Robust Cognitive Radio Cooperative Beamforming

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***Abstract*—We consider a cognitive radio (CR) relay network consisting of a cognitive source, a cognitive destination and a number of cognitive relay nodes that share spectrum with a primary transmitter and receiver. Due to poor channel conditions, the cognitive source is unable to communicate directly with the cognitive destination and hence employs the cognitive relays for assistance. We assume that perfect channel state information (CSI) for all links is not available to the CR. Under the assumption of partial and imperfect CSI at the CR system, we propose new robust CR cooperative relay beamformers where either the total relay transmit power or the cognitive destination signal-to-interference-and-noise ratio (SINR) is optimized subject to a constraint on the primary receiver outage probability. We formulate the robust total relay power minimisation and the cognitive destination SINR maximisation optimisation problems as a convex second order cone program and a semidefinite program, respectively. Cumulative distribution functions of primary receiver and cognitive destination receiver SINR for Rayleigh fading channels are presented.**

***Index Terms*—Cognitive radio, robust beamforming, cooperative beamforming, interference management, relay, convex optimisation, power control, beamforming, semidefinite relaxation, outage probability.**

I. Introduction

THE explosive growth in the use of wireless devices has motivated researchers to find new methods that enable the more efficient use of the radio spectrum resource. Cognitive radio (CR) [1] is a new paradigm for achieving this efficient use by managing the spectrum in a dynamic manner. CR modes of operation can be broadly grouped into two categories, interweave [2] and underlay [3]. The fundamental information-theoretic capacity limits of CR systems have been analysed in [3]–[9].

In an underlay CR system the secondary users (SUs) are only allowed to transmit if the interference at the primary user (PU) receiver can be maintained below some acceptable level. This is achieved by imposing either an average/peak interference constraint [3], [10], [11], or a minimum signal-to-interference-and-noise ratio (SINR) constraint [6]. The advantage of using the SINR-based scheme is that it allows the SU to optimise its transmissions based on the quality of the primary user transmitter $(\mathrm{PU_{Tx}})$ to the primary user receiver $(\mathrm{PU_{Rx}})$ link.

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The performance of underlay CR systems can be significantly improved by the use of multiple antennas. These performance improvements can also be realised by system employing multiple single antenna relay nodes through a technique known as cooperative relaying [12]–[15]. Geographically distributed relay nodes are cooperatively able to form a virtual antenna array and provide increased gains in capacity through distributed beamforming. In [12], it was shown that user cooperation could be used as a form of spatial diversity. This not only resulted in increased capacity for the users but also a more robust system where the users' rates were less affected by channel variations. Distributed beamforming designs in the form of convex optimisation problems were formulated in [13] and a semidefinite program (SDP) was introduced to obtain the optimum beamformers. Linear beamforming and power control for a two-hop relay broadcast channel for a cellular network utilising a multi-antenna relay was studied in [15]. It was found that the solutions obtained were extensions of the minimum-mean-squared-error (MMSE) and the zero-forcing (ZF) design criteria for downlink precoding in the traditional multiple-input single-output (MISO) broadcast channel without relay [16], [17].

Recently, there has been increasing attention to the use of cooperative beamforming in CR systems (see, e.g., [18]–[20]). The relay nodes are typically deployed by the CR system to aid a SU transmitter $(\mathrm{SU_{Tx}})$ to communicate with a distant SU receiver $(\mathrm{SU_{Rx}})$ when the link between the $\mathrm{SU_{Tx}}$ and $\mathrm{SU_{Rx}}$ is poor. Cooperative beamforming at the relays not only improves SU performance through beamforming but also allows more control over the interference generated at the $\mathrm{PU_{Rx}}$. The best beamformer performance is obviously obtained when perfect/full channel state information (CSI) is available and the design of CR cooperative relay beamformers under this assumption have been studied in [18]–[20]. In practical communication systems, this assumption may be over idealistic as perfect CSI for all links is rarely available. Channel estimation errors, limited CSI feedback and outdated channel estimates are some of the sources of the imperfections. The design of worst-case robust cooperative beamformers that are less susceptible to these imperfections have been investigated in [10], [14], [21]. Unfortunately, solutions obtained through the worst-case approach can be overly conservative because the true probability of worst-case errors may be extremely low [22].

In a CR relay network, CSI of the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$ and SU relays $(\mathrm{SU_{Rls}})$ to $\mathrm{PU_{Rx}}$ is generally difficult to acquire and some level of cooperation with the PU system may be required. The level of cooperation determines the quality of the CSI that is available to the SU. Therefore, in this paper, we consider a CR relay network where only partial and imperfect

CSI of the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$ and the SU relays to $\mathrm{PU_{Rx}}$ links is available to the CR system. We propose new robust CR cooperative relay beamformers where either the total relay transmit power or the cognitive destination SINR is optimised subject to a $\mathrm{PU_{Rx}}$ outage probability constraint. The problem posed in Section IV-B has been previously considered in [23]. This paper extends this previous work by formulating two new robust CR cooperative relay beamformers for the cases where i) only partial CSI is available for the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$ link and full CSI for other links; and ii) only partial CSI is available for the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$ link and the CSI for the SU relays to $\mathrm{PU_{Rx}}$ links is imperfect.

The contributions of this paper are as follows.

- We first formulate the CR relay cooperative beamforming problem under the assumption of full CSI at the CR system as total relay power minimisation and cognitive destination SINR maximisation problems.
- We show that the total relay power minimisation and the cognitive destination SINR optimisation problems can be transformed into a convex second order cone program (SOCP) [24] and a convex semidefinite program (SDP), respectively.
- We present robust beamformers that guarantee a certain $\mathrm{PU_{Rx}}$ outage probability for the scenarios where partial CSI is available for the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$ link and
  1) full CSI is available for all other links;
  2) partial CSI is available for the SU relays to $\mathrm{PU_{Rx}}$ links and full CSI is available for all other links;
  3) imperfect CSI is available for the SU relays to $\mathrm{PU_{Rx}}$ links and full CSI is available for all other links.
- We show that the robust total relay power minimisation and the robust cognitive destination SINR optimisation problems can be transformed into a convex second order cone program (SOCP) [24] and a convex semidefinite program (SDP), respectively.

The performance resulting from the optimisation problems outlined above is demonstrated by means of capacity cumulative distribution functions (CDFs) for various channel conditions. Although we only consider flat Rayleigh channels, the framework developed in this paper can be readily extended to other channel models such as Ricean or Nakagami.

In this paper, we assume both i) the proposed optimisation problems are solved by a central SU processing unit; and ii) a dedicated link, such as that in a distributed antenna system [25], [26], between this central SU processing unit and each relay node exists.

The rest of this paper is organised as follows. In Section II, the system model is introduced. The CR cooperative beamforming problem for full CSI is formulated in Section III. In Section IV, we present novel robust CR cooperative beamformers under varying levels of channel uncertainty. Simulation results are presented in Section V and conclusions in Section VI.

Notation: Upper (lower) bold face letters are used for matrices (vectors); $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, $\mathbb{E}\{\cdot\}$ and $\|\cdot\|$ denote complex conjugate, transpose, Hermitian transpose, expectation and Euclidean norm, respectively. $|\cdot|^2$ denotes the magnitude squared operator for scalars and element-wise magnitude squared for vectors. $(\cdot)^{1/2}$ denotes the square root operator for scalars and element-wise square root for vectors. $\min(\cdot)$ denotes the minimum element of a vector. $\mathrm{tr}(\cdot)$, $\mathcal{C}^{R\times 1}$, $\mathcal{C}^{R\times R}$, $\odot$, $\Re\{\cdot\}$ and $\Im\{\cdot\}$ denote the matrix trace operator, space of $R\times 1$ vectors with complex entries, space of $R\times R$ matrices with complex entries, element-wise product between vectors, the real part and the imaginary part. $\mathbf{W}\succeq 0$ denotes that $\mathbf{W}$ is a positive semidefinite matrix. The notation $\mathbf{x}\sim\mathcal{N}_{\mathcal{C}}(\mathbf{m},\mathbf{\Sigma})$ states that $\mathbf{x}$ contains entries of complex Gaussian random variables, with mean $\mathbf{m}$ and covariance $\mathbf{\Sigma}$.

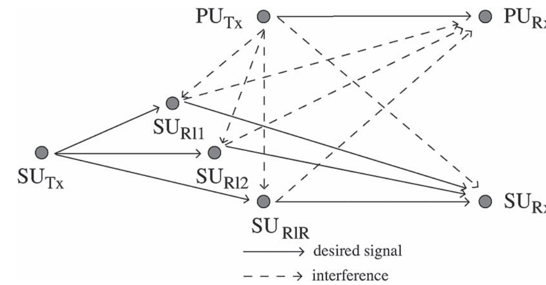

Fig. 1. System model.

II. System Model

Consider a CR relay network which consists of a secondary transmitter ($\mathrm{SU_{Tx}}$), a secondary receiver ($\mathrm{SU_{Rx}}$), R secondary relay ($\mathrm{SU_{Rl}}$) nodes and a $\mathrm{PU_{Tx}}$ and $\mathrm{PU_{Rx}}$ pair, as shown in Fig. 1. We assume that due to poor channel conditions between the $\mathrm{SU_{Tx}}$ and $\mathrm{SU_{Rx}}$, there is no reliable link between them. Hence, the $\mathrm{SU_{Tx}}$ employs the $\mathrm{SU_{Rls}}$ to communicate with the $\mathrm{SU_{Rx}}$. Since the PU and SU systems use the same frequency band, the $\mathrm{PU_{Rx}}$ experiences interference from the $\mathrm{SU_{Rl}}$ transmissions and both $\mathrm{SU_{Rl}}$ and $\mathrm{SU_{Rx}}$ experience interference from the $\mathrm{PU_{Tx}}$ transmissions. Furthermore, we assume that the link between the $\mathrm{SU_{Tx}}$ and $\mathrm{PU_{Rx}}$ is poor and the $\mathrm{SU_{Tx}}$ signal is sufficiently attenuated at the $\mathrm{PU_{Rx}}$ to be ignored. Including the $\mathrm{SU_{Tx}}$ interference at the $\mathrm{PU_{Rx}}$ changes the solutions but not their structure, hence, it has been omitted for simplicity. Each transmitter and receiver in the system are assumed to be equipped with a single antenna.

All links in the network are assumed to be independent, point-to-point, flat Rayleigh fading channels. The channel coefficients of the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$, $\mathrm{PU_{Tx}}$ to $\mathrm{SU_{Rl}}i$, $\mathrm{PU_{Tx}}$ to $\mathrm{SU_{Rx}}$, $\mathrm{SU_{Tx}}$ to $\mathrm{SU_{Rl}}i$, $\mathrm{SU_{Rl}}i$ to $\mathrm{SU_{Rx}}$ and $\mathrm{SU_{Rl}}i$ to $\mathrm{PU_{Rx}}$ links are denoted by h_{pp}, $h_{\mathrm{pr}}^{(i)}$, h_{ps}, $h_{\mathrm{s}}^{(i)}$, $h_{\mathrm{rs}}^{(i)}$ and $h_{\mathrm{rp}}^{(i)}$, respectively. The instantaneous channel powers of these links are represented by $g_{\mathrm{pp}}=|h_{\mathrm{pp}}|^2$, $g_{\mathrm{p}}^{(i)}=\left|h_{\mathrm{pr}}^{(i)}\right|^2$, $g_{\mathrm{ps}}=|h_{\mathrm{ps}}|^2$, $g_{\mathrm{sr}}^{(i)}=\left|h_{\mathrm{s}}^{(i)}\right|^2$, $g_{\mathrm{r}}^{(i)}=\left|h_{\mathrm{rs}}^{(i)}\right|^2$ and $g_{\mathrm{rp}}^{(i)}=\left|h_{\mathrm{r}}^{(i)}\right|^2$ and have the means: $\Omega_{\mathrm{pp}}=\mathbb{E}\{g_{\mathrm{pp}}\}$, $\Omega_{\mathrm{pr}}^{(i)}=\mathbb{E}\{g_{\mathrm{pr}}^{(i)}\}$, $\Omega_{\mathrm{ps}}=\mathbb{E}\{g_{\mathrm{ps}}\}$, $\Omega_{\mathrm{sr}}^{(i)}=\mathbb{E}\{g_{\mathrm{sr}}^{(i)}\}$, $\Omega_{\mathrm{rs}}^{(i)}=\mathbb{E}\{g_{\mathrm{r}}^{(i)}\}$ and $\Omega_{\mathrm{rp}}^{(i)}=\mathbb{E}\{g_{\mathrm{rp}}^{(i)}\}$.

We consider a secondary system that utilises a two-step amplify-and-forward (AF) protocol. During the first step, the $\mathrm{SU_{Tx}}$ broadcasts the signal $\sqrt{P_{\mathrm{s}}}s_{\mathrm{s}}$ to the relays, where P_{s} is the $\mathrm{SU_{Tx}}$ transmit power and s_{s} the information symbol. Simultaneously, the $\mathrm{PU_{Tx}}$ transmits the signal $\sqrt{P_{\mathrm{p}}s_{\mathrm{p}}^{(1)}}$, where P_{p} is the $\mathrm{PU_{Tx}}$ transmit power and $s_{\mathrm{p}}^{(1)}$ the information symbol. We assume that $\mathbb{E}\{|s_{\rm s}|^2\} = \mathbb{E}\{|s_{\rm p}^{(1)}|^2\} = 1$. The signal received at the ith relay is given by

$$x_i = \underbrace{\sqrt{P_{\rm s}}s_{\rm s}h_{\rm sr}^{*(i)}}_{\text{wanted signal}} + \underbrace{\sqrt{P_{\rm p}}s_{\rm p}^{(1)}h_{\rm pr}^{*(i)} + n_{\rm r}^{(i)}}_{\text{interference+noise}}, \tag{1}$$

where $n_{\rm r}^{(i)}$ is the additive white Gaussian noise (AWGN) with a variance of $\sigma_{\rm r}^2$ at the ith relay.

During the second step, the ith relay transmits the signal

$$\begin{aligned} y_i &= x_i w_i \\ &= \sqrt{P_{\rm s}}s_{\rm s}h_{\rm sr}^{*(i)}w_i + \sqrt{P_{\rm p}}s_{\rm p}^{(1)}h_{\rm pr}^{*(i)}w_i + n_{\rm r}^{(i)}w_i, \end{aligned} \tag{2}$$

where w_i is the complex beamforming weight applied by the ith relay. During this time, the $\mathrm{PU_{Tx}}$ transmits the signal $\sqrt{P_{\rm p}s_{\rm p}^{(2)}}$, where $s_{\rm p}^{(2)}$ is the information symbol and is assumed to be different to that transmitted in the first step. We assume that $\mathbb{E}\{|s_{\rm p}^{(2)}|^2\} = 1$. At the $\mathrm{SU_{Rx}}$, the received signal can be expressed as

$$\begin{aligned} z_s &= \sum_{i=1}^{R} y_i h_{\rm rs}^{*(i)} + \sqrt{P_{\rm p}}s_{\rm p}^{(2)}h_{\rm ps}^{*} \\ &= \underbrace{\sqrt{P_{\rm s}}s_{\rm s}[\mathbf{h}_{\rm sr}\odot\mathbf{h}_{\rm rs}]^H\mathbf{w}}_{\text{wanted signal}} + \underbrace{[\mathbf{n}_{\rm r}\odot\mathbf{h}_{\rm rs}]^H\mathbf{w} + n_{\rm s}}_{\text{noise}} \\ &\quad + \underbrace{\sqrt{P_{\rm p}}s_{\rm p}^{(2)}h_{\rm ps}^{*} + \sqrt{P_{\rm p}}s_{\rm p}^{(1)}[\mathbf{h}_{\rm pr}\odot\mathbf{h}_{\rm rs}]^H\mathbf{w}}_{\text{interference}}, \end{aligned} \tag{3}$$

and that at the $\mathrm{PU_{Rx}}$ as

$$\begin{aligned} z_p &= \sqrt{P_{\rm p}}s_{\rm p}^{(2)}h_{\rm pp} + \sum_{i=1}^{R} y_i h_{\rm rp}^{(i)} \\ &= \underbrace{\sqrt{P_{\rm p}}s_{\rm p}^{(2)}h_{\rm pp}}_{\text{wanted signal}} + \underbrace{[\mathbf{n}_{\rm r}\odot\mathbf{h}_{\rm rp}]^H\mathbf{w} + n_{\rm p}}_{\text{noise}} \\ &\quad + \underbrace{\sqrt{P_{\rm s}}s_{\rm s}[\mathbf{h}_{\rm sr}\odot\mathbf{h}_{\rm rp}]^H\mathbf{w}}_{\text{SU interference}} + \underbrace{\sqrt{P_{\rm p}}s_{\rm p}^{(1)}[\mathbf{h}_{\rm pr}\odot\mathbf{h}_{\rm rp}]^H\mathbf{w}}_{\text{self interference}}, \end{aligned} \tag{4}$$

where $\mathbf{h}_{\rm sr} \triangleq [h_{\rm sr}^{(1)}h_{\rm sr}^{(2)}\dots h_{\rm sr}^{(R)}]^T$, $\mathbf{h}_{\rm rs} \triangleq [h_{\rm rs}^{(1)}h_{\rm rs}^{(2)}\dots h_{\rm rs}^{(R)}]^T$, $\mathbf{h}_{\rm pr} \triangleq [h_{\rm pr}^{(1)}h_{\rm pr}^{(2)}\dots h_{\rm pr}^{(R)}]^T$, $\mathbf{h}_{\rm rp} \triangleq [h_{\rm rp}^{(1)}h_{\rm rp}^{(2)}\dots h_{\rm rp}^{(R)}]^T$, $\mathbf{w} \triangleq [w_1 w_2 \dots w_R]^T$, $\mathbf{n}_{\rm r} \triangleq [n_r^{(1)}n_r^{(2)}\dots n_r^{(R)}]^T$ and $n_{\rm s}$ and $n_{\rm p}$ are AWGN with powers $\sigma_{\rm s}^2$ and $\sigma_{\rm p}^2$ at the $\mathrm{SU_{Rx}}$ and $\mathrm{PU_{Rx}}$, respectively. Note that the relays also retransmit the PU's signal, hence, the $\mathrm{PU_{Rx}}$ also receives the $\mathrm{PU_{Tx}}$ symbol from the first step, which is treated as self interference in our analysis.

By assuming that $s_{\rm s}$, $s_{\rm p}^{(1)}$, $s_{\rm p}^{(2)}$, $n_{\rm r}^{(i)}\forall i$, $n_{\rm s}$ and $n_{\rm p}$ are all uncorrelated from each other and perfect CSI is available, and therefore considering the channel coefficients as deterministic constants, the total relay transmit power can be expressed as

$$\begin{aligned} P_T &= \sum_{i=1}^{R}\mathbb{E}\left\{|y_i|^2\right\} \\ &= \mathbf{w}^H\mathbf{E}\mathbf{w}, \end{aligned} \tag{5}$$

where $\mathbf{E} = P_{\rm s}\mathrm{diag}(|\mathbf{h}_{\rm sr}|^2) + P_{\rm p}\mathrm{diag}(|\mathbf{h}_{\rm pr}|^2) + \sigma_{\rm r}^2\mathbf{I}$. The ith relay's transmit power is given by $P_{Rl}^{(i)} = \mathbf{E}_{ii}|w_i|^2$. The SINR at the $\mathrm{SU_{Rx}}$ is expressed as

$$\begin{aligned} \gamma_{\rm s} &= \frac{P_{\rm s}\left|[\mathbf{h}_{\rm sr}\odot\mathbf{h}_{\rm rs}]^H\mathbf{w}\right|^2}{P_{\rm p}|h_{\rm ps}|^2 + P_{\rm p}\left|[\mathbf{h}_{\rm pr}\odot\mathbf{h}_{\rm rs}]^H\mathbf{w}\right|^2 + \sigma_{\rm r}^2\|\mathbf{h}_{\rm rs}\odot\mathbf{w}\|^2 + \sigma_{\rm s}^2} \\ &= \frac{\mathbf{w}^H\mathbf{Q}\mathbf{w}}{P_{\rm p}|h_{\rm ps}|^2 + \mathbf{w}^H(\mathbf{R}+\mathbf{V})\mathbf{w} + \sigma_{\rm s}^2}, \end{aligned} \tag{6}$$

where $\mathbf{Q} = P_{\rm s}[\mathbf{h}_{\rm sr}\odot\mathbf{h}_{\rm rs}][\mathbf{h}_{\rm sr}\odot\mathbf{h}_{\rm rs}]^H$, $\mathbf{R} = P_{\rm p}[\mathbf{h}_{\rm pr}\odot\mathbf{h}_{\rm rs}][\mathbf{h}_{\rm pr}\odot\mathbf{h}_{\rm rs}]^H$ and $\mathbf{V} = \sigma_{\rm r}^2\mathrm{diag}(|\mathbf{h}_{\rm rs}|^2)$. Using the following definition

$$I_p \triangleq P_{\rm s}\left|[\mathbf{h}_{\rm sr}\odot\mathbf{h}_{\rm rp}]^H\mathbf{w}\right|^2 + P_{\rm p}\left|[\mathbf{h}_{\rm pr}\odot\mathbf{h}_{\rm rp}]^H\mathbf{w}\right|^2 + \sigma_{\rm r}^2\|\mathbf{h}_{\rm rp}\odot\mathbf{w}\|^2,$$

the SINR at the $\mathrm{PU_{Rx}}$ can be expressed as

$$\begin{aligned} \gamma_{\rm p} &= \frac{P_{\rm p}|h_{\rm pp}|^2}{I_p + \sigma_{\rm p}^2} \\ &= \frac{P_{\rm p}|h_{\rm pp}|^2}{\mathbf{w}^H(\mathbf{B}+\mathbf{C}+\mathbf{D})\mathbf{w} + \sigma_{\rm p}^2}, \end{aligned} \tag{7}$$

where $\mathbf{B} = P_{\rm s}[\mathbf{h}_{\rm sr}\odot\mathbf{h}_{\rm rp}][\mathbf{h}_{\rm sr}\odot\mathbf{h}_{\rm rp}]^H$, $\mathbf{C} = P_{\rm p}[\mathbf{h}_{\rm pr}\odot\mathbf{h}_{\rm rp}][\mathbf{h}_{\rm pr}\odot\mathbf{h}_{\rm rp}]^H$ and $\mathbf{D} = \sigma_{\rm r}^2\mathrm{diag}(|\mathbf{h}_{\rm rp}|^2)$.

To guarantee a certain level of quality-of-service (QoS) to the primary user, in our beamformer design formulations under the assumption of perfect CSI, we impose the $\mathrm{PU_{Rx}}$ instantaneous SINR constraint $\gamma_{\rm p} \geq \gamma_{\rm T}$. This constraint is transformed into a probability based constraint in Section IV.

III. Beamformer Optimisation

In this section, we present two beamformer design optimisation problems. The first optimisation problem finds the optimum beamforming weight vector, $\mathbf{w}$, such that the total relay transmit power, P_T, is minimised subject to $\mathrm{PU_{Rx}}$ and $\mathrm{SU_{Rx}}$ QoS constraints, i.e., the $\mathrm{PU_{Rx}}$ and $\mathrm{SU_{Rx}}$ SINR are maintained above $\gamma_{\rm T}$ and $\gamma_{\rm s,min}$, respectively.

The second optimisation problem finds the optimum $\mathbf{w}$ that maximises the $\mathrm{SU_{Rx}}$ SINR subject to the $\mathrm{PU_{Rx}}$ QoS constraint and an individual maximum transmit power constraint, $P_{\rm Rl,max}^{(i)}$, on each relay node. In practice, the relay power constraint may be due either to regulatory or hardware limitations.

In our formulations, we assume that we are unable to control the PU's transmit power and the PU transmits at a constant power of $P_{\rm p}$. In this section, the beamformers are designed under the assumption that perfect CSI for all links are available at the SU system. This allows us to obtain fundamental limits on performance. However, in practice, the channel would need to be estimated, hence the performance results obtained in this section provide an upper bound. In Section IV, we consider the case when perfect CSI is not available.

A. Relay Power Minimisation

The total relay transmit power minimisation problem can be mathematically represented as

$$\min_{\mathbf{w}} \quad \mathbf{w}^H \mathbf{E} \mathbf{w} \tag{8a}$$
$$\text{s.t.} \quad \gamma_{\mathrm{p}} \geq \gamma_{\mathrm{T}} \tag{8b}$$
$$\gamma_{\mathrm{s}} \geq \gamma_{\mathrm{s,min}}. \tag{8c}$$

Problem (8) is a nonconvex optimisation problem; however, it can be reformulated into a convex optimisation problem by choosing $[\mathbf{h}_{\mathrm{sr}} \odot \mathbf{h}_{\mathrm{rs}}]^H \mathbf{w}$ to be real and positive without loss of generality [23]. Hence, the relay power minimisation problem can be stated as the following convex second-order cone program (SOCP) [23]

$$\min_{\mathbf{w}} \quad \mathbf{w}^H \mathbf{E} \mathbf{w} \tag{9a}$$
$$\text{s.t.} \quad \sqrt{P_{\mathrm{p}} |h_{\mathrm{pp}}|^2} \geq \sqrt{\gamma_{\mathrm{T}}} \left\| \begin{matrix} \sqrt{P_{\mathrm{s}}} [\mathbf{h}_{\mathrm{sr}} \odot \mathbf{h}_{\mathrm{rp}}]^H \mathbf{w} \\ \sqrt{P_{\mathrm{p}}} [\mathbf{h}_{\mathrm{pr}} \odot \mathbf{h}_{\mathrm{rp}}]^H \mathbf{w} \\ \sigma_{\mathrm{r}} [\mathbf{h}_{\mathrm{rp}} \odot \mathbf{w}] \\ \sigma_{\mathrm{p}} \end{matrix} \right\| \tag{9b}$$
$$\sqrt{P_{\mathrm{s}}} [\mathbf{h}_{\mathrm{sr}} \odot \mathbf{h}_{\mathrm{rs}}]^H \mathbf{w} \geq \sqrt{\gamma_{\mathrm{s,min}}} \left\| \begin{matrix} \sqrt{P_{\mathrm{p}}} h_{\mathrm{ps}} \\ \sqrt{P_{\mathrm{p}}} [\mathbf{h}_{\mathrm{pr}} \odot \mathbf{h}_{\mathrm{rs}}]^H \mathbf{w} \\ \sigma_{\mathrm{r}} [\mathbf{h}_{\mathrm{rs}} \odot \mathbf{w}] \\ \sigma_{\mathrm{s}} \end{matrix} \right\|. \tag{9c}$$

In the interest of brevity, the further constraints $\Re\{[\mathbf{h}_{\mathrm{sr}} \odot \mathbf{h}_{\mathrm{rs}}]^H \mathbf{w}\} > 0$ and $\Im\{[\mathbf{h}_{\mathrm{sr}} \odot \mathbf{h}_{\mathrm{rs}}]^H \mathbf{w}\} = 0$, are not explicitly stated in any of the SOCPs in the following sections.

B. Secondary Receiver SINR Maximisation

The $\mathrm{SU_{Rx}}$ SINR maximisation problem is expressed as

$$\max_{\mathbf{w}} \quad \frac{\mathbf{w}^H \mathbf{Q} \mathbf{w}}{\mathbf{w}^H (\mathbf{R} + \mathbf{V}) \mathbf{w} + P_{\mathrm{p}} |h_{\mathrm{ps}}|^2 + \sigma_{\mathrm{s}}^2} \tag{10a}$$
$$\text{s.t.} \quad \mathbf{E}_{ii} |w_i|^2 \leq P_{\mathrm{Rl,max}}^{(i)}, \quad i = 1 \ldots R \tag{10b}$$
$$\mathbf{w}^H \gamma_{\mathrm{T}} (\mathbf{B} + \mathbf{C} + \mathbf{D}) \mathbf{w} + \gamma_{\mathrm{T}} \sigma_{\mathrm{p}}^2 - P_{\mathrm{p}} |h_{\mathrm{pp}}|^2 \leq 0. \tag{10c}$$

Problem (10) is a nonconvex optimisation problem; however, it can be transformed into an optimisation problem which has the structure of a linear-fractional program [24]. Using the definition $\mathbf{W} \triangleq \mathbf{w}\mathbf{w}^H$, problem (10) can be restated as

$$\max_{\mathbf{W}} \quad \frac{\mathrm{tr}(\mathbf{Q}\mathbf{W})}{\mathrm{tr}\left((\mathbf{R} + \mathbf{V})\mathbf{W}\right) + P_{\mathrm{p}} |h_{\mathrm{ps}}|^2 + \sigma_{\mathrm{s}}^2} \tag{11a}$$
$$\text{s.t.} \quad \mathbf{E}_{ii} \mathbf{W}_{ii} \leq P_{\mathrm{Rl,max}}^{(i)}, \quad i = 1 \ldots R \tag{11b}$$
$$\gamma_{\mathrm{T}}\, \mathrm{tr}\left((\mathbf{B} + \mathbf{C} + \mathbf{D})\mathbf{W}\right) + \gamma_{\mathrm{T}} \sigma_{\mathrm{p}}^2 - P_{\mathrm{p}} |h_{\mathrm{pp}}|^2 \leq 0 \tag{11c}$$
$$\mathbf{W} \succeq 0 \tag{11d}$$
$$\mathrm{rank}(\mathbf{W}) = 1. \tag{11e}$$

Since the rank constraint (11e) is a nonconvex constraint, problem (11) is a nonconvex optimisation problem. However, it can be relaxed into a convex optimisation problem by using semidefinite relaxation (SDR) [24], [27], [28], i.e., remove the rank constraint. In [23], the relaxed form of problem (11) was solved in an iterative manner by solving a number of convex feasibility problems. Since the relaxed form of problem (11) has the same structure as a linear-fractional program, the Charnes-Cooper transformation [24] can be used to solve it efficiently without needing an iterative procedure. To proceed, we first define the pair

$$\tilde{\mathbf{W}} = \frac{\mathbf{W}}{\mathrm{tr}\left((\mathbf{R} + \mathbf{V})\mathbf{W}\right) + P_{\mathrm{p}} |h_{\mathrm{ps}}|^2 + \sigma_{\mathrm{s}}^2},$$
$$t = \frac{1}{\mathrm{tr}\left((\mathbf{R} + \mathbf{V})\mathbf{W}\right) + P_{\mathrm{p}} |h_{\mathrm{ps}}|^2 + \sigma_{\mathrm{s}}^2}.$$

Using these definitions, the relaxed form of problem (11) can be stated as

$$\max_{\tilde{\mathbf{W}}, t} \quad \mathrm{tr}(\mathbf{Q}\tilde{\mathbf{W}}) \tag{12a}$$
$$\text{s.t.} \quad \mathbf{E}_{ii} \tilde{\mathbf{W}}_{ii} \leq t P_{\mathrm{Rl,max}}^{(i)}, \quad i = 1 \ldots R \tag{12b}$$
$$\gamma_{\mathrm{T}}\, \mathrm{tr}\left((\mathbf{B} + \mathbf{C} + \mathbf{D})\tilde{\mathbf{W}}\right) + t\left(\gamma_{\mathrm{T}} \sigma_{\mathrm{p}}^2 - P_{\mathrm{p}} |h_{\mathrm{pp}}|^2\right) \leq 0 \tag{12c}$$
$$\tilde{\mathbf{W}} \succeq 0 \tag{12d}$$
$$\mathrm{tr}\left((\mathbf{R} + \mathbf{V})\tilde{\mathbf{W}}\right) + t\left(P_{\mathrm{p}} |h_{\mathrm{ps}}|^2 + \sigma_{\mathrm{s}}^2\right) = 1 \tag{12e}$$
$$t \geq 0. \tag{12f}$$

Problem (12) is a convex optimisation problem and can be solved using interior point methods. After solving this problem, the beamforming matrix is obtained by dividing $\tilde{\mathbf{W}}$ by t, i.e., $\mathbf{W} = \tilde{\mathbf{W}}/t$. The optimum beamforming vector, $\mathbf{w}^*$, is given by the principle eigenvector of $\mathbf{W}$.

IV. Robust Beamformer Optimisation

So far we have assumed that perfect CSI of all links is available at the SU system. Unfortunately, in practise, perfect CSI for all links is seldom available and the assumption of perfect CSI may be unrealistic. For our analysis, we assume that the channels for the $\mathrm{SU_{Tx}}$ to $\mathrm{SU_{Rl}}$ and $\mathrm{SU_{Rl}}$ to $\mathrm{SU_{Rx}}$ links are accurately known through the SU's channel estimation procedure and those between the $\mathrm{PU_{Tx}}$ and $\mathrm{SU_{Rl}}$ can be accurately measured, for example, through knowledge of the PU pilot symbols. In this section we formulate a number of robust optimisation problems based on varying levels of uncertainty on the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$ and $\mathrm{SU_{Rl}}$ to $\mathrm{PU_{Rx}}$ links. In a cognitive radio system, this may correspond to the level of cooperation between the primary and secondary systems. Generally, CSI of the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$ link would be the most difficult to obtain since this link is fully isolated from the SU system. The SU would have to rely on the PU to provide this information and the CSI quality would depend on the level of cooperation between the two systems. In our robust beamformer formulations, we assume that the SU system has only partial CSI for the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$ link, specifically, we assume that only the mean channel power, Ω_{pp}, of this link is provided by the PU. CSI of the $\mathrm{SU_{Rl}}$

to $\mathrm{PU_{Rx}}$ link would also be difficult to acquire and cooperation with the PU would be needed. However, if the PU system had a bidirectional link, then the SU could estimate the CSI of the $\mathrm{PU_{Rx}}$ to $\mathrm{SU_{Rl}}$ link when the $\mathrm{PU_{Rx}}$ assumes the role of a transmitter. In this paper, we design robust beamformers based on the quality of the CSI of this link that is available to the SU. We focus on three levels of quality: i) perfect CSI; ii) imperfect CSI; and iii) highly quantised CSI in the form of mean channel powers.

In our formulation we consider the PU outage probability as a QoS parameter. The outage probability constraint is generally referred to as a soft constraint and tends to be more flexible than a worst-case constraint [22]. In the system under consideration, outage occurs when the PU SINR, γ_{p}, falls below the PU SINR threshold, γ_{T}. The outage probability is expressed as

$$\begin{aligned}\mathrm{P_o} &= \Pr\{\gamma_{\mathrm{p}} \leq \gamma_{\mathrm{T}}\}\\ &= \Pr\left\{\frac{P_{\mathrm{p}}|h_{\mathrm{pp}}|^2}{\mathbf{w}^H(\mathbf{B}+\mathbf{C}+\mathbf{D})\mathbf{w}+\sigma_{\mathrm{p}}^2} \leq \gamma_{\mathrm{T}}\right\}. \end{aligned}\tag{13}$$

Hence, given a maximum allowable outage probability, $\mathrm{P}_{o,\max}$, constraints (9b) and (10c) are replaced with

$$\Pr\left\{\frac{P_{\mathrm{p}}|h_{\mathrm{pp}}|^2}{\mathbf{w}^H(\mathbf{B}+\mathbf{C}+\mathbf{D})\mathbf{w}+\sigma_{\mathrm{p}}^2} \leq \gamma_{\mathrm{T}}\right\} \leq \mathrm{P_{o,max}}. \tag{14}$$

A. *Partial CSI Availability for the* $\mathrm{PU_{Tx}}$ *to* $\mathrm{PU_{Rx}}$ *Link*

In this section, we assume that perfect CSI is available for all links except for the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$ link. We assume that only the mean channel power, Ω_{pp}, of the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$ link is available, i.e., instantaneous channel realisation is not available. Since h_{pp} is a zero-mean Gaussian random variable, $|h_{\mathrm{pp}}|^2$ is exponentially distributed and therefore the outage probability can be expressed as

$$\mathrm{P_o} = 1-\exp\left(-\frac{\gamma_{\mathrm{T}}\left(\mathbf{w}^H(\mathbf{B}+\mathbf{C}+\mathbf{D})\mathbf{w}+\sigma_{\mathrm{p}}^2\right)}{P_{\mathrm{p}}\Omega_{\mathrm{pp}}}\right). \tag{15}$$

Using (15), the PU outage probability constraint (14) can then be stated as

$$\mathbf{w}^H(\mathbf{B}+\mathbf{C}+\mathbf{D})\mathbf{w}+\sigma_{\mathrm{p}}^2+\frac{P_{\mathrm{p}}\Omega_{\mathrm{pp}}}{\gamma_{\mathrm{T}}}\log(1-\mathrm{P_{o,max}}) \leq 0, \tag{16}$$

or equivalently as the following SOCP constraint

$$\sqrt{-P_{\mathrm{p}}\Omega_{\mathrm{pp}}\log(1-\mathrm{P_{o,max}})} \geq \sqrt{\gamma_{\mathrm{T}}}\left\|\begin{matrix}\sqrt{P_{\mathrm{s}}}[\mathbf{h}_{\mathrm{sr}}\odot\mathbf{h}_{\mathrm{rp}}]^H\mathbf{w}\\ \sqrt{P_{\mathrm{p}}}[\mathbf{h}_{\mathrm{pr}}\odot\mathbf{h}_{\mathrm{rp}}]^H\mathbf{w}\\ \sigma_{\mathrm{r}}[\mathbf{h}_{\mathrm{rp}}\odot\mathbf{w}]\\ \sigma_{\mathrm{p}}\end{matrix}\right\|. \tag{17}$$

The robust $\mathrm{SU_{Rl}}$ power minimisation problem in this scenario is therefore expressed as the following SOCP

$$\min_{\mathbf{w}} \quad \mathbf{w}^H\mathbf{E}\mathbf{w}, \quad \text{s.t.} \quad \text{(9c) and (17)}. \tag{18}$$

It is straightforward to show that the robust $\mathrm{SU_{Rx}}$ SINR maximisation problem is essentially the same as the relaxed form of problem (11) but with the instantaneous $\mathrm{PU_{Rx}}$ SINR constraint (11c) replaced by the PU outage probability constraint as shown below

$$\begin{aligned}\max_{\mathbf{W}} \quad & \frac{\mathrm{tr}(\mathbf{QW})}{\mathrm{tr}\left((\mathbf{R}+\mathbf{V})\mathbf{W}\right)+P_{\mathrm{p}}|h_{\mathrm{ps}}|^2+\sigma_{\mathrm{s}}^2}\\ \text{s.t.} \quad & \text{(11b) and (11d)}\\ & \mathrm{tr}\left((\mathbf{B}+\mathbf{C}+\mathbf{D})\mathbf{W}\right)+\sigma_{\mathrm{p}}^2\\ & \quad +\frac{P_{\mathrm{p}}\Omega_{\mathrm{pp}}}{\gamma_{\mathrm{T}}}\log(1-\mathrm{P_{o,max}}) \leq 0. \end{aligned}\tag{19}$$

The solution of problem (19) can be found using the method described in Section III-B.

B. *Partial CSI Availability for the* $\mathrm{PU_{Tx}}$ *to* $\mathrm{PU_{Rx}}$ *and* $\mathrm{SU_{Rl}}$ *to* $\mathrm{PU_{Rx}}$ *Links*

In this section, we summarise our main findings from [23] for the scenario where full CSI is available for all links except for the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$ and $\mathrm{SU_{Rl}}$ to $\mathrm{PU_{Rx}}$ links. The assumption is that only the mean channel powers, Ω_{pp} and $\Omega_{\mathrm{rp}}^{(i)}\forall i$, of the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$ and $\mathrm{SU_{Rl}}$ to $\mathrm{PU_{Rx}}$ links are available.

The PU outage probability expression can be rewritten as follows

$$\mathrm{P_o} = \Pr\left\{P_{\mathrm{p}}|h_{\mathrm{pp}}|^2-\gamma_{\mathrm{T}}\mathbf{w}^H(\mathbf{B}+\mathbf{C}+\mathbf{D})\mathbf{w} \leq \gamma_{\mathrm{T}}\sigma_{\mathrm{p}}^2\right\}. \tag{20}$$

In (20), $P_{\mathrm{p}}|h_{\mathrm{pp}}|^2$ is known to have an exponential distribution with a mean of $P_{\mathrm{p}}\Omega_{\mathrm{pp}}$. Using Lemma 1 in [23], $\gamma_{\mathrm{T}}\mathbf{w}^H(\mathbf{B}+\mathbf{C}+\mathbf{D})\mathbf{w}$ was shown to be the sum of $R+2$ exponentially distributed independent random variables with the rate parameters $\lambda_i = 1/(\gamma_{\mathrm{T}}\sigma_{\mathrm{r}}^2\Omega_{\mathrm{rp}}^{(i)}\mathbf{W}_{ii})$, $i=1\ldots R$, $\lambda_{R+1}=1/\mathrm{tr}(\mathbf{\Sigma}_B\mathbf{W})$ and $\lambda_{R+2}=1/\mathrm{tr}(\mathbf{\Sigma}_C\mathbf{W})$. Here, $\mathbf{W}=\mathbf{w}\mathbf{w}^H$, $\mathbf{\Sigma}_B=\gamma_{\mathrm{T}}P_{\mathrm{s}}\mathrm{diag}(\mathbf{\Omega}_{\mathrm{rp}}\odot|\mathbf{h}_{\mathrm{sr}}|^2)$, $\mathbf{\Sigma}_C=\gamma_{\mathrm{T}}P_{\mathrm{p}}\mathrm{diag}(\mathbf{\Omega}_{\mathrm{rp}}\odot|\mathbf{h}_{\mathrm{pr}}|^2)$ and $\mathbf{\Omega}_{\mathrm{rp}}=\left[\Omega_{\mathrm{rp}}^{(1)}\Omega_{\mathrm{rp}}^{(2)}\ldots\Omega_{\mathrm{rp}}^{(R)}\right]^T$.

Hence, the PDF in (20) is that of a difference between an exponential random variable and the sum of $R+2$ exponentially distributed random variables, and therefore the outage probability constraint can be expressed as [23]

$$\prod_{i=1}^{R+2}\left(1+\frac{1}{P_{\mathrm{p}}\Omega_{\mathrm{pp}}\lambda_i}\right) \leq \frac{\exp\left(-\frac{\gamma_{\mathrm{T}}\sigma_{\mathrm{p}}^2}{P_{\mathrm{p}}\Omega_{\mathrm{pp}}}\right)}{1-\mathrm{P_{o,max}}}. \tag{21}$$

Note that constraint (21) is nonconvex (the term on the left hand side is in fact concave), and is difficult to handle. For this reason, the geometric-arithmetic mean inequality was used to replace the left hand side of (21) with its upper bound. The tightened convex outage probability constraint is thus given by [23]

$$\begin{aligned}&\frac{1}{P_{\mathrm{p}}\Omega_{\mathrm{pp}}}\mathbf{w}^H\left(\mathbf{\Sigma}_B+\mathbf{\Sigma}_C+\gamma_{\mathrm{T}}\sigma_{\mathrm{r}}^2\mathrm{diag}(\mathbf{\Omega}_{\mathrm{rp}})\right)\mathbf{w}\\ &\quad+(R+2)\left(1-\left(\frac{\exp\left(-\frac{\gamma_{\mathrm{T}}\sigma_{\mathrm{p}}^2}{P_{\mathrm{p}}\Omega_{\mathrm{pp}}}\right)}{1-\mathrm{P_{o,max}}}\right)^{\frac{1}{R+2}}\right) \leq 0, \end{aligned}\tag{22}$$

and the equivalent SOCP constraint by

$$\sqrt{(R+2)\left(\left(\frac{\exp\left(-\frac{\gamma_{\mathrm{T}}\sigma_{\mathrm{p}}^2}{P_{\mathrm{p}}\Omega_{\mathrm{pp}}}\right)}{1-\mathrm{P}_{\mathrm{o,max}}}\right)^{\frac{1}{R+2}}-1\right)} \geq \sqrt{\frac{\gamma_{\mathrm{T}}}{P_{\mathrm{p}}\Omega_{\mathrm{pp}}}}\left\|\begin{matrix}\sqrt{P_{\mathrm{s}}}[\boldsymbol{\Omega}_{\mathrm{rp}}^{1/2}\odot\mathbf{h}_{\mathrm{sr}}\odot\mathbf{w}]\\ \sqrt{P_{\mathrm{p}}}[\boldsymbol{\Omega}_{\mathrm{rp}}^{1/2}\odot\mathbf{h}_{\mathrm{pr}}\odot\mathbf{w}]\\ \sigma_{\mathrm{r}}[\boldsymbol{\Omega}_{\mathrm{rp}}^{1/2}\odot\mathbf{w}]\end{matrix}\right\|, \quad (23)$$

where $\boldsymbol{\Omega}_{\mathrm{rp}}^{1/2}$ is the element-wise square root of the vector $\boldsymbol{\Omega}_{\mathrm{rp}}$.

The robust $\mathrm{SU_{Rl}}$ power minimisation SOCP can therefore be expressed as

$$\min_{\mathbf{w}} \quad \mathbf{w}^H\mathbf{E}\mathbf{w}, \quad \text{s.t.} \quad \text{(9c) and (23)}. \quad (24)$$

By directly using constraint (22), the robust $\mathrm{SU_{Rx}}$ SINR maximisation problem can be expressed as

$$\max_{\mathbf{W}} \quad \frac{\mathrm{tr}(\mathbf{QW})}{\mathrm{tr}\left((\mathbf{R}+\mathbf{V})\mathbf{W}\right)+P_{\mathrm{p}}|h_{\mathrm{ps}}|^2+\sigma_{\mathrm{s}}^2}$$
$$\text{s.t.} \quad \text{(11b), (11d), and (22)}, \quad (25)$$

which can be solved using the methods described in Section III-B.

Using the outage probability upper bound results in tightening of the constraint. In the $\mathrm{SU_{Rl}}$ power minimisation problem, this tightening may result in some feasible problems appearing infeasible. Likewise, the $\mathrm{SU_{Rx}}$ SINR maximisation problem may become infeasible or the solution obtained may be suboptimal since the power allocated to the beamformer would be less than what would have been allocated if the original constraint was used. Iterative algorithms to obtain the optimum solutions of (24) and (25) were proposed in [23]. However, through extensive numerical simulations, it was found that the solutions obtained by directly solving problems (24) and (25) with the tightened outage probability constraint are very close to the optimum and, in practice, it is not necessary to use the iterative algorithms.

C. Partial CSI Availability for the $\mathrm{PU_{Tx}}$ *to* $\mathrm{PU_{Rx}}$ *Link and Imperfect CSI Availability for the* $\mathrm{SU_{Rl}}$ *to* $\mathrm{PU_{Rx}}$ *Links*

In this section, we assume that full CSI is available for all links except for the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$ and $\mathrm{SU_{Rl}}$ to $\mathrm{PU_{Rx}}$ links. We assume that only the mean channel power, Ω_{pp}, of the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$ link is available and that $\mathrm{SU_{Rl}}$ to $\mathrm{PU_{Rx}}$ link CSI is imperfect. This imperfection may be due to estimation errors or other factors such as quantisation. Perfect CSI for all other links is available. Our aim is to design a beamformer that is robust against CSI imperfections due to estimation errors for one particular realisation of the $\mathrm{SU_{Rl}}$ to $\mathrm{PU_{Rx}}$ channel. The $\mathrm{SU_{Rl}}$ to $\mathrm{PU_{Rx}}$ Rayleigh channel, having been instantiated becomes a deterministic unknown. We model this unknown as having non-zero mean, equal to the channel estimate, and small variance, corresponding to the channel uncertainty. (By contrast, in Section IV-B the Rayleigh channel has zero mean, and large variance, equal to the channel power). Adopting the imperfect CSI model of [29], [30], we have

$$\mathbf{h}_{\mathrm{rp}} = \tilde{\mathbf{h}}_{\mathrm{rp}} + \rho\mathbf{e}, \quad (26)$$

where $\tilde{\mathbf{h}}_{\mathrm{rp}}$ is the imperfect $\mathrm{SU_{Rl}}$ to $\mathrm{PU_{Rx}}$ link CSI estimate and $\mathbf{e}$ is the zero mean estimation error vector with independently distributed complex Gaussian entries and the diagonal covariance matrix $\boldsymbol{\Sigma}_{\mathrm{e}} = (\|\boldsymbol{\Omega}_{\mathrm{rp}}^{1/2}\|^2/R)\mathbf{I}$, i.e., $\mathbf{e}\sim\mathcal{N}_{\mathcal{C}}(\mathbf{0},\boldsymbol{\Sigma}_{\mathrm{e}})$. We assume that $\tilde{\mathbf{h}}_{\mathrm{rp}}$ is obtained using an unbiased maximum likelihood estimator, hence, over the ensemble of all realisations of the $\mathrm{SU_{Rl}}$ to $\mathrm{PU_{Rx}}$ channel, $\tilde{\mathbf{h}}_{\mathrm{rp}}$ is distributed as $\mathcal{N}_{\mathcal{C}}(\mathbf{0},\mathrm{diag}(\boldsymbol{\Omega}_{\mathrm{rp}})-\rho^2\boldsymbol{\Sigma}_{\mathrm{e}})$. For the purpose of constructing an optimisation problem, an instance of $\tilde{\mathbf{h}}_{\mathrm{rp}}$ is drawn from this distribution and treated as a deterministic constant. $0\leq\rho\leq(\min(\boldsymbol{\Omega}_{\mathrm{rp}})/(\|\boldsymbol{\Omega}_{\mathrm{rp}}^{1/2}\|^2/R))^{1/2}$ determines the quality of the CSI, which is perfect when $\rho=0$ and completely uncertain when $\rho=(\min(\boldsymbol{\Omega}_{\mathrm{rp}})/(\|\boldsymbol{\Omega}_{\mathrm{rp}}^{1/2}\|^2/R))^{1/2}$. Since $\min(\boldsymbol{\Omega}_{\mathrm{rp}})\leq\|\boldsymbol{\Omega}_{\mathrm{rp}}^{1/2}\|^2/R$, the maximum value of ρ is 1, which occurs when all elements of $\boldsymbol{\Omega}_{\mathrm{rp}}$ are equal.

Note that our definition of the error covariance matrix implies that the entries are i.i.d.; however, if the entries have different variances—for instance, the quality of the CSI estimate obtained at each relay node may be different from each other—then the definition can easily be modified to accommodate this without affecting the analysis that follows.

To find an expression for the outage probability (20), we first note that using (26), $\gamma_{\mathrm{T}}\mathbf{w}^H(\mathbf{B}+\mathbf{C}+\mathbf{D})\mathbf{w}$ can be expressed as

$$\begin{aligned}
&\gamma_{\mathrm{T}}\mathbf{w}^H(\mathbf{B}+\mathbf{C}+\mathbf{D})\mathbf{w}\\
&= 2\gamma_{\mathrm{T}}P_{\mathrm{s}}\rho\Re\left\{\mathbf{w}^H[\mathbf{h}_{\mathrm{sr}}\odot\tilde{\mathbf{h}}_{\mathrm{rp}}][\mathbf{h}_{\mathrm{sr}}\odot\mathbf{e}]^H\mathbf{w}\right\}\\
&+ 2\gamma_{\mathrm{T}}P_{\mathrm{p}}\rho\Re\left\{\mathbf{w}^H[\mathbf{h}_{\mathrm{pr}}\odot\tilde{\mathbf{h}}_{\mathrm{rp}}][\mathbf{h}_{\mathrm{pr}}\odot\mathbf{e}]^H\mathbf{w}\right\}\\
&+ 2\gamma_{\mathrm{T}}\sigma_{\mathrm{r}}^2\rho\Re\left\{\mathbf{w}^H\mathrm{diag}\left(\left(\tilde{\mathbf{h}}_{\mathrm{rp}}^H\right)^T\odot\mathbf{e}\right)\mathbf{w}\right\}\\
&+ \gamma_{\mathrm{T}}P_{\mathrm{s}}\rho^2\mathbf{w}^H[\mathbf{h}_{\mathrm{sr}}\odot\mathbf{e}][\mathbf{h}_{\mathrm{sr}}\odot\mathbf{e}]^H\mathbf{w}\\
&+ \gamma_{\mathrm{T}}P_{\mathrm{p}}\rho^2\mathbf{w}^H[\mathbf{h}_{\mathrm{pr}}\odot\mathbf{e}][\mathbf{h}_{\mathrm{pr}}\odot\mathbf{e}]^H\mathbf{w}\\
&+ \gamma_{\mathrm{T}}\sigma_{\mathrm{r}}^2\rho^2\mathbf{w}^H\mathrm{diag}\left(|\mathbf{e}|^2\right)\mathbf{w}\\
&+ \gamma_{\mathrm{T}}\mathbf{w}^H\Big[P_{\mathrm{s}}[\mathbf{h}_{\mathrm{sr}}\odot\tilde{\mathbf{h}}_{\mathrm{rp}}][\mathbf{h}_{\mathrm{sr}}\odot\tilde{\mathbf{h}}_{\mathrm{rp}}]^H\\
&\qquad + P_{\mathrm{p}}[\mathbf{h}_{\mathrm{pr}}\odot\tilde{\mathbf{h}}_{\mathrm{rp}}][\mathbf{h}_{\mathrm{pr}}\odot\tilde{\mathbf{h}}_{\mathrm{rp}}]^H+\sigma_{\mathrm{r}}^2\mathrm{diag}\left(|\tilde{\mathbf{h}}_{\mathrm{rp}}|^2\right)\Big]\mathbf{w}.
\end{aligned} \quad (27)$$

The terms on the right hand side of (27) are denoted by $r_1, r_2, \ldots, r_7$ PDFs of which are given by the following lemma.

Lemma 1: r_1, r_2 and r_3 are zero mean Gaussian random variables with variances, σ_1^2, σ_2^2 and σ_1^3 given by

$$\begin{aligned}
\sigma_1^2 &= 2\gamma_{\mathrm{T}}^2P_{\mathrm{s}}^2\mathrm{tr}\left(\left(\mathbf{h}_{\mathrm{sr}}\mathbf{h}_{\mathrm{sr}}^H\odot\rho^2\boldsymbol{\Sigma}_{\mathrm{e}}\right)\mathbf{W}\right)\\
&\quad\mathrm{tr}\left([\mathbf{h}_{\mathrm{sr}}\odot\tilde{\mathbf{h}}_{\mathrm{rp}}][\mathbf{h}_{\mathrm{sr}}\odot\tilde{\mathbf{h}}_{\mathrm{rp}}]^H\mathbf{W}\right)
\end{aligned} \quad (28)$$

$$\begin{aligned}
\sigma_2^2 &= 2\gamma_{\mathrm{T}}^2P_{\mathrm{p}}^2\mathrm{tr}\left(\left(\mathbf{h}_{\mathrm{pr}}\mathbf{h}_{\mathrm{pr}}^H\odot\rho^2\boldsymbol{\Sigma}_{\mathrm{e}}\right)\mathbf{W}\right)\\
&\quad\mathrm{tr}\left([\mathbf{h}_{\mathrm{pr}}\odot\tilde{\mathbf{h}}_{\mathrm{rp}}][\mathbf{h}_{\mathrm{pr}}\odot\tilde{\mathbf{h}}_{\mathrm{rp}}]^H\mathbf{W}\right),
\end{aligned} \quad (29)$$

$$\sigma_3^2 = 2\gamma_{\mathrm{T}}^2\sigma_{\mathrm{r}}^4\left(\mathrm{vec}(\mathbf{W}^H)\right)^H\boldsymbol{\Sigma}_{\tilde{E}}\mathrm{vec}(\mathbf{W}^H), \quad (30)$$

where $\mathbf{W} = \mathbf{w}\mathbf{w}^H$ and $\mathbf{\Sigma}_{\tilde{E}} = \mathbb{E}\{\mathrm{vec}(\tilde{\mathbf{E}})\mathrm{vec}(\tilde{\mathbf{E}})^H\}$ is an $R^2 \times R^2$ diagonal matrix with entries on the main diagonal given by, $\mathbf{\Sigma}_{\tilde{E}_{jj}} = \rho^2 \mathbf{\Sigma}_{\mathrm{e}_{ii}} |\tilde{h}_{\mathrm{rp}}^{(i)}|^2, i = 1 \ldots R, j = i(R+1) - R$, and zeros everywhere else.

r_4 and r_5 are exponentially distributed random variables with means, μ_4 and μ_5 given by

$$\mu_4 = \gamma_{\mathrm{T}} P_{\mathrm{s}} \mathrm{tr}\left(\left(\mathbf{h}_{\mathrm{sr}}\mathbf{h}_{\mathrm{sr}}^H \odot \rho^2 \mathbf{\Sigma}_{\mathrm{e}}\right)\mathbf{W}\right), \tag{31}$$

$$\mu_5 = \gamma_{\mathrm{T}} P_{\mathrm{p}} \mathrm{tr}\left(\left(\mathbf{h}_{\mathrm{pr}}\mathbf{h}_{\mathrm{pr}}^H \odot \rho^2 \mathbf{\Sigma}_{\mathrm{e}}\right)\mathbf{W}\right). \tag{32}$$

r_6 is a sum of R independent exponentially distributed random variables with rate parameters $\lambda_i = 1/(\gamma_{\mathrm{T}}\sigma_{\mathrm{r}}^2 \rho^2 \mathbf{\Sigma}_{\mathrm{e}_{ii}} \mathbf{W}_{ii})$, $i = 1 \ldots R$ and the mean and variance, μ_6 and σ_6^2, respectively, given by

$$\mu_6 = \left[\prod_{i=1}^{R} \lambda_i\right] \sum_{j=1}^{R} \frac{1}{\lambda_j^2 \prod_{k=1,k\neq j}^{N}(\lambda_k - \lambda_j)}, \tag{33}$$

$$\sigma_6^2 = \left[\prod_{i=1}^{R} \lambda_i\right] \sum_{j=1}^{R} \frac{2}{\lambda_j^3 \prod_{k=1,k\neq j}^{N}(\lambda_k - \lambda_j)} - \mu_6^2. \tag{34}$$

r_7 is a deterministic constant.

Proof: The proof is given in Appendix A.

Due to the correlation between the terms of (27), its exact PDF is difficult to handle. However, we propose an accurate approximation of the PDF which is easier to handle based on the following observation. In a practical cognitive radio system, the PU requires a very reliable link, hence the outage probability specified will generally be very small. To satisfy the stringent outage probability constraint, both σ_1^2 and σ_2^2 must also be small. Notice that the expression for σ_1^2 contains the term $P_{\mathrm{s}}\mathrm{tr}\left((\mathbf{h}_{\mathrm{sr}}\mathbf{h}_{\mathrm{sr}}^H \odot \rho^2 \mathbf{\Sigma}_{\mathrm{e}})\mathbf{W}\right)$, which can be rewritten as $P_{\mathrm{s}} \sum_{i=1}^{R} \rho^2 \mathbf{\Sigma}_{\mathrm{e}_{ii}} |h_{\mathrm{s}}^{(i)}|^2 \mathbf{W}_{ii}$. This term represents the SU interference that is generated at the $\mathrm{PU_{Rx}}$ due to CSI errors, and its level can only be controlled by adjusting the beamformer transmit power. Hence, as the $\mathrm{SU_{Tx}}$ to $\mathrm{SU_{Rl}}$ link gets stronger, the beamformer weights will be scaled down to achieve the outage probability constraint. Note that this term also appears in μ_4, which is used in our final approximation, (42), of the PU outage probability constraint and its magnitude is controlled by controlling the magnitude of μ_4. We note that the beamformer is able to control interference from the $P_{\mathrm{s}}\mathrm{tr}\left([\mathbf{h}_{\mathrm{sr}} \odot \tilde{\mathbf{h}}_{\mathrm{rp}}][\mathbf{h}_{\mathrm{sr}} \odot \tilde{\mathbf{h}}_{\mathrm{rp}}]^H \mathbf{W}\right)$ part of σ_1^2 through both amplitude and phase control and is able to keep it sufficiently low to satisfy the outage probability constraint. Again, note that this term appears in the deterministic constant r_7, which is used in (42). Hence, the magnitude of this term is controlled by controlling the magnitude of r_7.

In the $\mathrm{SU_{Rx}}$ SINR maximisation problem (10), the individual relay transmit power constraints also limit the beamformer weight magnitudes, which in turn limit the levels of σ_1^2 and σ_2^2. From the definition of $\mathbf{E}$ and (10b), we see that for a fixed value of $P_{\mathrm{Rl,max}}^{(i)}$, the maximum value the ith beamformer weight magnitude can take decreases as either the $\mathrm{SU_{Tx}}$ or $\mathrm{PU_{Tx}}$ to the ith relay link gets stronger.

The expression for σ_2^2 contains two terms that represent PU self interference the level of which is controlled in a similar way to that described above, i.e., by controlling the levels of μ_5 and r_7, both of which appear in (42). Since both σ_1^2 and σ_2^2 are expected to be small, the PDF of r_1 and r_2 will be concentrated around zero and can be neglected.

Note that σ_{r}^2 is generally small—for instance, a receiver with a 2 MHz bandwidth and a noise figure (NF) of 30 dB operating at a room temperature of 293 K has an effective noise power of approximately -80 dBm—σ_3^2 is very small and therefore, the PDF of r_3 is concentrated around zero and can be safely ignored. Similarly, both μ_6 and σ_6^2 are very small and the PDF of r_6 is also concentrated near zero and can be neglected.

From the above discussion, we see that the PDF of (27) can be approximated as the sum of two correlated exponentially distributed random variables r_4 and r_5. Next, we show that the correlation between r_4 and r_5 is small and therefore they can be treated as independent random variables. By letting $\mathbf{H}_1 = \mathbf{h}_{\mathrm{sr}}\mathbf{h}_{\mathrm{sr}}^H \odot \rho^2 \mathbf{e}\mathbf{e}^H$ and $\mathbf{H}_2 = \mathbf{h}_{\mathrm{pr}}\mathbf{h}_{\mathrm{pr}}^H \odot \rho^2 \mathbf{e}\mathbf{e}^H$, the covariance between r_4 and r_5 is given by

$$\begin{aligned}\mathrm{Cov}(r_4, r_5) &= \gamma_{\mathrm{T}}^2 P_{\mathrm{s}}^2 \mathbb{E}\left\{\mathrm{tr}\left(\mathbf{W}\mathbf{H}_1\right)\mathrm{tr}\left(\mathbf{W}\mathbf{H}_2\right)^*\right\} - \mu_4\mu_5 \\ &= \gamma_{\mathrm{T}}^2 P_{\mathrm{s}}^2 \,\mathrm{vec}(\mathbf{W}^H)^H \mathbb{E}\left\{\mathrm{vec}(\mathbf{H}_1)\mathrm{vec}(\mathbf{H}_2)^H\right\}\mathrm{vec}(\mathbf{W}^H) \\ &\quad - \mu_4\mu_5 \\ &= \gamma_{\mathrm{T}}^2 P_{\mathrm{s}}^2 \sum_{i=1}^{R}\sum_{j=1}^{R} \left(\mathbf{h}_{\mathrm{sr}}\mathbf{h}_{\mathrm{sr}}^H\right)_{ij} \left(\mathbf{h}_{\mathrm{pr}}\mathbf{h}_{\mathrm{pr}}^H\right)_{ij}^* \rho^4 \mathbf{\Sigma}_{\mathrm{e}_{ii}} \mathbf{\Sigma}_{\mathrm{e}_{jj}} |\mathbf{W}_{ij}|^2.\end{aligned} \tag{35}$$

It is evident from (35) that for small values of $\rho\mathbf{\Sigma}_{\mathrm{e}_{ii}} \forall i$, the covariance is low. Recall that when the $\mathrm{SU_{Tx}}$ to $\mathrm{SU_{Rl}}$ and $\mathrm{PU_{Tx}}$ to $\mathrm{SU_{Rl}}$ links are strong, the beamformer weights are scaled down to meet the outage probability constraint. In this scenario, $|\mathbf{W}_{ij}|^2 \forall i, j$ will be small and the covariance will tend to be low. Therefore, in our analysis, we treat r_4 and r_5 as independent random variables.

Hence, $\gamma_{\mathrm{T}}\mathbf{w}^H(\mathbf{B} + \mathbf{C} + \mathbf{D})\mathbf{w}$ can be approximated as

$$\gamma_{\mathrm{T}}\mathbf{w}^H(\mathbf{B} + \mathbf{C} + \mathbf{D})\mathbf{w} \approx r_4 + r_5 + r_7, \tag{36}$$

and the outage probability can be approximated by

$$\mathrm{P_o} \approx \Pr\left\{P_{\mathrm{p}}|h_{\mathrm{pp}}|^2 - (r_4 + r_5) \leq \gamma_{\mathrm{T}}\sigma_{\mathrm{p}}^2 + r_7\right\}. \tag{37}$$

Note that the PDF in (37) is the difference between an exponentially distributed random variable and the sum of two independent exponentially distributed random variables. In Fig. 2, we show a comparison of the empirical CDF obtained through Monte Carlo simulations and the approximation (37) for $\rho = 0.5$ in three channel conditions where the signal to interference channel power ratios (SICR) are set to 8 dB, 3 dB and 0.8 dB, i.e., $\Omega_{\mathrm{s}}^{(i)}/\Omega_{\mathrm{pr}} = \Omega_{\mathrm{rs}}^{(i)}/\Omega_{\mathrm{ps}} = \Omega_{\mathrm{pp}}/\Omega_{\mathrm{rp}}^{(i)} = \{8, 3, 0.8\}$ dB $\forall i$. In all three cases, there are 8 relay nodes, $P_{\mathrm{p}} = P_{\mathrm{s}} = 30$ dBm, $P_{\mathrm{Rl,max}}^{(i)} = 30$ dBm $\forall i$, $\gamma_{\mathrm{T}} = 5$ dB, $\gamma_{\mathrm{s,min}} = 0$ dB, noise power at each receiver is assumed to be -80 dBm, the maximum $\mathrm{PU_{Rx}}$ outage probability, $\mathrm{P}_{o,\max}$, is set to 5% and $\mathbf{\Sigma}_{\mathrm{e}_{ii}} = \|\mathbf{\Omega}_{\mathrm{rp}}^{1/2}\|^2/8$, $\forall i$. Due to space constraints, the empirical and

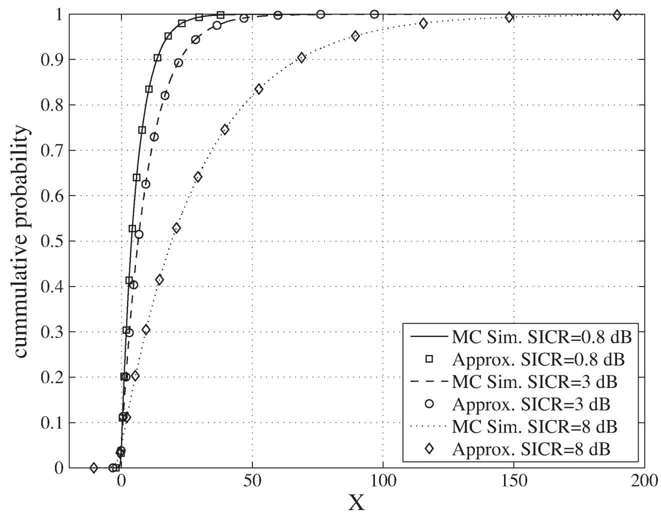

Fig. 2. Empirical and Approximated CDF of (37).

approximated CDF for each channel condition is shown only for one realisation of the channel vectors, where the vectors have been scaled to obtain the required SICR. However, the approximation holds for any realisation of the channel vectors, since no assumptions have been made about channel vectors in its derivation. The empirical and approximated CDF for each channel condition is obtained by first designing a robust beamformer for $\mathrm{SU_{Rx}}$ SINR maximisation problem (44) and then using the resulting beamformer in Monte Carlo simulations and in the analytical expression for the approximation. It is evident that the approximation accurately represents the empirical CDF. Similar results are obtained for the robust $\mathrm{SU_{Rl}}$ transmit power minimisation problem (43).

Using the approximation in (37), the outage probability is expressed as

$$\mathrm{P_o}=1-\exp\left(-\frac{\gamma_\mathrm{T}\sigma_\mathrm{p}^2+r_7}{P_\mathrm{p}\Omega_\mathrm{pp}}\right)\left(\frac{1}{1+\frac{\mu_4}{P_\mathrm{p}\Omega_\mathrm{pp}}}\right)\left(\frac{1}{1+\frac{\mu_5}{P_\mathrm{p}\Omega_\mathrm{pp}}}\right), \tag{38}$$

and the outage probability constraint is given by

$$\exp\left(\frac{r_7}{P_\mathrm{p}\Omega_\mathrm{pp}}\right)\left(1+\frac{\mu_4}{P_\mathrm{p}\Omega_\mathrm{pp}}\right)\left(1+\frac{\mu_5}{P_\mathrm{p}\Omega_\mathrm{pp}}\right) \le \frac{\exp\left(-\frac{\gamma_\mathrm{T}\sigma_\mathrm{p}^2}{P_\mathrm{p}\Omega_\mathrm{pp}}\right)}{1-\mathrm{P_{o,max}}}. \tag{39}$$

It is worth noting that, when there are no $\mathrm{SU_{Rl}}$ to $\mathrm{PU_{Rx}}$ link CSI errors, constraint (39) reduces to constraint (16). This is expected since the only channel uncertainty remaining is in the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$ link, which was analysed in Section IV-A.

We use the geometric-arithmetic mean inequality and rewrite (39) as

$$\exp\left(\frac{r_7}{P_\mathrm{p}\Omega_\mathrm{pp}}\right)+\left(1+\frac{\mu_4}{P_\mathrm{p}\Omega_\mathrm{pp}}\right)+\left(1+\frac{\mu_5}{P_\mathrm{p}\Omega_\mathrm{pp}}\right) \le 3\left(\frac{\exp\left(-\frac{\gamma_\mathrm{T}\sigma_\mathrm{p}^2}{P_\mathrm{p}\Omega_\mathrm{pp}}\right)}{1-\mathrm{P_{o,max}}}\right)^{\frac{1}{3}}. \tag{40}$$

Note that (40) is a non-convex constraint and is difficult to handle. However, the assumptions that were made to obtain the approximate outage probability expression also imply that r_7 is small. Thus, $\exp(r_7/(P_\mathrm{p}\Omega_\mathrm{pp})) \approx (1+r_7/(P_\mathrm{p}\Omega_\mathrm{pp}))$, allowing us to write the outage probability constraint as the convex constraint

$$\frac{1}{P_\mathrm{p}\Omega_\mathrm{pp}}(r_7+\mu_4+\mu_5)+3\left(1-\left(\frac{\exp\left(-\frac{\gamma_\mathrm{T}\sigma_\mathrm{p}^2}{P_\mathrm{p}\Omega_\mathrm{pp}}\right)}{1-\mathrm{P_{o,max}}}\right)^{\frac{1}{3}}\right) \le 0. \tag{41}$$

or equivalently as the SOCP

$$\sqrt{3\left(\left(\frac{\exp\left(-\frac{\gamma_\mathrm{T}\sigma_\mathrm{p}^2}{P_\mathrm{p}\Omega_\mathrm{pp}}\right)}{1-\mathrm{P_{o,max}}}\right)^{\frac{1}{3}}-1\right)} \ge \sqrt{\frac{\gamma_\mathrm{T}}{P_\mathrm{p}\Omega_\mathrm{pp}}}\left\|\begin{matrix}\sqrt{P_\mathrm{s}}[\mathbf{h}_\mathrm{sr}\odot\tilde{\mathbf{h}}_\mathrm{rp}]^H\mathbf{w}\\ \sqrt{P_\mathrm{p}}[\mathbf{h}_\mathrm{pr}\odot\tilde{\mathbf{h}}_\mathrm{rp}]^H\mathbf{w}\\ \sigma_\mathrm{r}[\tilde{\mathbf{h}}_\mathrm{rp}\odot\mathbf{w}]\\ \sqrt{P_\mathrm{s}}\left[\mathrm{diag}\left(\left[\mathbf{h}_\mathrm{sr}\mathbf{h}_\mathrm{sr}^H\odot\rho^2\boldsymbol{\Sigma}_\mathrm{e}\right]^{\frac{1}{2}}\right)\odot\mathbf{w}\right]\\ \sqrt{P_\mathrm{p}}\left[\mathrm{diag}\left(\left[\mathbf{h}_\mathrm{pr}\mathbf{h}_\mathrm{pr}^H\odot\rho^2\boldsymbol{\Sigma}_\mathrm{e}\right]^{\frac{1}{2}}\right)\odot\mathbf{w}\right]\end{matrix}\right\|. \tag{42}$$

The robust $\mathrm{SU_{Rl}}$ power minimisation problem with the tightened outage probability SOCP constraint can therefore be expressed as

$$\min_{\mathbf{w}} \quad \mathbf{w}^H\mathbf{E}\mathbf{w}, \quad \text{s.t.} \quad (42) \text{ and } (9c). \tag{43}$$

The robust $\mathrm{SU_{Rx}}$ SINR maximisation problem can be expressed as

$$\begin{aligned}\max_{\mathbf{W}} \quad & \frac{\mathrm{tr}(\mathbf{QW})}{\mathrm{tr}\left((\mathbf{R}+\mathbf{V})\mathbf{W}\right)+P_\mathrm{p}|h_\mathrm{ps}|^2+\sigma_\mathrm{s}^2}\\ \text{s.t.} \quad & (11\text{b}) \text{ and } (11\text{d})\\ & \frac{1}{P_\mathrm{p}\Omega_\mathrm{pp}}(\tilde{r}_7+\mu_4+\mu_5)\\ & \quad \le 3\left(\left(\frac{\exp\left(-\frac{\gamma_\mathrm{T}\sigma_\mathrm{p}^2}{P_\mathrm{p}\Omega_\mathrm{pp}}\right)}{1-\mathrm{P_{o,max}}}\right)^{\frac{1}{3}}-1\right),\end{aligned} \tag{44}$$

where

$$\tilde{r}_7 \triangleq \gamma_\mathrm{T}\mathrm{tr}\left(\left(P_\mathrm{s}[\mathbf{h}_\mathrm{sr}\odot\tilde{\mathbf{h}}_\mathrm{rp}][\mathbf{h}_\mathrm{sr}\odot\tilde{\mathbf{h}}_\mathrm{rp}]^H+\sigma_\mathrm{r}^2\mathrm{diag}\left(|\tilde{\mathbf{h}}_\mathrm{rp}|^2\right) + P_\mathrm{p}[\mathbf{h}_\mathrm{pr}\odot\tilde{\mathbf{h}}_\mathrm{rp}][\mathbf{h}_\mathrm{pr}\odot\tilde{\mathbf{h}}_\mathrm{rp}]^H\right)\mathbf{W}\right).$$

Problem (44) can be solved using the method described in Section III-B.

Since problems (43) and (44) have the same form as (24) and (25), respectively, the iterative algorithms proposed in [23] can be used to improve on the solutions obtained by solving

(43) and (44). However, through our extensive numerical simulations, we have found that the improvements are marginal and do not motivate the use of the iterative algorithms.

V. Simulation Results and Discussion

We illustrate the performance of our proposed methods through numerical simulations in i.i.d. Rayleigh flat-fading channels. We consider a system with 8 relay nodes. In all simulations we have set $P_{\rm p} = P_{\rm s} = 30$ dBm, $P^{(i)}_{\rm Rl,max} = 30$ dBm $\forall\, i$, $\gamma_{\rm T} = 5$ dB and the noise power at each receiver is assumed to be -80 dBm, i.e., $\sigma^2_{\rm p} = \sigma^2_{\rm r} = \sigma^2_{\rm s} = -80$ dBm. The maximum $\mathrm{PU_{Rx}}$ outage probability, $\mathrm{P}_{o,\max}$, is set to 5%. Channel powers of the direct paths, i.e., $\Omega_{\rm pp}$, $\Omega^{(i)}_{\rm sr}\forall i$ and $\Omega^{(i)}_{\rm rs}\forall i$, are set to 10 dB. For our simulations we have set the SICR of all receivers to 5 dB. Simulations for the total relay power minimisation problem have $\gamma_{\rm s,min} = 0$ dB. According to CSI error model (26), $\mathbf{\Sigma}_{{\rm e}_{ii}} = \|\mathbf{\Omega}^{1/2}_{\rm rp}\|^2/8 = 5$ dB, $\forall\, i$. To illustrate the impact of CSI errors and the effectiveness of our proposed method, we present simulation results for four different values of ρ, namely, 0.05, 0.2, 0.3 and 0.5.

The results obtained from our methods are compared against the full CSI, worst-case and non-robust designs. As the name suggests, the worst-case beamformer guarantees that the SINR at the $\mathrm{PU_{Rx}}$ is above the threshold $\gamma_{\rm T}$ in the worst-case channel condition. Since instantaneous realisation of $h_{\rm pp}$ is not available for the beamformer design of Section IV-A, our worst-case design solves problems (8) and (10) based on the expected value of (7). Note that (7) is at its minimum when $|h_{\rm pp}|^2 = \Omega_{\rm pp} - \epsilon_1$ for some appropriately chosen value of $\epsilon_1 \geq 0$. The worst-case beamformer ensures that this minimum value is always above the threshold $\gamma_{\rm T}$. To provide a fair comparison with the methods proposed in this paper, ϵ_1 is chosen such that $\Pr\{|h_{\rm pp}|^2 \geq \Omega_{\rm pp} - \epsilon_1\} = 1 - \mathrm{P}_{o,\max}$. Similarly, the expected value of (7) is used to design the worst-case beamformer of Section IV-B since instantaneous realisations of both $h_{\rm pp}$ and $\mathbf{h}_{\rm rp}$ are not available. In this case the expected value of (7) is at its minimum when $|h_{\rm pp}|^2 = \Omega_{\rm pp} - \epsilon_1$ and $|h^{(i)}_{\rm r}|^2 = \Omega^{(i)}_{\rm rp} + \epsilon_2 \forall i$, for some appropriately chosen values of $\epsilon_1, \epsilon_2 \geq 0$. ϵ_1 and ϵ_2 are chosen such that $\Pr\{|h_{\rm pp}|^2 \geq \Omega_{\rm pp} - \epsilon_1\} \prod_{i=1}^{R} \Pr\{|h^{(i)}_{\rm r}|^2 \leq \Omega^{(i)}_{\rm rp} + \epsilon_2\} = 1 - \mathrm{P}_{o,\max}$.

To derive the worst-case beamformer of Section IV-C, we use channel uncertainty model (26), with $\rho = 1$. Here, $\mathbf{e}$ is the error vector which has a norm bound of ϵ_3, i.e., $\|\mathbf{e}\| \leq \epsilon_3$. The worst-case beamformer will ensure that the $\mathrm{PU_{Rx}}$ SINR is always above $\gamma_{\rm T}$ for all CSI error vectors satisfying $\|\mathbf{e}\| \leq \epsilon_3$ and $|h_{\rm pp}|^2 \geq \Omega_{\rm pp} - \epsilon_1$. Using (26) and the worst-case value of $|h_{\rm pp}|^2$ in (7), the $\mathrm{PU_{Rx}}$ SINR constraint can be expressed as

$$\begin{aligned} &-\tilde{\mathbf{h}}^H_{\rm rp}\mathbf{F}\tilde{\mathbf{h}}_{\rm rp} - \tilde{\mathbf{h}}^H_{\rm rp}\mathbf{F}\mathbf{e} - \mathbf{e}^H\mathbf{F}\tilde{\mathbf{h}}_{\rm rp} - \mathbf{e}^H\mathbf{F}\mathbf{e} \\ &\quad -\sigma^2_{\rm p} + \frac{P_{\rm p}(\Omega_{\rm pp} - \epsilon_1)}{\gamma_{\rm T}} \geq 0, \\ \text{s.t.}\quad & 1 - \left\|\frac{\mathbf{e}}{\epsilon_3}\right\|^2 \geq 0, \end{aligned} \tag{45}$$

where $\mathbf{F} = P_{\rm s}(\mathbf{h}_{\rm sr}\mathbf{h}^H_{\rm sr} \odot \mathbf{W}) + P_{\rm p}(\mathbf{h}_{\rm pr}\mathbf{h}^H_{\rm pr} \odot \mathbf{W}) + \sigma^2_{\rm r}(\mathbf{I} \odot \mathbf{W})$. The S-Procedure [11] can be used to combine the two constraints in (45) into one convex constraint. The S-Procedure states that

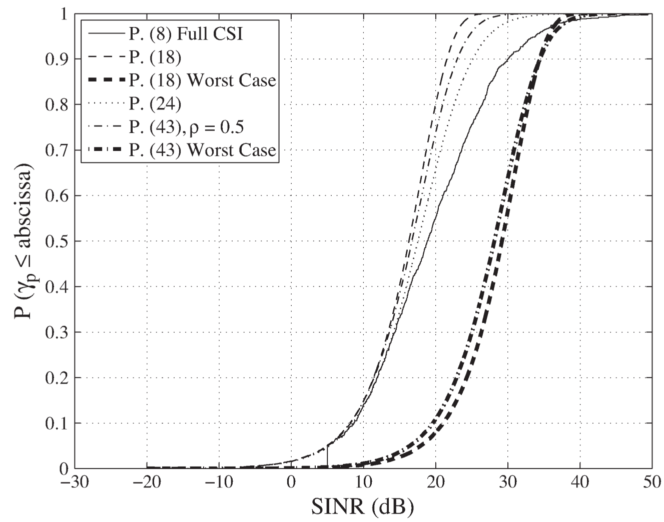

Fig. 3. SINR at the $\mathrm{PU_{Rx}}$ for the total relay power minimisation problem.

$$\begin{aligned} \exists_{s\geq 0}| &-\tilde{\mathbf{h}}^H_{\rm rp}\mathbf{F}\tilde{\mathbf{h}}_{\rm rp} - \tilde{\mathbf{h}}^H_{\rm rp}\mathbf{F}\mathbf{e} - \mathbf{e}^H\mathbf{F}\tilde{\mathbf{h}}_{\rm rp} - \mathbf{e}^H\mathbf{F}\mathbf{e} \\ &-\sigma^2_{\rm p} + \frac{P_{\rm p}(\Omega_{\rm pp} - \epsilon_1)}{\gamma_{\rm T}} \geq s\left(1 - \left\|\frac{\mathbf{e}}{\epsilon_3}\right\|^2\right), \end{aligned} \tag{46}$$

which can be rewritten as the quadratic

$$\exists_{s\geq 0}|[1 \quad \mathbf{e}^H]\mathbf{G}\begin{bmatrix}1\\ \mathbf{e}\end{bmatrix} \geq 0, \tag{47}$$

where $\mathbf{G}$ is defined as

$$\mathbf{G} = \begin{bmatrix} -\tilde{\mathbf{h}}^H_{\rm rp}\mathbf{F}\tilde{\mathbf{h}}_{\rm rp} - \sigma^2_{\rm p} + \frac{P_{\rm p}(\Omega_{\rm pp}-\epsilon_1)}{\gamma_{\rm T}} - s & -\tilde{\mathbf{h}}^H_{\rm rp}\mathbf{F} \\ -\mathbf{F}\tilde{\mathbf{h}}_{\rm rp} & -\left(\mathbf{F} - \frac{s}{\epsilon_3^2}\mathbf{I}\right) \end{bmatrix}.$$

Note that ensuring (47) is the same as ensuring that $\mathbf{G} \succeq 0$. Hence, the worst-case $\mathrm{PU_{Rx}}$ SINR constraint becomes a convex matrix positive semidefinite constraint. Problems (8) and (10) are transformed into worst-case robust problems by replacing the instantaneous $\mathrm{PU_{Rx}}$ SINR constraints with $\mathbf{G} \succeq 0$ and the introduction of the auxiliary variable s. ϵ_1 and ϵ_3 are chosen such that $\Pr\{|h_{\rm pp}|^2 \geq \Omega_{\rm pp} - \epsilon_1\}\Pr\{\|\mathbf{e}\| \leq \epsilon_3\} = 1 - \mathrm{P}_{o,\max}$.

Our proposed robust beamformer of Section IV-C is also compared against a non-robust beamformer. The non-robust beamformer is designed by treating CSI of $\mathbf{h}_{\rm rp}$ as perfect by ignoring the effects of CSI errors.

In Fig. 3, results are provided for the CDF of the $\mathrm{PU_{Rx}}$ SINR obtained through solving the SU total relay power minimisation problem (8), and the corresponding proposed robust problems (18), (24) and (43). Results are also provided for the worst-case beamformer designs. It can be seen that the required 5% probability of $\mathrm{PU_{Rx}}$ SINR being below 5 dB is satisfied by all three robust optimisation schemes proposed in this paper. Being very conservative, the worst-case designs result in almost zero PU outage probability. A feasible solution for the worst-case beamformer of Section IV-B could not be found, hence

TABLE I
SU BLOCKING PROBABILITIES AND MEAN RELAY POWER FOR TOTAL RELAY POWER MINIMISATION PROBLEM

Problem	Blocking Probability (%)	Mean Total Relay Power (dBm)
(8), Full CSI	0.2	−41.0
(18)	0.2	−41.3
(24)	41	−46.5
(43), $\rho = 0.05$	0.2	−41.3
(43), $\rho = 0.2$	0.2	−41.3
(43), $\rho = 0.3$	2.1	−41.6
(43), $\rho = 0.5$	15	−43.6
Sec. IV-A Worst-Case	0.3	−41.6
Sec. IV-B Worst-Case	100	–
Sec. IV-C Worst-Case	84	−54.0

results are not shown on the figure. This is because the worst-case method aggressively protects the $\mathrm{PU_{Rx}}$ and is not able to find a power allocation which guarantees QOS to the $\mathrm{PU_{Rx}}$ in the worst-case scenario.

Table I summarises the SU blocking probabilities and the mean total relay power of the various total relay power minimisation problems discussed in this paper. SU blocking probability is defined as the probability that the SU is not able to access the channel, i.e., the probability that the optimisation problem is infeasible due to either SU or PU QoS constraints not being able to be satisfied. We see that increasing channel uncertainty increases the SU blocking probability. The results also show that it is not vital to have the full CSI for the $\mathrm{PU_{Tx}}$ to $\mathrm{PU_{Rx}}$ link. Knowledge of the mean channel power of this link only is sufficient to obtain the same SU blocking probability as for the full CSI scenario. It is evident that the worst-case beamformers tend to have much higher SU blocking probabilities than the robust beamformers proposed in this paper; for instance, the worst-case beamformers of Section IV-B and C result in blocking probabilities of 100% and 84%, respectively, which would render them impractical. The results also show that the mean total relay power decreases with increasing channel uncertainty. This is because the channel uncertainty causes the beamformers to become more conservative and the beamformer power is reduced to control interference at the $\mathrm{PU_{Rx}}$.

In Fig. 4, results are provided for the CDF of the $\mathrm{PU_{Rx}}$ SINR obtained through solving the $\mathrm{SU_{Rx}}$ SINR maximisation problem (12), and the corresponding proposed robust problems (19), (25) and (44). Results are also provided for the worst-case designs and a non-robust beamformer design for problem (44). The non-robust beamformer treats $\mathbf{h}_{\mathrm{rp}}$ CSI as perfect and ignores the effect of CSI errors in the design process. We see that the outage probability for the full CSI solution is zero. Results show that the 5% $\mathrm{PU_{Rx}}$ outage probability requirement is satisfied by all three robust solutions proposed in this paper. The non-robust solution achieves a $\mathrm{PU_{Rx}}$ outage probability which is greater than 5% because the outage probability constraint is not respected by this design. Again, the worst-case designs result in very conservative solutions that attain $\mathrm{PU_{Rx}}$ outage probabilities which are close to zero.

In Fig. 5, the output $\mathrm{SU_{Rx}}$ SINR CDF results for the $\mathrm{SU_{Rx}}$ SINR maximisation problem (12), and the corresponding proposed robust problems (19), (25) and (44) are provided. Results for the worst-case beamformers are also plotted. We see that

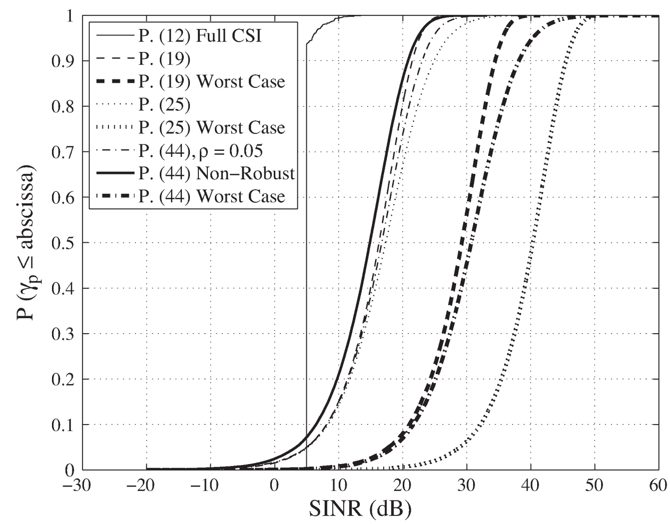

Fig. 4. SINR at the $\mathrm{PU_{Rx}}$ for the $\mathrm{SU_{Rx}}$ SINR maximisation problem.

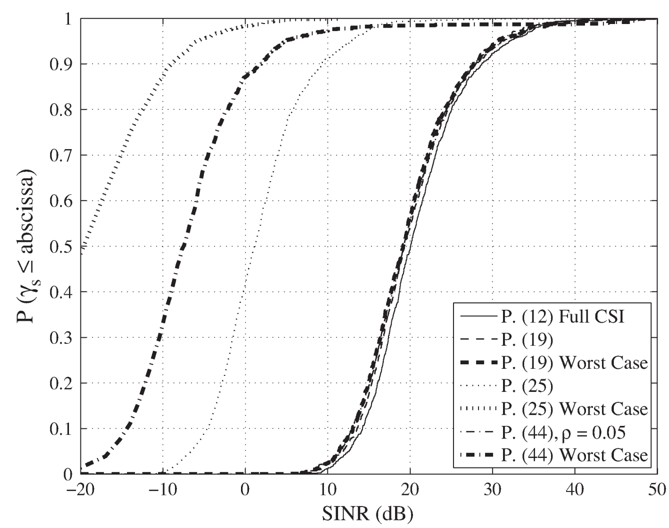

Fig. 5. SINR at the $\mathrm{SU_{Rx}}$ for the $\mathrm{SU_{Rx}}$ SINR maximisation problem.

problems (19) and (44) ($\rho = 0.05$, see Fig. 7 for results for various values of ρ) result in almost the same performance which is very close to the full CSI scenario. The performance loss due to partial CSI on the $\mathrm{SU_{Rl}}$ to $\mathrm{PU_{Rx}}$ link, problem (25), is clearly visible. The worst-case beamformer for problem (19) results in almost the same performance as the robust design proposed in this paper; however, the worst-case designs for problems (25) and (44) result in performance that is inferior to our proposed methods.

In Fig. 6, the CDF of the $\mathrm{PU_{Rx}}$ SINR obtained through solving (44) for various values of ρ is provided. The outage probability requirement is satisfied by designs for all three values of ρ. We see that the solutions for $\rho = 0.3$ dB and $\rho = 0.05$ dB result in the same PU performance.

In Fig. 7, the CDF of the $\mathrm{SU_{Rx}}$ SINR obtained through solving (44) for various values of ρ is provided. As a reference, the CDFs of the $\mathrm{SU_{Rx}}$ SINR for problems (19) and (25) are also plotted. As expected, the $\mathrm{SU_{Rx}}$ performance degrades with increasing CSI error variance. As the CSI error variance increases, the CDF curves are seen to move away from the CDF curve of problem (19) and towards that of problem (25).

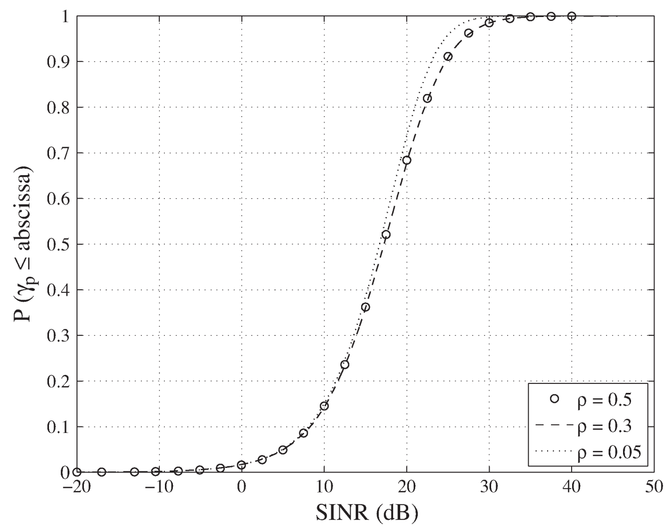

Fig. 6. SINR at the $\mathrm{PU_{Rx}}$ for various CSI error level ρ for the $\mathrm{SU_{Rx}}$ SINR maximisation problem.

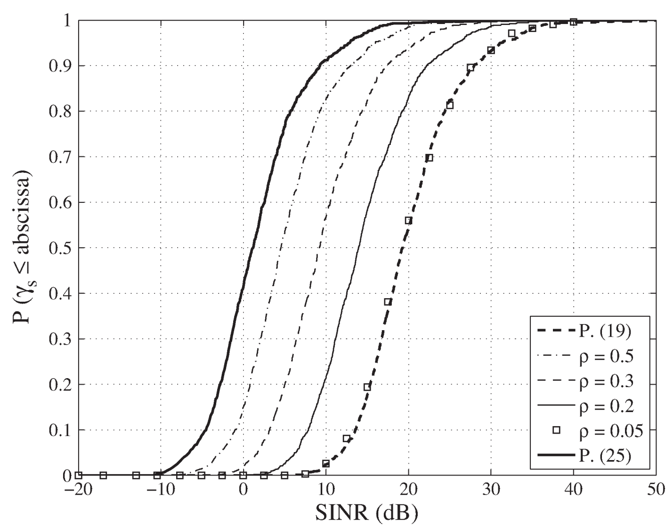

Fig. 7. SINR at the $\mathrm{SU_{Rx}}$ for various CSI error level ρ for the $\mathrm{SU_{Rx}}$ SINR maximisation problem.

VI. Conclusion

In this paper, we have studied robust cooperative beamformers for a CR relay network that guarantee a certain $\mathrm{PU_{Rx}}$ outage probability under the assumption of partial and imperfect CSI. We have shown that the total relay power minimisation problem can be solved using a SOCP and that the cognitive destination SINR maximisation problem can be stated as a convex semidefinite program (SDP) using probabilistic constraints. Simulation results have shown how the achieved robustness varies with CSI uncertainty.

Appendix A
Distributions of $r_1, r_2, r_3, r_4, r_5, r_6$ and r_7

Both r_1 and r_2 have the general form $\mathbf{g}^H\mathbf{u}\mathbf{x}^H\mathbf{g}$, where $\mathbf{x} \in \mathcal{C}^{R\times 1}$ is a complex Gaussian random vector with the distribution $\mathcal{N}_\mathcal{C}(\mathbf{0}, \mathbf{\Sigma})$ and $\mathbf{u}, \mathbf{g} \in \mathcal{C}^{R\times 1}$ are deterministic vectors. It is easily shown that $\mathbf{g}^H\mathbf{u}\mathbf{x}^H\mathbf{g}$ is a zero mean complex Gaussian random variable with variance $\mathrm{tr}(\mathbf{\Sigma}\mathbf{G})\mathrm{tr}(\mathbf{U}\mathbf{G})$, where $\mathbf{G} = \mathbf{g}\mathbf{g}^H$ and $\mathbf{U} = \mathbf{u}\mathbf{u}^H$. Hence, we see that r_1 and r_2 are zero mean Gaussian random variables with variances given by (28) and (29), respectively.

Since r_3 is a linear combination of zero mean Gaussian random variables, it is also a zero mean Gaussian random variable. By defining $\tilde{\mathbf{E}} = \rho\mathrm{diag}((\tilde{\mathbf{h}}_{\mathrm{rp}}^H)^T \odot \mathbf{e})$, the variance can be expressed as

$$\sigma_3^2 = 2\gamma_{\mathrm{T}}^2\sigma_{\mathrm{r}}^4\mathbb{E}\left\{\mathrm{tr}\,(\mathbf{W}\tilde{\mathbf{E}})\mathrm{tr}\,(\mathbf{W}\tilde{\mathbf{E}})^*\right\}. \tag{48}$$

Invoking [31, Theorem 1.2.22. (ii)], which states that $\mathrm{tr}\,(\mathbf{W}\tilde{\mathbf{E}}) = (\mathrm{vec}(\mathbf{W}^H))^H\mathrm{vec}(\tilde{\mathbf{E}})$ and because $\mathbf{\Sigma}_\mathrm{e}$ is a diagonal matrix, (48) can be rewritten as (30).

Using [23, Lemma 1], r_4 and r_5 are recognised as exponentially distributed random variables with means given by (31) and (32), respectively.

It is easy to show that r_6 can be expressed as

$$r_6 = \gamma_{\mathrm{T}}\sigma_{\mathrm{r}}^2\rho^2\sum_{i=1}^{R}|w_i|^2|e_i|^2. \tag{49}$$

Since the entries of $\mathbf{e}$ are independently distributed Gaussian random variables, $|e_i|^2\forall i$, are independently distributed exponential random variables and therefore, (49) is a sum of R independent exponentially distributed random variables whose mean and variance is known to have the forms given by (33) and (34), respectively.

Since the expression of r_7 does not contain any random variables, it is a deterministic constant. This completes the proof. □

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