MATH 452

General Topology

Assignment 7

1. Let G be a topological group. Prove that $\mathcal{U}_L$, as defined at 13.11, is a uniformity that defines the topology of G. [13.12 may help here; in effect, one has to prove various facts about $\mathfrak{N}(e)$ that are equivalent in this case to the axioms of a uniformity.]

2. Show that, in a topological group G, the mapping

$$x \mapsto x^{-1} : (G, \mathcal{U}_L) \longrightarrow (G, \mathcal{U}_R)$$

is a uniform homeomorphism.

3. Prove that the uniformity defined on a Tikhonov space Ω by the pseudometrics d_f mentioned in the proof of 13.16 does indeed induce the original topology.

4. If Ω is a uniform space, A is a subspace of Ω (with the subspace topology), Ψ is a complete separated uniform space, and $f : A \longrightarrow \Psi$ is uniformly continuous, prove that there is a unique continuous mapping $g : \mathrm{cl}_\Omega(A) \longrightarrow \Psi$ such that $g|A = f$, and that g is also uniformly continuous.

5. Suppose that A is a precompact subset of the uniform space Ω. Prove that $\mathrm{cl}_\Omega(A)$ is also precompact. Deduce that the completion of a precompact uniform space is compact.

6. Show that any product of complete uniform spaces is complete.

7. Prove that any product of precompact uniform spaces is precompact.