2 - P_{\mathrm{p}}|h_{\mathrm{pp}}^{(2)}|^2 \leq 0 \tag{12c}$$

$$\mathbf{W} \succeq 0 \tag{12d}$$

$$\mathrm{rank}\left(\mathbf{W}\right) = 1 \tag{12e}$$

Due to the rank constraint (12e) being a nonconvex constraint, problem (12) is a nonconvex optimisation problem. Application of semidefinite relaxation (SDR) [14] allows the problem to be relaxed into a convex optimisation problem, i.e., removing the rank constraint. The Charnes-Cooper transformation [14] can be used to solve the relaxed form of problem (12). By defining the pair

$$\tilde{\mathbf{W}} = \frac{\mathbf{W}}{\mathrm{tr}\left(\left(\mathbf{R}+\mathbf{V}\right)\mathbf{W}\right) + P_{\mathrm{p}}|h_{\mathrm{ps}}|^2 + \sigma_{\mathrm{s}}^2},$$

$$t = \frac{1}{\mathrm{tr}\left(\left(\mathbf{R}+\mathbf{V}\right)\mathbf{W}\right) + P_{\mathrm{p}}|h_{\mathrm{ps}}|^2 + \sigma_{\mathrm{s}}^2},$$

the relaxed form of problem (12) can be stated as

$$\max_{\tilde{\mathbf{W}},t} \quad \mathrm{tr}\left(\mathbf{Q}\tilde{\mathbf{W}}\right) \tag{13a}$$

$$\text{s.t.} \quad \mathbf{E}_{ii}\tilde{\mathbf{W}}_{ii} \leq tP_{\mathrm{Rl,max}}^{(i)}, \quad i = 1 \ldots R \tag{13b}$$

$$\gamma_{\mathrm{T}}\,\mathrm{tr}\left(\left(\mathbf{B}+\mathbf{C}+\mathbf{D}\right)\tilde{\mathbf{W}}\right) + t(\gamma_{\mathrm{T}}\sigma_{\mathrm{p}}^2 - P_{\mathrm{p}}|h_{\mathrm{pp}}^{(2)}|^2) \leq 0 \tag{13c}$$

$$\tilde{\mathbf{W}} \succeq 0 \tag{13d}$$

$$\mathrm{tr}\left(\left(\mathbf{R}+\mathbf{V}\right)\tilde{\mathbf{W}}\right) + t(P_{\mathrm{p}}|h_{\mathrm{ps}}|^2 + \sigma_{\mathrm{s}}^2) = 1 \tag{13e}$$

$$t \geq 0 \tag{13f}$$

Problem (13) is a convex optimisation problem and can be solved using interior point methods. After solving this problem, the beamforming matrix is recovered as $\mathbf{W} = \tilde{\mathbf{W}}/t$. If $\mathbf{W}$ is rank-one, then the optimum beamforming vector, $\mathbf{w}^*$, can be chosen to be the principle eigenvector of $\mathbf{W}$. The Gaussian randomisation technique [15] can be used to recover a good rank-one approximation when the rank is higher than one. Similar to other works on beamforming (see, for example [6]), in our extensive numerical simulations, we have never encountered a solution that had a rank higher than one.

IV. Robust Beamformer Design

In practise, perfect CSI for all links is seldom available and the assumption of perfect CSI may be unrealistic. For our analysis, we assume that the channels for the $\mathrm{SU_{Tx}}$ to $\mathrm{SU_{Rl}}$ and $\mathrm{SU_{Rl}}$ to $\mathrm{SU_{Rx}}$ links are accurately known through the SU's channel estimation procedure and those between the $\mathrm{PU_{Tx}}$ and $\mathrm{SU_{Rl}}$ can be accurately measured, for example, through knowledge of the PU pilot symbols. We assume that only partial CSI in the form of mean channel powers for the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$ and $\mathrm{SU_{Tx}}$ to $\mathrm{PU_{Rx}}$ links is available, i.e., only Ω_{pp} and Ω_{sp} are known. Furthermore, we assume that an imperfect CSI estimate of the $\mathrm{SU_{Rl}}$ to $\mathrm{PU_{Rx}}$ link is available.

Our robust formulations are based on the PU outage probability. The PU is in outage when the SINR at the PU, $\gamma_{\rm p}$, falls below the PU SINR threshold, $\gamma_{\rm T}$. The aim is to constrain this outage probability to be less than or equal to some maximum allowable probability, $\mathrm{P}_{\rm o,max}$. The outage probability in the first transmission step is expressed as

$$\mathrm{P_o}^{(1)} = \Pr\left\{P_{\rm p}|h_{\rm pp}^{(1)}|^2 - \gamma_{\rm T}P_{\rm s}|h_{\rm sp}|^2 \leq \gamma_{\rm T}\sigma_{\rm p}^2\right\}. \quad (14)$$

The PDF in (14) is that of a difference between two independent exponential random variables and can be shown to have the following form

$$f(\psi) = \begin{cases} \frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}\exp\left(-\lambda_1\psi\right) & \text{if } \psi \geq 0 \\ \frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}\exp\left(\lambda_2\psi\right) & \text{if } \psi < 0 \end{cases} \quad (15)$$

where $\lambda_1 = 1/(P_{\rm p}\Omega_{\rm pp})$ and $\lambda_2 = 1/(\gamma_{\rm T}P_{\rm s}\Omega_{\rm sp})$. Using (15) and utilising the fact that $\gamma_{\rm T}\sigma_{\rm p}^2 \geq 0$, (14) can be rewritten as

$$\begin{aligned}\mathrm{P_o}^{(1)} &= 1 - \int_{\gamma_{\rm T}\sigma_{\rm p}^2}^{\infty}\frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}\exp\left(-\lambda_1\psi\right)\mathrm{d}\psi \\ &= 1 - \frac{P_{\rm p}\Omega_{\rm pp}}{\gamma_{\rm T}P_{\rm s}\Omega_{\rm sp}+P_{\rm p}\Omega_{\rm pp}}\exp\left(-\frac{\gamma_{\rm T}\sigma_{\rm p}^2}{P_{\rm p}\Omega_{\rm pp}}\right). \quad (16)\end{aligned}$$

We note that the optimum $\mathrm{SU_{Tx}}$ transmit power will satisfy the outage probability constraint with equality. Hence, using (16) the robust power allocation for the $\mathrm{SU_{Tx}}$ is given by

$$P_{\rm s} = \begin{cases} 0 & \frac{P_{\rm p}\Omega_{\rm pp}}{\gamma_{\rm T}} < -\frac{\sigma_{\rm p}^2}{\log\left(1-\mathrm{P_{o,max}}\right)}, \\ \min\left(P_t, P_{\rm s,max}\right) & \text{otherwise} \end{cases} \quad (17)$$

where P_t is given by

$$P_t = \frac{P_{\rm p}\Omega_{\rm pp}\left(\exp\left(-\frac{\gamma_{\rm T}\sigma_{\rm p}^2}{P_{\rm p}\Omega_{\rm pp}}\right) - (1-\mathrm{P_{o,max}})\right)}{\gamma_{\rm T}\Omega_{\rm sp}(1-\mathrm{P_{o,max}})} \quad (18)$$

Observe that this robust power allocation is based entirely on deterministic constants and only needs to be performed when one of these constants change.

We model the imperfect $\mathrm{SU_{Rl}}$ to $\mathrm{PU_{Rx}}$ link CSI estimate as [16]

$$\tilde{\mathbf{h}}_{\rm rp} = \sqrt{(1-\rho^2)}\mathbf{h}_{\rm rp} + \rho\mathbf{e}, \quad (19)$$

where $\tilde{\mathbf{h}}_{\rm rp}$ is the imperfect $\mathrm{SU_{Rl}}$ to $\mathrm{PU_{Rx}}$ link CSI estimate and $\mathbf{e}$ is the zero mean estimation error vector with independently distributed complex Gaussian entries and the diagonal covariance matrix $\mathbf{\Sigma}_{\rm e} = \mathrm{diag}(\mathbf{\Omega}_{\rm rp})$, i.e., $\mathbf{e} \sim \mathcal{N}_{\mathcal{C}}(\mathbf{0}, \mathbf{\Sigma}_{\rm e})$. $0 \leq \rho \leq 1$ determines the quality of the CSI, which is perfect when $\rho = 0$ and has maximum uncertainty when $\rho = 1$.

The PU outage probability in the second transmission step is given by

$$\mathrm{P_o}^{(2)} = \Pr\left\{\frac{P_{\rm p}|h_{\rm pp}|^2}{\mathbf{w}^H(\mathbf{B}+\mathbf{C}+\mathbf{D})\mathbf{w}+\sigma_{\rm p}^2} \leq \gamma_{\rm T}\right\}. \quad (20)$$

Using (19), $\gamma_{\rm T}\mathbf{w}^H(\mathbf{B}+\mathbf{C}+\mathbf{D})\mathbf{w}$ can be expressed as

$$\begin{aligned}\gamma_{\rm T}\mathbf{w}^H(\mathbf{B}+\mathbf{C}+\mathbf{D})\mathbf{w} = \frac{\gamma_{\rm T}}{1-\rho^2}\Big(\\ -2P_{\rm s}\rho\Re\{\mathbf{w}^H[\mathbf{h}_{\rm sr}\odot\tilde{\mathbf{h}}_{\rm rp}][\mathbf{h}_{\rm sr}\odot\mathbf{e}]^H\mathbf{w}\} \\ -2P_{\rm p}\rho\Re\{\mathbf{w}^H[\mathbf{h}_{\rm pr}\odot\tilde{\mathbf{h}}_{\rm rp}][\mathbf{h}_{\rm pr}\odot\mathbf{e}]^H\mathbf{w}\} \\ -2\sigma_{\rm r}^2\rho\Re\{\mathbf{w}^H\mathrm{diag}((\tilde{\mathbf{h}}_{\rm rp}^H)^T\odot\mathbf{e})\mathbf{w}\} \\ +P_{\rm s}\rho^2\mathbf{w}^H[\mathbf{h}_{\rm sr}\odot\mathbf{e}][\mathbf{h}_{\rm sr}\odot\mathbf{e}]^H\mathbf{w} \\ +P_{\rm p}\rho^2\mathbf{w}^H[\mathbf{h}_{\rm pr}\odot\mathbf{e}][\mathbf{h}_{\rm pr}\odot\mathbf{e}]^H\mathbf{w} \\ +\sigma_{\rm r}^2\rho^2\mathbf{w}^H\mathrm{diag}(|\mathbf{e}|^2)\mathbf{w} \\ +\mathbf{w}^H\Big[P_{\rm s}[\mathbf{h}_{\rm sr}\odot\tilde{\mathbf{h}}_{\rm rp}][\mathbf{h}_{\rm sr}\odot\tilde{\mathbf{h}}_{\rm rp}]^H \\ +P_{\rm p}[\mathbf{h}_{\rm pr}\odot\tilde{\mathbf{h}}_{\rm rp}][\mathbf{h}_{\rm pr}\odot\tilde{\mathbf{h}}_{\rm rp}]^H + \sigma_{\rm r}^2\mathrm{diag}(|\tilde{\mathbf{h}}_{\rm rp}|^2)\Big]\mathbf{w}\Big)\end{aligned} \quad (21)$$

In the interest of brevity, the terms on the right hand side of (21) are denoted by $r_1, r_2, \ldots, r_7$. Due to the correlation between the terms of (21), its exact PDF is difficult to handle. However, we propose an accurate approximation of the PDF which is easier to handle based on the following observation. In a practical wireless receiver, the noise variance, $\sigma_{\rm r}^2$, is generally very small, for instance, a receiver with a 2 MHz bandwidth and a noise figure (NF) of 30 dB operating at a room temperature of 293 K has an effective noise power of approximately -80 dBm. Hence, random variables that contain the term $\sigma_{\rm r}^2$, namely r_3 and r_6, can be safely ignored.

It is easily shown that r_1 and r_2 are zero mean Gaussian random variables with variances, σ_1^2 and σ_2^2 given by

$$\begin{aligned}\sigma_1^2 = 2\gamma_{\rm T}^2P_{\rm s}^2\,\mathrm{tr}\left((\mathbf{h}_{\rm sr}\mathbf{h}_{\rm sr}^H\odot(\rho/(1-\rho^2))^2\mathbf{\Sigma}_{\rm e})\mathbf{W}\right) \\ \mathrm{tr}\left([\mathbf{h}_{\rm sr}\odot\tilde{\mathbf{h}}_{\rm rp}][\mathbf{h}_{\rm sr}\odot\tilde{\mathbf{h}}_{\rm rp}]^H\mathbf{W}\right) \quad (22) \\ \sigma_2^2 = 2\gamma_{\rm T}^2P_{\rm p}^2\,\mathrm{tr}\left((\mathbf{h}_{\rm pr}\mathbf{h}_{\rm pr}^H\odot(\rho/(1-\rho^2))^2\mathbf{\Sigma}_{\rm e})\mathbf{W}\right) \\ \mathrm{tr}\left([\mathbf{h}_{\rm pr}\odot\tilde{\mathbf{h}}_{\rm rp}][\mathbf{h}_{\rm pr}\odot\tilde{\mathbf{h}}_{\rm rp}]^H\mathbf{W}\right), \quad (23)\end{aligned}$$

where $\mathbf{W} = \mathbf{w}\mathbf{w}^H$.

Using [17, Lemma 1], r_4 and r_5 are recognised as exponentially distributed random variables with means given by

$$\mu_4 = \gamma_{\rm T}P_{\rm s}\,\mathrm{tr}\left((\mathbf{h}_{\rm sr}\mathbf{h}_{\rm sr}^H\odot(\rho^2/(1-\rho^2))\mathbf{\Sigma}_{\rm e})\mathbf{W}\right), \quad (24)$$

$$\mu_5 = \gamma_{\rm T}P_{\rm p}\,\mathrm{tr}\left((\mathbf{h}_{\rm pr}\mathbf{h}_{\rm pr}^H\odot(\rho^2/(1-\rho^2))\mathbf{\Sigma}_{\rm e})\mathbf{W}\right). \quad (25)$$

r_7 is a deterministic constant.

In a practical cognitive radio system, the PU requires a very reliable link, hence the outage probability specified will generally be very small. In order to satisfy the stringent outage probability constraint, both σ_1^2 and σ_2^2 must also be small. Notice that the expression for σ_1^2 contains the term $P_{\rm s}\mathrm{tr}\left((\mathbf{h}_{\rm sr}\mathbf{h}_{\rm sr}^H\odot(\rho/(1-\rho^2))^2\mathbf{\Sigma}_{\rm e})\mathbf{W}\right)$, which can be rewritten as $P_{\rm s}\sum_{i=1}^{R}(\rho/(1-\rho^2))^2\mathbf{\Sigma}_{{\rm e}_{ii}}|h_{\rm sr}^{(i)}|^2\mathbf{W}_{ii}$. This term represents the SU interference that is generated at the $\mathrm{PU_{Rx}}$ due to CSI errors, and its level can only be controlled by adjusting the beamformer transmit power. Hence, as the $\mathrm{SU_{Tx}}$ to $\mathrm{SU_{Rl}}$ link gets stronger, the beamformer weights will be scaled down in order to achieve the outage probability constraint. Note that this term also appears in μ_4, which is used in our final approximation, (26), of the PU outage probability constraint and its magnitude is controlled by controlling the magnitude of

μ_4. We note that the beamformer is able to control interference from the $P_{\rm s}{\rm tr}([\mathbf{h}_{\rm sr}\odot\tilde{\mathbf{h}}_{\rm rp}][\mathbf{h}_{\rm sr}\odot\tilde{\mathbf{h}}_{\rm rp}]^H\mathbf{W})$ part of σ_1^2 through both amplitude and phase control and is able to keep it sufficiently low to satisfy the outage probability constraint. Again, note that this term appears in the deterministic constant r_7, which is used in (26). Hence, the magnitude of this term is controlled by controlling the magnitude of r_7.

The individual relay transmit power constraints in the beamforming problem (11) also limit the beamformer weight magnitudes, which in turn limit the levels of σ_1^2 and σ_2^2. From the definition of $\mathbf{E}$ and (11b), we see that for a fixed value of $P^{(i)}_{\rm Rl,max}$, the ith relay's maximum achievable beamformer weight magnitude decreases as either the $\mathrm{SU_{Tx}}$ or $\mathrm{PU_{Tx}}$ to the ith relay link gets stronger.

The expression for σ_2^2 contains two terms that represent PU self interference, the level of which is controlled in a similar way to that described above, i.e., by controlling the levels of μ_5 and r_7, both of which appear in (26). Since both σ_1^2 and σ_2^2 are expected to be small, the PDF of r_1 and r_2 will be concentrated around zero and can be neglected.

From the above discussion, we see that the PDF of (21) can be approximated as the sum of two correlated exponentially distributed random variables r_4 and r_5. For small values of ρ, the correlation between r_4 and r_5 is small and therefore they can be treated as independent random variables.

The PU outage probability can be approximated as

$$\begin{aligned}\mathrm{P_o}^{(2)} &\approx \Pr\left\{P_{\rm p}|h_{\rm pp}|^2-(r_4+r_5)\le\gamma_{\rm T}\sigma_{\rm p}^2+r_7\right\} \quad (26)\\ &=1-\Bigg[\left(\frac{1}{1+\frac{\mu_4}{P_{\rm p}\Omega_{\rm pp}}}\right)\left(\frac{1}{1+\frac{\mu_5}{P_{\rm p}\Omega_{\rm pp}}}\right)\\ &\qquad \exp\left(-\frac{\gamma_{\rm T}\sigma_{\rm p}^2+r_7}{P_{\rm p}\Omega_{\rm pp}}\right)\Bigg]\end{aligned}$$

and the outage probability constraint is given by

$$\exp\left(\frac{r_7}{P_{\rm p}\Omega_{\rm pp}}\right)\left(1+\frac{\mu_4}{P_{\rm p}\Omega_{\rm pp}}\right)\left(1+\frac{\mu_5}{P_{\rm p}\Omega_{\rm pp}}\right) \le \frac{\exp\left(-\frac{\gamma_{\rm T}\sigma_{\rm p}^2}{P_{\rm p}\Omega_{\rm pp}}\right)}{1-\mathrm{P_{o,max}}}. \quad (27)$$

Note that (27) is a nonconvex constraint. In order to transform this constraint into a convex constraint, we first use the geometric-arithmetic mean inequality[1] and rewrite (27) as

$$\exp\left(\frac{r_7}{P_{\rm p}\Omega_{\rm pp}}\right)+\left(1+\frac{\mu_4}{P_{\rm p}\Omega_{\rm pp}}\right)+\left(1+\frac{\mu_5}{P_{\rm p}\Omega_{\rm pp}}\right) \le 3\left(\frac{\exp\left(-\frac{\gamma_{\rm T}\sigma_{\rm p}^2}{P_{\rm p}\Omega_{\rm pp}}\right)}{1-\mathrm{P_{o,max}}}\right)^{\frac{1}{3}} \quad (28)$$

[1]Note that the use of the geometric-arithmetic mean inequality results in tightening of the constraint. This may cause the beamforming problem to become infeasible or the solution obtained may be sub-optimal since the power allocated to the beamformer would be less than what would have been allocated if the original constraint was used. We have developed an algorithm that iteratively finds the minimum outage probability that meets the original constraint; however, through our extensive numerical simulations we have found that the solution obtained by directly solving the problem with the tightened constraint is very close to the optimum and, in practice, it is not necessary to use the iterative algorithm.

which is still a nonconvex constraint. However, the assumptions that were made to obtain the approximate outage probability expression also imply that r_7 is small. Thus, $\exp(r_7/(P_{\rm p}\Omega_{\rm pp}))\approx(1+r_7/(P_{\rm p}\Omega_{\rm pp}))$, allowing us to write the outage probability constraint as the convex constraint

$$\frac{1}{P_{\rm p}\Omega_{\rm pp}}(\tilde{r}_7+\mu_4+\mu_5)+3\left(1-\left(\frac{\exp\left(-\frac{\gamma_{\rm T}\sigma_{\rm p}^2}{P_{\rm p}\Omega_{\rm pp}}\right)}{1-\mathrm{P_{o,max}}}\right)^{\frac{1}{3}}\right) \le 0, \quad (29)$$

where

$$\tilde{r}_7\triangleq\frac{\gamma_{\rm T}}{1-\rho^2}{\rm tr}\Big(\big(P_{\rm s}[\mathbf{h}_{\rm sr}\odot\tilde{\mathbf{h}}_{\rm rp}][\mathbf{h}_{\rm sr}\odot\tilde{\mathbf{h}}_{\rm rp}]^H+\sigma_{\rm r}^2{\rm diag}(|\tilde{\mathbf{h}}_{\rm rp}|^2) + P_{\rm p}[\mathbf{h}_{\rm pr}\odot\tilde{\mathbf{h}}_{\rm rp}][\mathbf{h}_{\rm pr}\odot\tilde{\mathbf{h}}_{\rm rp}]^H\big)\mathbf{W}\Big).$$

Finally, the beamforming problem (12) can be converted into a robust beamforming problem by replacing constraint (12c) with the robust probability based constraint (29). Again, the Charnes-Cooper transformation can be used to solve the resulting robust beamforming problem.

Although our outage probability approximation is based on the assumption that ρ is small, we have empirically found that the approximation is accurate up to ρ values as large as 0.8 in various channel conditions.

V. Simulation Results and Discussion

We illustrate the performance of our proposed methods through numerical simulations in i.i.d. Rayleigh flat-fading channels. We consider a system with 4 relay nodes. In all simulations we have set $P_{\rm p}=P_{\rm s,max}=30$ dBm, $P^{(i)}_{\rm Rl,max}=30$ dBm $\forall i$, $\gamma_{\rm T}=5$ dB and the noise power at each receiver is assumed to be -80 dBm. The maximum $\mathrm{PU_{Rx}}$ outage probability, $\mathrm{P_{o,max}}$, is set to 5%. Channel powers of the direct paths, i.e., $\Omega_{\rm pp}$, $\Omega^{(i)}_{\rm sr}$ $\forall i$ and $\Omega^{(i)}_{\rm rs}$ $\forall i$, are set to 10 dB. For our simulations we have set the signal-to-interference-channel-ratio of all receivers to 5 dB, i.e., $\Omega^{(i)}_{\rm sr}/\Omega_{\rm pr}=\Omega^{(i)}_{\rm rs}/\Omega_{\rm ps}=\Omega_{\rm pp}/\Omega^{(i)}_{\rm rp}=\Omega_{\rm pp}/\Omega_{\rm sp}=5$ dB. According to CSI error model (19), $\mathbf{\Sigma}_{\mathrm{e}_{ii}}=\Omega^{(i)}_{\rm rp}=5$ dB, $\forall i$. To illustrate the impact of CSI errors and the effectiveness of our proposed method, we present simulation results for various values of ρ. Our proposed robust beamformer is also compared against a non-robust beamformer. The non-robust beamformer is designed by treating CSI of $\mathbf{h}_{\rm rp}$ as perfect by ignoring the effects of CSI errors. Due to space constraints, results for a limited set of parameters are presented; however, through extensive numerical simulations, we have verified that our proposed methods perform just as well in a wide range of channel scenarios and QoS constraints.

In Fig. 2, results are provided for the CDF of the SINR at the $\mathrm{PU_{Rx}}$ for the full CSI and robust designs. The CDFs presented are ensemble CDFs obtained through solving multiple realisations of the proposed designs. In each realisation of the problem, new instances of the required channels are generated and the power allocation and beamforming problems are solved. Results are also provided for a non-robust beamformer design.

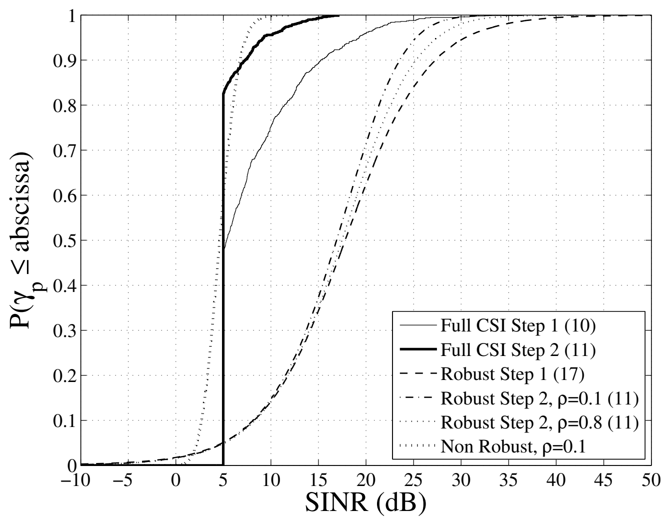

Fig. 2. SINR at the PU_{Rx}.

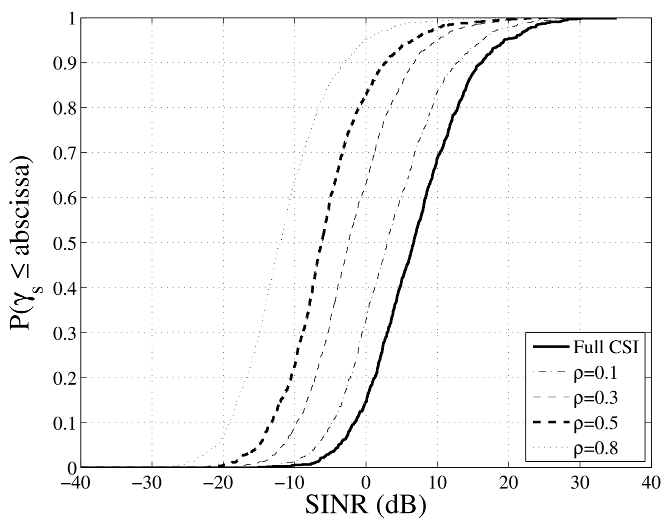

Fig. 3. SINR at the SU_{Rx}.

We see that the outage probability for the full CSI solution is zero in both transmission steps. Results show that the 5% PU_{Rx} outage probability requirement is satisfied by the robust SU_{Tx} power allocation problem (17). We see that the robust beamformer satisfies the 5% outage probability requirement for $\rho = 0.1$ and also for $\rho = 0.8$. This demonstrates that the proposed outage probability approximation is also accurate in very large channel uncertainties. The non-robust beamforming solution achieves an outage probability which is almost 60% because the outage probability constraint is not respected by this design.

In Fig. 3, the CDF of the SINR at the SU_{Rx} for the full CSI beamformer and its robust counterpart is plotted. To illustrate the impact of CSI errors, results for various values of ρ are provided. As expected, the full CSI design results in the best SU_{Rx} performance and the performance degrades with increasing CSI error variance.

VI. Conclusions

In this paper, we have studied a robust cooperative beamformer for a CR relay network under the assumption of partial and imperfect CSI. We have proposed a robust SU_{Tx} power allocation design based on partial CSI that maximises the SINR at the cognitive relay nodes while satisfying the outage probability requirement of the PU_{Rx}. We have shown that the robust SU_{Rx} SINR maximisation beamforming problem under the assumption of partial and imperfect CSI can be stated as a problem that has the form of a linear-fractional program. Using the Charnes-Cooper transformation, the robust beamforming problem can be transformed into a convex SDP. Our simulation results have shown that the proposed methods achieve the required robustness in various levels of channel uncertainty.

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