MATH 452

General Topology

Assignment 4

1. Prove that the product of *uncountably many nontrivial* metric spaces is not metrizable. [“Nontrivial” here means “having more than one point”.]

2. Prove Lemma 6.20 and Lemma 6.21.

3. Prove Lemma 7.6: any subspace of a T_i-space is T_i, for $i = 0, 1, 2, 3$.

4. Prove that the product of (any number of) T_i-spaces is T_i, for $i = 0, 1, 2, 3$.

5. A topological space Ω is *separable* if there is a countable set S such that $\mathrm{cl}(S) = \Omega$. If Ω is a separable metric space, show that any subspace of Ω is separable, and that Ω must be second countable.

6. Let $(x_d)_{d\in D}$ be a net in the Hausdorff topological space Ω. Show that, if it has a limit, then that limit is unique; that is, if $x_d \underset{D}{\rightarrow} x$ and $x_d \underset{D}{\rightarrow} y$, then $x = y$.

7. Conversely, prove that a topological space in which every convergent net has only one limit must be Hausdorff.

8. Prove that, if Ω and Ψ are topological spaces and $f : \Omega \longrightarrow \Psi$ is continuous at $x \in \Omega$, and if the net $(x_d)_{d\in D}$ converges to x in Ω, then $(f(x_d))_{d\in D}$ converges to $f(x)$ in Ψ.

9. Prove directly (i.e. not via nets) that a topological space is Hausdorff if and only if every convergent filter has only one limit.

10. Recall the topology $\mathcal{G}$ of Assignment 1, question 5. Is it normal?