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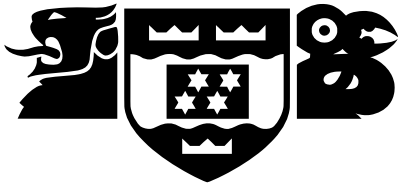

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Improved Continuous PSO for Feature Selection using Statistical Clsutering and Genetic Operations

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Abstract

Feature selection is an important problem in classification. However, it is a difficult task due to the large search space in the original dataset as well as the complex interaction between features. Statistical clustering methods considers feature interaction to group similar features in the same feature cluster. This work firstly introduces a new representation for continuous particle swarm optimisation (PSO), which takes the advantage of statistical clustering to select a small number of features while still achivers high classification performance. Another hybrid PSO based algorithm is also proposed in this project, which embeds genetic operators into standard PSO to avoid premature convergence problem. Experimental results show that by using the statistical clustering information in PSO for feature selection, small subsets of features are evolved, which achieve significant improvement over using all features in terms of the classification performance. Furthermore, the hybrid PSO algorithm is found to better explore the search space than the standard PSO for feature selection.

Acknowledgments

I wish to acknowledge the research into the statistical clustering of data undertaken by Ivy Liu, which has been used to cluster features that are used in the PSO algorithm presented in this project. I would like to express the deepest appreciation to my two supervisors, Bing Xue and Mengjie Zhang, who spend poured countless hours on encouraging and supporting me during this project.

Contents

Chapter 1

Introduction

1.1 Introduction to Feature Selection

Classification is one of the most important tasks in machine learning, which aims to predict the class label of an instance based on the value of features. In the learning process, a set of instances, called training set, is used to train a classification algorithm, which is tested on unseen dataset, called testing set. In many problems, a large number of features is used to well describe the instances. Unfortunately, due to "the curse of dimensionality"[10], the larger a set of features is, the longer time the training process takes. In addition, relevant features are often unknown without prior knowledge. Therefore, a large number of features often contain irrelevant or redundant features, which are not useful for classification. Those features might lower the quality of the whole feature set [42], because they usually conceal the useful information from the relevant features. Feature selection methods [13] are used to remove those redundant and irrelevant features, which will not only speed up the learning/classification process but also maintain or even increase the classification performance over using all features. However, due to the complex interaction between features and the huge search space, it is hard to develop a good feature selection approach.
The main goal of feature selection is finding a small feature subset from a large set of original features to achieve similar or even better classification performance than using all features. In feature selection, suppose there are n features introduced, then the total number of possible subsets is 2^n. It can be seen that over the large search space, the exhaustive search is too slow to perform in most situations. In order to reduce the searching time, some greedy algorithms such as sequential forward selection [35] and sequential backward selection [22] are developed. However, these methods easily get stuck at the local optima. Because of the global search ability, evolutionary computation (EC) techniques, such as genetic programming (GP) [24], genetic algorithm (GAs) [41] and particle swarm optimization (PSO) [33, 40], have been applied to solve the feature selection problem. Compared with GA and GP, PSO is more preferable because it is simple and easy to implement. In addition, PSO not only uses fewer parameter but also converge more quickly.
Feature selection can be viewed as a multi-objective problem because it needs to maximize the classification accuracy and simultaneously minimize the dimensionality of the selected subset. However, with fewer features being used for classification, the classification accuracy is likely decreased. Therefore, those two objectives often conflict with each other and the searching process needs to consider the trade-off between them. Furthermore, statistical clustering methods (SCm) [19] can be applied as a preprocessing step of selecting process, where the similar features are grouped together in the same cluster. There are two main kinds of PSO, which are continuous PSO [15, 30] and binary PSO [17]. Most of PSO-based feature selection algorithms use binary PSO as its binary representation can naturally

present a feature subset. But the research in [36] shows that continuous PSO can achieve better performance than binary PSO.Therefire, this work will develop a new representation in continuous PSO to utilise the benefits of feature clustering information.

1.2 Objectives

The overall goal of this works is to develop a new PSO based feature selection algorithm, which selects a small feature subset while achieving similar or even better clasification performance than using all features. The performance of feature selection methods will be evaluated on a number of datasets with different numbers of features, classes and instances. In particular, the overall goal of this project can be divided into two main objectives:

- Objective 1: to develop new representation and updating methods for continuous PSO (CPSO), which can effectively utilise the statistical clustering information to select a smaller number of features and achieve better classification performance than using all features.
- Objective 2: to combine genetic operators with continuous PSO. By introducing genetic operators such as mutation or crossover, the diversity of swarm is expected to be ensured to better explore the search space to further improve the performance.

1.3 Organisation

The remainder of this report is organised as follows. Chapter 2 provdies background information. Objective 1 is discussed in Chapter 3 and 4. Chapter 5 presents a new hybrid PSO based algorithm, which aims to solve the second objective. Chapter 6 provides a discussion of the major conclusions drawn from this project and some remaining future work.

Chapter 2

Background

2.1 Machine Learning and Classification

Machine learning is a major field of Artificial Intelligence, which aims to develop a system that is capable to learn from data. Based on the desired output of the algorithm, machine learning algorithms can be grouped into three main catergories: supervised learning, unsupervised learning, and reinforcement learning. Classification is one of the most important problems in machine learning, which is to assign of a class label to a given input instance.

2.2 Feature Selection

In classification problems, a large number of features are usually used to well decribe the instances of datasets. However, due to "the curse of dimensionality"[10], having a large number of features results in a long classifier training time, a complex classifier structure and poor preditive performance. In addition, a large set of features might contain redundant and/or irrelevant features, which would not improve or even decrease the classification accuracy. Feature selection is a technique, which selects a small subet of features from the original features. It can not only improve the classification performance but also reduce the training time. According to the evaluation criterion, existing feature selection methods can be fallen into two categories: wrapper approaches and filter approaches [7, 18]. In a wrapper approach, a learning algorithm is used to calculate the fitness value of the selected features. Meanwhile, a filter approach is done in an independent way of learning algorithms. Therefore, wrapper methods usually can achieve better classification accuracy than filters. But wrappers may produce a feature subset with poor generality, which is only good for specific a learning algorithm. In addition, compared with wrappers, filter methods are usually less expensive in terms of the computation complexity. This work focuses mainly on wrapper feature selection.

2.3 Particle Swarm Optimisation (PSO)

Particle Swarm Optimisation (PSO) [16] is an evolutionary computation method, which is inspired by social behaviours such as bird flocking and fish schooling. In PSO, a problem is opimized by using a population, called swarm, of candidate solutions, which are called particles. In order to find the optimal solution, each particle moves around the search space by updating its position as well as velocity. Particularly, the current position of particle is represented by a vector $x_i = (x_{i1}, x_{i2}, \ldots, x_{iD})$, where D is the dimensionality of the search space. These positions are updated by using another vector, called velocity $v_i = (v_{i1}, v_{i2}, \ldots, v_{iD})$,

which is limited by a predefined maximum velocity, v_{max} and $v_{id} \in [-v_{max}, v_{max}]$. During the search process, each particle maintains a record of the position of its previous best performance, called *pbest*. The best position of its neighbours is also recoreded, which is called *gbest*. The position and velocity of each particle are updated according to the following equations:

$$v_{id}^{t+1} = w * v_{id}^{t} + c_1 * r_{i1} * (p_{id} - x_{id}^{t}) + c_2 * r_{i2} * (p_{gd} - x_{id}^{t}) \tag{2.1}$$

$$x_{id}^{t+1} = x_{id}^{t} + v_{id}^{t+1} \tag{2.2}$$

where t denotes the t^{th} iteration in the search process, d is the d^{th} dimension in the search space, i is the index of particle, w is inertia weight, c_1 and c_2 are acceleration constants, r_{i1} and r_{i2} are random values uniformly distributed in [0,1], p_{id} and p_{gd} represent the position entry of *pbest* and *gbest* in the d^{th} dimesion, respectively.

2.4 Related Work on Feature Selection

2.4.1 Traditional Feature Selection Methods

A basic version of feature selection is feature ranking [7], where a score is assigned to each feature according to an evaluation criterion. Feature selection can be performed by selecting the features with the highest scores. However, this type of algorithm ignores the interaction between features. Additionally, the features with the highest scores are usually similar. Therefore, these algorithms tend to selecting redundant features.

Sequential search techniques are also applied to solve feature selection problems. In particular, squential forward selection (SFS) [35] and sequential backward selection (SBS) [22] are proposed. At each step of selection process, SFS (or SBS) adds (or removes) a feature from an empty (full) feature set. Alothough these local search techniques achieve better performance than the feature ranking method, they might suffer "nesting" problem, in which once a feature is added (or removed) from the feature set, it cannot be removed (or added) later. In order to avoid nesting effect, Stearns [31] proposed a "plus-l-takeaway-r" method in which SFS was applied l times forward and then SBS was applied for r back tracking steps. However, it is challenge to determine the best values of (l,r). This problem is addressed by sequential backward floating selection (SBFS) and sequential forward floating selection (SFFS), propsoed by Pudil et al.[27]. In SBFS ad SFFS, the values (l, r) are dynamically determined rather than being fixed in the "plus-l-takeaway-r" method.

2.4.2 EC Approaches(Non-PSO) for Feature Selection

EC techniques are well known because of their global search ability. EC algorithms have been applied to feature selection problems, such as GAs [43], GP [25]. Zhu et al.[43] proposed a hybrid feature selection approach, which combines both local search and GA. In this algorithm, a filter method is used to rank features individually. Basing on the ranking information, GA delete or add a feature to achieve better fitness value, which is the classification accuracy. The experiments showed that this algorithm outperforms the GA alone and other algorithms.

Yuan et al. [41] proposed a two-phase feature selection approach based on GA. The proposed algorithm combined both filter and wrapper approach. Particularly, the first phase is a filter approach, which uses inconsistency criterion to remove irrelevant features. The second phase is a wrapper approach, which uses a feedforward neural network to remove rhe redundant features. The proposed algorithm intended to reduce the computation cost at wrapper step by reducing the size of feauture set in the filter step. However, due the

the complex interaction between features, the first phase might remove the features, which should be included in the best feature set.

Neshatian and Zhang [25] proposed a wrapper GP-based apporach, which evaluates and rank feature subset in binary classification tasks. Expreiments show that the proposed methods detected subset of relevant features in different situations, where other methods had difficulties.

2.4.3 PSO-based Feature Selection Methods

Many EC algorithms have been used for feature selection, such as GAs, GP or PSO. PSO is preferable because it is easier to implement and uses fewer parameters than GAs and GP. Two PSO based filter feature selection algorithms were proposed in [4], where mutual information and entropy are used in the fitness function to evaluate the relevance and redundancy of the selected feature subset. The experiments show that the proposed methods significantly reduce the number of features whilst achieve similar or better classification than using all features.

In PSO, premature convergence is a common problem, in which the swarm converges quickly to a local optima. To avoid premature convergence, Chuang et al. [5] proposed a new *gbest* updating mechanism, which resets *gbest* elements to zero if it maintains the same value after several iterations. However, the performance of this algorithm is not compared with other PSO based algorithms. Another binary PSO based algorithm, which also aims to avoid premature convergence, is proposed by Bin et el. [2]. At each iteration of this algorithm, the swarm is divided into two groups, named "leaders" and "followers". The "leaders" have better fitness value. The "followers" update their positions and velocities based on "leaders"' update. The experimental results showed that the propoed update strategy better utilises the social behaviour phenomenon than the standard binary PSO. Another new *gbest* updating mechanism is developed by Xue et al. [39], which regards not only classification accuracy but also the number of selected features. The proposed algorithms can increase the classification accuracy and simultaneously reduce both the number of selected features and the computational time.
Based on the statistical clusering method and PSO, Lane et al. [19] proposed a feature selection approach, which selects one feature from each cluster. Although the number of features is significantly reduced to be the same as number of clusters, the classification accuracy is still imporved. The results demonstrate that feature clusters, provided by the statistical clusering method , is useful information for feature seletion. Therefore, this work will take the advantage of such information to further develop a new PSO based approach for feature selection.

Chapter 3

A New Representation in PSO for Feature Selection

This chapter addresses Objective 1, which is to develop a new representation in PSO for feature selection that can make use of the statistical clustering information for feature selection. Since the statistical clustering method groups similar features in one feature cluster, selecting a small number of representative features from each cluster is expected to provide the same or similar information to using all features in that cluster. Hence the number of features is reduced while the classification performace would be maintained or even improved. Additionally, in most of current PSO-based feature selection methods, each feature is encoded as one dimension in search space, which results in a high dimensional the search space. This chapter introduces a new representation, which has a lower dimensionality to utilise the statistical information to select a small number of relevant features from the large original set of features.

3.1 Development of A New Representation for PSO-based Feature Selection Approaches

With standard PSO for feature selection, the dimensionality of each particle equals to the number of features, which is large in most situations. Due to the "curse of dimensionality", it might take a long time to select a good subset of features from such a high dimensional search space. In addition, in the original set of features, many features are similar, which can be considered redundant features. Those features are likely to be selected together since they have same effects on the classification process. However, if those redundant features are selected, the classification performance will not be improved meanwhile the training time might be longer due to the large number of features. A new representation, based on statistical feature clustering, is introduced to address the redundancy problem as well as to improve the classification performance.

3.1.1 Feature Clustering Information

Removing redundant features is an essential task in feature selection problems. In the traditional representation, all features play the same role which allows the presentation of redundant features in the solution. The statistical clustering algorithms proposed by Pledger and Arnold [26] and Matechou et. al. [23] are used to group features into different groups, called feature clusters. Features in the same cluster are considered similar and features in different

clusters are dissimilar to each other. The technical detail of statistical clustering methods is not described here due to the page limit and the scope of this project.

The statistical clustering information can be utilised to minimise the chance of selecting redundant features. Particularly, in the new representation, all features which are in the same cluster, have to compete with each other to be selected in the optimal solution. Each cluster possesses a finite number of entries in the position of each particle. Those entries are indices of the selected features from that cluster.

In the new representation, the length of the new representation is determined according to the number of clusters and the total number of features. There are a limited number of entries, which correspond to a feature cluster to determine the selection of features from this cluster. In particular, the number of entries belonged to the j^{th} cluster is the maximum number of selected features from a cluster, which is called mSN_j. This important parameter will be defined in the later section. Being smaller than the number of features in a cluster is one of the conditions to select mSN_j, which is $mSB_j \leq N_j$, where N_j is the number of features in the j^{th} cluster. By applying the clustering information, the new representation introduces a shorter position vector for each particle. In particular, the length of the new representation is $\sum_j mSN_j$, which is smaller than $\sum_j N_j$. On the other hand, $\sum_j N_j$ is the total number of original features, which is also the length of the traditional representation in PSO for feature selection. So the new representation not only minimises the chance of selecting redundant features, but also shorter the position vector, which hopefully leads to shorter computation time.

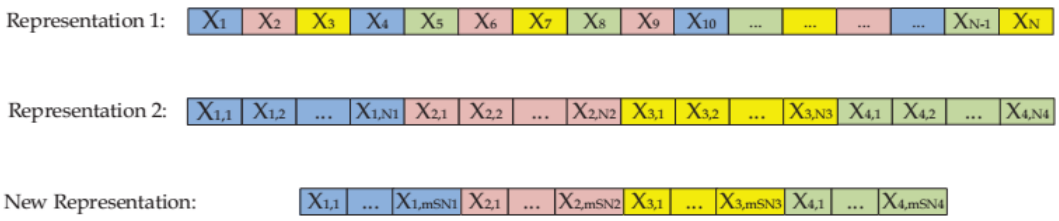


Figure 3.1: Example of N features that are grouped into 4 clusters with N_1, N_2,N_3 and N_4 features, respectively, then $N = N_1 + N_2 + N_3 + N_4$. mSN_j is the predefined maximum number of features seletcted from cluster j and $mSN_1 < N_1, \ldots, mSN_4 < N_4$

The comparison between the standard representation and the new representation is given in the Figures 3.1. Note that all features from the same cluster are represented by the same colour. Representation 1 shows the traditional way of using PSO for feature selection without considering the feature clustering information. Representation 2 the the proposed new representation consider the feature clustering information. Representation 2 is different from Representation 1 by putting features in the same cluster together, which is proposed by [20]. The new representation is represented in Rpresentation 3, which is different from Representation 1 and 2 in two main aspects. Firstly, the dimensionality of the new representation is smaller than the two other representations. The second difference is the meaning of each element in the position vector. In Representation 1 and 2, each element (e.g. x_i or $x_{j,k}$) determines whether the corresponding feature is selected or not. In the new representation, each element shows which feature is selected from the corresponding cluster.

3.1.2 Indexing Features in Feature Clusters

Feature clusters will be used as an input to get rid of the redundant features or irrelevant features in a new PSO-based algorithm. In order to allow a particle to refer to features within

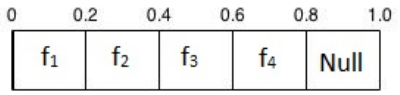


Figure 3.2: Interval for selection features (Not PSO positions)

a cluster, all features in the cluster need to be indexed. The chance of selecting a feature is affected by its index, the indexing method ensures fairness between features. This requirement is done by assigning sub-intervals with identical length to all features in the same cluster. In addition, a virtual feature, called "null" feature, is introduced to each cluster, which allows the selection of zero feature from a cluster, in case all features in that cluster are irrelevant or redundant.

Suppose the j^{th} cluster C_j contains N_j features, which is $C_j = \{f_1, f_2, \ldots, f_{N_j}\}$. A close interval [0,1] is split into $(N_j + 1)$ sub-intervals that are assigned to features within C_j by the follwing rule:

$$\begin{cases} \text{Feature } f_i : [\frac{i-1}{N_j+1}, \frac{i}{N_j+1}] \text{ where } i \in [1, N_j] \\ \text{Null Feature} : [\frac{N_j}{N_j+1}, 1] \end{cases} \tag{3.1}$$

So each feature is assigned to a sub-interval, whose length is $\frac{1}{N_j+1}$. For example, suppose a cluster C_j consists of 4 features, $C_j = \{f_1, f_2, f_3, f_4\}$, each feature is indexed as shown in Figure 3.2. As can be seen in Figure 3.2, the interval [0,1] is further divied into five intervals, where four of them corresponds to the four features while the last interval corresponds to the "Null" feature, i.e. no feature is selected. Suppose that its $mSN_j = 2$ and the position values are $\{x_{1,1} = 0.5, x_{1,2} = 0.96\}$. As $x_{1,1} \in [0.4, 0.6]$, which is the interval of feature f_3, f_3 will be seleted. Similarly $x_{1,2} \in [0.8, 1.0]$ that belongs to $Null$ feature, which means that no feature is selected. So that entry values are iterpreted as selecting only feature f_3 from the cluster. As can be seen from the above example, the use of "null" feature allows the algorithm to choose less than mSN_j features, so mSN_j plays a role as an "upper limit"number of the selected features. Equation 3.2 shows a general case of how to determine which feature or no feature is selected from cluster j, where x is the position value in a dimension.

$$Feature = \begin{cases} f_k, \text{ if } x \in [\frac{k-1}{N_j+1}, \frac{k}{N_j+1}] \text{ where } k \in [1, N_j] \\ \text{Null Feature, if } x \in [\frac{N_j}{N_j+1}, 1] \end{cases} \tag{3.2}$$

3.1.3 Define Upper Limit mSN_j

The "upper limit", mSN_j, is an important parameter for this algorithm, which controls the number of selected features from each cluster. Basically, mSN_j needs to satisfy the following conditions:

1. $mSN_j \leq |cluster|$
2. mSN_j increases with respect to $|cluster|$, because the larger the cluster is, the more information it provides.

According to the above criteria, the upper limit number of selected features from each cluster is calculated by the following equation:

$$mSN_j = \sqrt{|cluster|} \tag{3.3}$$

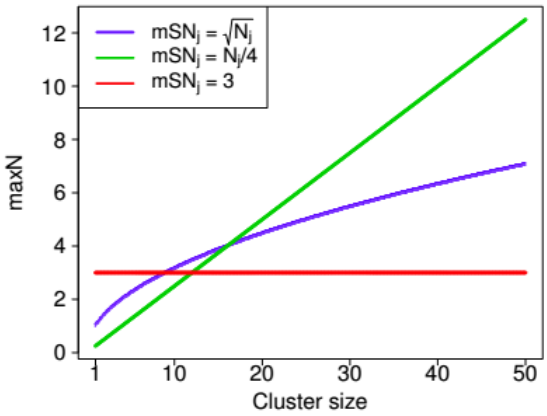


Figure 3.3: Three different ways of determining mSN_j

Figure 3.3 compares three different ways to determine mSN_j which are Equation 3.3, a constant function and a liner function, respectively plotted by using green, blue and red lines. As can be seen from the figure, Formula 3.3 allows selecting more features from a cluster that contains a large number of features, which cannot be done by the constant function (blue line). On the other hand, Equation 3.3 is preferred over linear scaling, since it defines a smaller number of selected features from large feature clusters, which likely consists of redundant features. The smaller mSN_j will reduce the chance of selecting those redundant features.

Overall, the pseu-docode of the new algorithm with the new representation (PSOCC), which bases on continuous PSO based algorithm and feature clustering information, is shown in Algorithm 1

Algorithm 1 : Pseudo-code of PSOCC

begin
indexing features in each cluster;
define mSN_j for each cluster according to Equation 3.3;
randomly initialise the position and velocity of each particle;
while *Maximum iterations* is not reached **do**
    evaluate the fitness of each particle according to its classification accuracy;
    for $i = 1$ to *Population size* **do**
        update *pbest* and *gbest* of particle i;
    end for
    for $i = 1$ to *Population size* **do**
        update v_i of particle i according to Equation 2.1;
        update x_i of particle i according to Equation 2.2;
    end for
end while
calculate the training and testing classification accuracy of the selected feature subset;
return the position of *gbest*, the training and testing classification accuracies;
end

Dataset	#features	#clusters	#classes	#instances
Wine	13	6	3	178
Vehicle	18	6	4	846
Ionosphere	34	11	2	351
Sonar	60	12	2	208
Musk1	166	14	2	476
Arrhythmia	279	15	16	452
Madelon	500	11	2	4400
Multiple Features	649	15	10	2000

Table 3.1: Datasets

3.2 Experimental Design

3.2.1 Benchmark Techniques

In order to examine the performance of the proposed representation, a traditional wrapper feature selection approaches (LFS [12]) and two PSO based feature selection algorithm (PSOFS [37] and PSO42 [39]) are used for comparison purposes in the experiments. The traditional algorithm, LFS, limits the number of features being cosidered during the forward selection process, which is quite similar to the upper limit number of selected features from each cluster. The algorithm PSOFS selects features by using standard continuous PSO. The other PSO-based algorithm, PSO42, introduces a new initialisation strategy and updating mechanism. Both PSOFS and PSO42 use the traditional representation.

3.2.2 Datasets and Parameter Settings

Eight datasets (Table 3.1) chosen from the UCI machine learning repository [1] are used in the experiments. These datasets have different numbers of fetures, classes and instances. For each dataset, all instaces are randomly divided into a training set and a test set, which contains 70% and 30% of the instances, respectively. Up to 500 training instances are used in the statistica clustering method to group features into different clusters, where the number of clusters are listed in the second colum in Table 3.1. In the experiments, the classification/learning algorithm is K-nearest neighbour (KNN) where K = 5.
The parameters of PSO are set as follows [34]: w = 0.7298, $c_1 = c_2$ = 1.49618, v_{max} = 6.0, population size is 30, the maximum number of iterations is 100. The fully connected topology is used. The algorithm POCC is run 30 independent times on each dataset. A statistical significance test, Wilcoxon signed-rank test, is performed to compare between the algorithm's classification accuracies of different algorithms. The significance level of the Wilcoxon test was set as 0.05.

3.3 Experimental Results

Table 4.1 shows the experemental results of the PSOCC algorithm. In this table, "All" means that all available features are used, "Ave-size" shows the average number of selected features, "Best", "Ave-Test-Acc", "Std-Test-Acc" illustrate the best, average and standard deviation of the testing accuracies over the 30 independent runs. T represents the results of the significance tests between the testing accuracy of PSOCC and other algorithms. "+" or "-" means that the algorithm PSOCC achieved significantly better or worse classification performance than other algorithms, "=" means there is no significant difference between them.

As can be seen from Table 3.2, on the eight datasets, PSOCC achieves similar classification accruacy to all features on two datasets and significantly higher accuracy than all features on six datasets. Furthermore, PSOCC also selects a small feature subsets from the original one. On most of datasets, the number of features is reduced by 70%. Especically, on the large datasets such as Madelon and Multiple Features, nearly 90% of the features are removed from the original feature sets meanwhile the classification accuracy is significantly increased. The results show that PSOCC successfully removes redundant/irrelevant features and simultaneously maintains or even improves the classification performance.

Dataset	Method	Ave-Size	Best	Ave-Test-Acc	Std-Test-Acc	T
	All	13		76.54		+
	LFS	7		74.07		+
Wine	PSOFS	7.93	98.77	95.6	1.7953	+
	PSO42	6.73	98.77	94.86	1.8628	+
	PSOCC	4.75	100	96.70	3.10	
	All	18		83.86		+
	LFS	9		83.07		+
Vehicle	PSOFS	9.5	87.01	85.03	0.8899	=
	PSO42	10.33	87.01	85.44	0.8372	-
	PSOCC	5.87	86.22	84.72	0.8720	
	All	34		83.81		+
	LFS	4		86.67		+
Ionosphere	PSOFS	12.47	93.33	88.41	2.3079	=
	PSO42	3.13	91.43	86.69	1.6444	+
	PSOCC	9.7	91.43	88.63	1.6765	
	All	60		76.19		+
	LFS	3		77.78		+
Sonar	PSOFS	26.1	84.13	77.3	3.5765	+
	PSO42	11.23	84.13	77.94	3.2104	=
	PSOCC	14.33	84.13	78.94	4.0185	
	All	166		83.92		=
	LFS	10		85.31		-
Musk1	PSOFS	85.93	88.81	84.61	2.0568	=
	PSO42	77.3	89.51	84.87	2.7042	=
	PSOCC	35.03	90.21	83.12	3.4196	
	All	279		94.46		+
	LFS	11		94.46		+
Arrhythmia	PSOFS	118.73	95.14	94.56	0.3517	=
	PSO42	69.77	95.59	94.77	0.4495	+
	PSOCC	44.17	95.59	94.96	0.38	
	All	500		70.9		+
	LFS	7		64.62		+
Madelon	PSOFS	259.07	78.97	76.35	1.0909	+
	PSO42	206.57	84.23	78.81	3.1171	+
	PSOCC	54.39	85.13	83.40	2.0368	
	All	649		98.63		+
Multiple	LFS	18		99.0		+
features	PSOFS	297.07	99.2	99.0	0.0934	-
	PSO42	314.5	99.2	99.0	0.0935	-
	PSOCC	51.07	99.23	98.84	0.1751	

Table 3.2: Experimental Results of PSOCC

According to Table 3.2, comparing with PSOFS, PSOCC selects a smaller number of features. On five of the eight datasets, PSOCC achieves similar or higher classification accu-

racy than PSOFS. Although on two datasets, (Vehicle and Multiple Features), the classification accuracy of PSOCC is lower than PSOFS, PSOCC selects fewer features than PSOFS. Especially, on the dataset Multiple Features, PSOCC only picks 51 features from the original 649 features, which is six times smaller than PSOFS, meanwhile the classification accuracy is reduced only 0.16%. Comparing with PSO42, on most of datasets, PSOCC selects a smaller number of features, and the classification performace is smilar or even better. Only on two datasets, (Sonar and Ionosphere), PSOCC selects a larger number of features, but PSOCC achieves significantly higher classification accuracy than PSO42. The results show that PSOCC with the new representation can use the clustering information to envolve better features subets with a smaller number of features and similar or even higher classfication accuracy than two existing PSO-based algorithms, PSOFS and PSO42.

Comparing between LFS and PSOCC, Table 3.2 shows that LFS selects a smaller number of features, but PSOCC achieves higher classification performance. The only exception is the dataset Musk1, where PSOCC achieves lower accuracy than LFS. But PSOCC's best solution achieves higher accuracy than LFS. The results show that PSOCC with the new representation can better explore the search space to find the better subsets than the traditional method, LFS.

3.4 Summary

The goal of Objective 1 was to develop a new PSO representation, which utilises the feature clusterinng information to envolve a small subset of features that maintains or improves the classification accuracy. This goal has been achieved by introuducing the upper limited number of selected features and indexing features method, which allowed to encode clustering information inside a particle's position. The algorithm with the new representation was compared with two PSO-based algorithms (PSOFS, PSO42) and a traditional methods (LFS). The results showed that the new algorithm successfully reduced the number of features and simultaneously maintained or increase the classification accuracy.

From the results in Table 3.2, it can be seen that for some datasets like Musk1 and Multiple Features, PSOCC selects a small number of features, which results in poor classification accuracy. Although the upper limited number of features decreases the computation time, it also reduces the chance to select a good feature subset. For example, in case all features in a cluster are important, but the upper limited number of selected features will not allow selecting all features from that cluster. In the following chapter, an improved version of PSOCC is proposed, which uses Gaussian distribution to better explore the search space to further improve the performance.

Chapter 4

Applying Gaussian Distribution in PSO

In Chapter 3, a new representation is proposed in PSO for feature selection to utilise the statistical clustering information to reduce the number of features selected and increase the classification performance. In that representation scheme, each element in the position vector indicates which feature is selected form its corresponding cluster. In particular, suppose a feature F in a cluster is assigned with an interval $[a, b]$, where $0 \leq a < b \leq 1$. If the element in the position vector has value c, which falls into the interval $[a, b]$, the feature F will be selected. This transformation rule introduces a limitation of the new representation. For instance, suppose that in two different particles p_1, p_2, the elements values are c_1 and c_2 respectively, where $a < c_1 < c_2 < b$. According to the transformation rule, both p_1 and p_2 selects the elements corresponding feature. Although the element in p_1 and p_2 are assigned with two different values, the corresponding selected features are identical. This fact significantly affects on the search ability of PSO. Particularly, a small change of an element within the position vector might not lead to any change in the selected features as well as the fitness value. The purpose of using continuous PSO in the new representation schema is to utilise the smooth movement of particle in the search space. However, the transformation rule accidently diminishes smooth property of the search space. This chapter aims to solve this limitation by adding more meaning to the element of a position vector and creating a new transformation rule, which allows particles to move smoother in the search space. The new transformation rule is expected to better utilise the continuous search space and achieve higher classification accuracy than the representation developed in Chapter 3.

4.1 Developemnt of New Representation Schema in Continuous PSO

4.1.1 Gaussian Distribution for Feature Selection

Before introducing a new meaning of entries in the position vector, it is worth to have a short review on the new representation, proposed in Chapter 3. In the new representation, the position value in a dimension determines which feature is selected from a certain cluster. In other word, the position value plays a role as the index of a feature in a cluster. For the cluster i that contains N_i features, an interval [0, 1] is equally divided into $(N_i + 1)$ sub-intervals. The length of a sub-interval is called step. Therefore, the step of a cluster can be calculated by the Equation 4.1.

$$step_i = \frac{1}{N_i + 1} \tag{4.1}$$

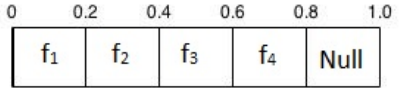


Figure 4.1: Indexing features

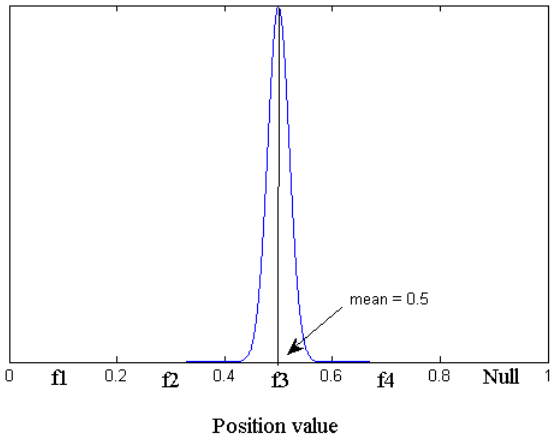


Figure 4.2: The Gaussian distribution when position value is 0.5.

where N_i is totalnumber of features in the ith cluster. If the entrys value falls into a features interval, this feature is selected. Although, the position value in a dimension could varies in the interval [0, 1], its meaning is still an index of a feature. Each position entrys value has exactly one corresponding feature. Suppose the i^{th} feature f_i in the j^{th} cluster is assigned to the interval [0.2,0.4]. Two different position entrys values 0.25 and 0.35 have exactly same interpretation, which is "feature f_i is selected". Therefore, when the position value is changed from 0.25 to 0.35, the set of selected feature is not changed. So even the search space is in continuous form, the particle does not move smooth.

In order to solve the above limitation, the position entry value is used to calculate the probability that a feature being selected. To achieve this, a Gaussian distribution is introduced in the interpreting process. This Gaussian distribution will determine which feature is selected within a certain cluster. The determining rule is shown later in the following section. By using the Gaussian distribution, all features in the cluster have chance to be selected, while in Chapter 3, only the corresponding feature will be selected. The parameters of the Gaussian distribution are defined, so that the corresponding feature has the biggest chance of being selected. The further from the corresponding feature a feature is, the less chance that feature is selected. Therefore, the position value is chosen as a mean for the Gaussian distribution, which ensures that the selection of the corresponding feature is given a higher probability than the selection of other features with in the cluster.

The standard deviation of the Gaussin distribution is set to $0.1 * step$, where $step$ is calculated using the Equation 4.1. This standard deviation ensures the probability that the corresponding feature being selected is more than 99%. The rest 1% is distibuted to other features with respect to how far the feature is from the corresponding feature.

An example of applying Gaussian distribution on the new representation is outlined below. In this example, suppose a cluster contains 4 features f_1, f_2, f_3, f_4. Therefore, an interval [0,1] is divided into 5 sub-intervals, which are respectively assigned to 4 features as being shown in the Figure 4.1. The step value is $step = \frac{1}{4+1} = 0.2$.

Suppose that the position value of a dimension is 0.5, the corresponding feature is f_3. The Gaussian distribution with this position value is shown in Figure 4.2. As can be seen

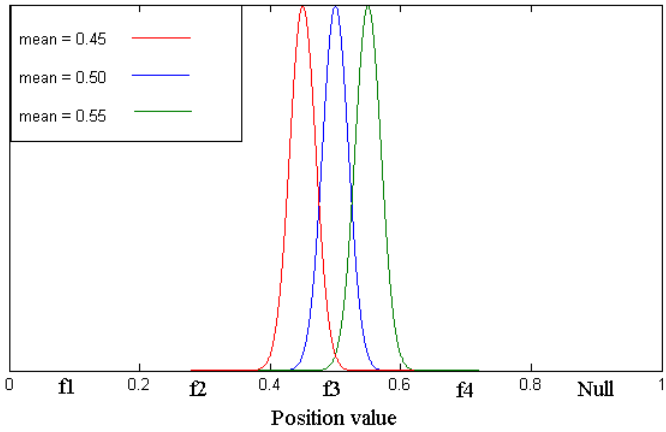


Figure 4.3: The differences introuduces by different position values.

from the figure, it is about more than 99% that feature f_3 is selected. In addition, f_2 and f_4 have more chance to be selected than f_1 or $Null$ (i.e. no feature is selected), because they are closer to f_3.

Algorithm 2 : Pseudo-code of GPSOCC

```
1: begin
2: indexing features in each cluster;
3: define N_sc for each cluster according to Equation 3.3
4: initialize the best feature set (BFS) with the worst fitness. 3.3;
5: randomly initialise the position and velocity of each particle;
6: while Maximum iterations is not reached do
7:     for each particle in the swarm do
8:         transform from the position to set of selected features
9:         evaluate the fitness based on the selected features
10:        if the fitness is better than BFS's fitness then
11:            update BFS
12:        end if
13:    end for
14:    for i = 1 to Population size do
15:        update pbest and gbest of particle i;
16:    end for
17:    for i = 1 to Population size do
18:        update v_i of particle i according to Equation 2.1;
19:        update x_i of particle i according to Equation 2.2;
20:    end for
21: end while
22: calculate the training and testing classification accuracy using BFS
23: return BFS, the training and testing classification accuracies;
24: end
```

Since the position value relates to the probability that a feature is selected, a change in position entry results in the different probablilities. To better illustrate this assertion, the above example is resued, in which the position entry takes 3 different values 0.45, 0.5 and 0.55. According to the trasnfromation rule in Chapter 3, these values indicate the same meaning, that is "feature f_3 is selected". However, by using Gaussian Distribution, these

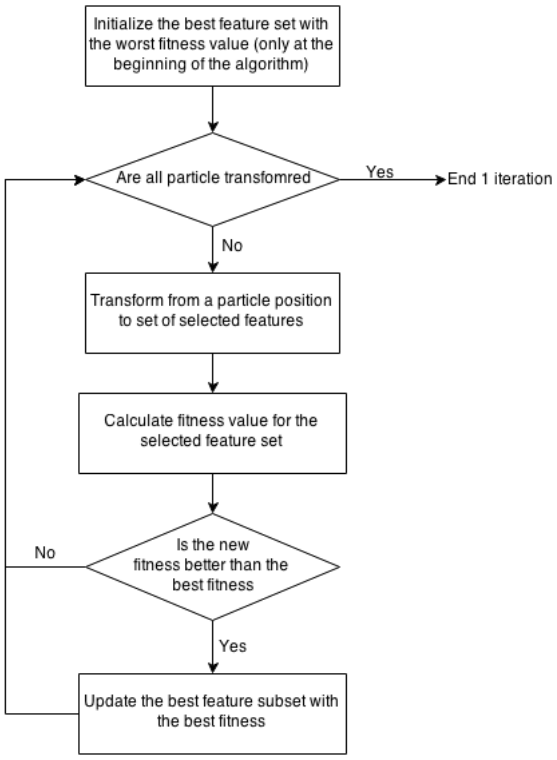


Figure 4.4: Updating process of the best feature set.

different values have different meanings. The differences are shown in Figure 4.3. At the beginning, the position value is 0.5, which is also a mean of the Gaussian Distribution. As can be seen, the chances that f_2 and f_4 being selected are equal. If the position value is updated to 0.55, which is still in the interval of feature f_3. The Gaussian distribution moves toward feature f_4. Therefore the feauture f_4 is more likely to be selected than feature f_2. On the contrary, the chance of feature f_2 being selected is more than feature f_4 when the position value is updated to 0.45. Therefore, althoguh these three different values are in the same features interval, they introduce different probability that features within the cluster being selected.

4.1.2 How to Select Features

Introducing the Gaussian distribution makes the position value has more meaning than just an index of selected feature in a cluster. This section discusses how to translate from the position value to the selected feature with Gaussian distribution. Firstly, a Gaussian distribution is built by using the position value as its mean. After that, a random number is generated from the distribution. Notice that this number should fall into the range [0,1]. That is why the standard deviation is calculated as the Equation , which is small enough to ensure that the random number is in the interval [0,1] in most cases. This random number is used to determine which feature is selected from a certain cluster. The selection rule is similar to the selection rule in PSOCC, except that the random number is used instead of the position value. The chance that the random number falls into a features interval can be viewed as the porbability that feature is selected.

Since the transformation process from the position to the selected future set uses a Gaussian distribution, the future set is not deterministic. In particular, the same position might results in different feature sets at different times. Therefore, it is necessary to keep track of

the best selected feature set, which is used as a solution at the end of each run. This selected set is updated during the fitness calculation process. The updating process is shown in Figure 4.4. The pseudo-code of the proposed algorithm, GPSOCC, is shown in Algorithm 2.

4.2 Experimental Design

Dataset	Method	Ave-Size	Ave-Train ± Std-Train	Ave-Test ± Std-Test	T
Wine	All	13		76.54	+
	GPSO1	5.4	96.71 ± 7.77E-14	96.59 ± 2.76	+
	PSOCC	4.75	95.05 ± 0.58	96.70 ± 3.10	+
	GPSOCC	4.60	97.37 ± 0.42	97.70 ± 2.52	
Vehicle	All	18		83.86	+
	GPSO1	8.94	86.11 ± 0.20	84.30 ± 0.62	+
	PSOCC	5.87	84.61 ± 0.56	84.72 ± 0.87	=
	GPSOCC	7.30	90.10 ± 0.40	84.74 ± 0.49	
Ionosphere	All	34		83.81	+
	GPSO1	7.66	91.59 ± 0.47	89.50 ± 1.68	-
	PSOCC	9.7	90.04 ± 0.99	88.63 ± 1.68	-
	GPSOCC	3.17	93.90 ± 0.67	86.89 ± 1.8	
Sonar	All	60		76.19	+
	GPSO1	17.64	86.74 ± 0.94	78.19 ± 4.14	=
	PSOCC	14.33	87.01 ± 2.00	78.94 ± 4.02	=
	GPSOCC	10.17	90.67 ± 1.6	78.25 ± 2.96	
Musk1	All	166		83.92	+
	GPSO1	39.64	90.02 ± 0.60	84.95 ± 2.73	-
	PSOCC	35.03	89.78 ± 1.25	83.12 ± 3.41	=
	GPSOCC	38.93	93.22 ± 1.37	83.29 ± 2.48	
Arrhythmia	All	279		94.46	+
	GPSO1	45.5	94.87 ± 0.09	94.85 ± 0.34	+
	PSOCC	44.17	95.11 ± 0.20	94.96 ± 0.38	+
	GPSOCC	42.03	95.75 ± 0.18	95.12 ± 0.34	
Madelon	All	500		70.9	+
	GPSO1	36.08	85.45 ± 0.73	85.68 ± 1.10	-
	PSOCC	54.39	83.73 ± 1.74	83.40 ± 2.00	+
	GPSOCC	51.17	89.20 ± 1.41	84.06 ± 1.65	
Multiple features	All	649		98.63	+
	GPSO1	91.4	99.38 ± 0.38	99.01 ± 0.13	=
	PSOCC	51.07	99.17 ± 0.09	98.84 ± 0.18	=
	GPSOCC	51	99.36 ± 0.07	98.86 ± 0.17	

Table 4.1: Experimental Results of GPSOCC

To examine the performance of the proposed algorithm, GPSOCC, a set of experiments have been conducted. The experimental design is the same as in Objective 1. In addition, the performance of GPSOCC is compared with PSOCC, which is the proposed algorithm in Chapter 3 and a binary PSO based feature selection algorithm, GPSO1 [20], which also uses statistical feature clustering information and Gaussian distribution.

4.3 Experimental Results

Table 4.1 shows the experemental results of the GPSOCC algorithm, where "All" means that all the available features are used for classification. "Ave-size" shows the average number of selected features over the 30 runs. "Ave-Train", "Std-Train", "Ave-Test", "Std-Test" illustrate the average and standard deviation of the training and testing accuracies over the 30 independent runs. T shows the results of the statistical significant tests between the accuracy of GPSOCC and other algorithms. "+" or "-" means that the algorithm GPSOCC achieved significantly better or worse classification performance than other algorithms, "=" means there is no significant difference between them.

From Table 4.1 , it can be seen that the number of features selected by GPSOCC is much smaller than the total number of features, but using the selected features only, the 5NN classification algorithm achieved significantly better or similar classification accuracy. For example, on the Madelon dataset, GPSOCC selects on average 51 features from the original 500 features, but achieve a significant increase in classification accuracy of 14%. The results suggests that GPSOCC can be successfully used for feature selection to reduce the dimensionality of the data and significatly increase the classification performance over using all features.

In comparison with PSOCC, GPSOCC achieves a higher classification performance in three datasets and has similar performace on four others. The only dataset that the PSOCC outperforms GPSOCC is the Ionosphere datatset. However the number of features selected by GPSOCC is about three times smaller than the number of features selected by PSOCC. In addition, on all datasets, GPSOCC achieves higher traning accuracy than PSOCC. For example, on the Madelon dataset, GPSOCC selects less features than PSOCC but GPSOCC achieves higher training accuracy, 89.2%, which is about 6% better than PSOCCs tranining accuracy. The results suggests that the Gaussian distribution can help PSO to better explore the continuous search space by applying smooth movement for each particle.

Comparing GPSOCC with GPSO1, GPSOCC selects less features than GPSO on most of datasets, except on the Madelon dataset. In terms of testing accuracy, GPSOCC achieves higher classification performace on 3 datasets and has similar peroformance on two other datasets. On all datasets, GPSOCC s traning accuracies are much better than GPSO1s training accuracy, which illustrates that GPSOCC explores the search space better than GPSO1.

4.4 Summary

The goal of this chapter is to further improve the new PSO representation, proposed in the chapter 2. The goal is successfully achieved by introducing a Gaussian distribution, which uses the position values as its mean to determine the probability of each feature in a certain cluster being selected. A number of experiments have been conducted to compare the new algorithm GPSOCC with its predecessor, PSOCC and GPSO1, which also useses statistical cluster information and Gaussian distribution. The results shows that by using the Gaussian distribution, GPSOCC can explore the serch space better than two other PSO algorithms. However, on some datasets, GPSOCC sacrifices its classification accuracy to achieve smaller set of selected features. In the following chapter, a new updating mechanism with genetic operators (crossover and mutaion) is developed to help PSO explore the search spcae better.

Chapter 5

A Hybrid PSO for Feature Selection

In recent years, several evolutionary algorithms have been developed for feature selection problems in which Genetic Algorithm (GA) and Particle Swarm Optimisation (PSO) are the two most popular algorithms. GA [11] is a heuristic search technique that mimics the process of natural evolution, which includes some pseudo-biological operators such as inheritance, mutation, selection and crossover. In term of searching ability, GA is very good at exploring the entire search space becuase of the pseudo-biological operators. However, GA is not really good at precisely locate local optimal solution in the search region. Another drawback of GA is its expensive computation cost.

Some comparisions between GA and PSO have been done in [8, 3, 14], which show that PSO usually achieve the same performance as GA but with much cheaper computation cost. However, premature convergence is a drawback of PSO, in which the entire population is trapped in local optima. This problem appears frequently in solving high dimensional problems such as feature selection, whose search space contains too many local optima. Some studies have been done to overcome this limitation. One way is increasing the exploration ability by applying dynamic coefficient, which is proposed in [32, 28]. Another solution is introduced in [6], which resets *gbest* if the *gbest* value does not change after three iterations. In [21], a multi-swarm PSO is proposed to avoid premature convergence problem, in which the population is splitted into many small sized sub-swarms to achieve better performance. Every fixed number of generations, the population is grouped randomly to exchange the information as well as to prevent early convergence inside each sub-swarm. Some hybrid techniqes, which combine GA and PSO, have been done with low level of intergration. For example, in [29], the combination of GA and PSO is done by using the result of one algorithm as an input of the other algorithm. Another stronger co-opeartion of GA and PSO is proposed by A.Gandelli [9], where the intergration is done during the entire run. Particularly, the whole population is splitted into two sub-populations, which are evolved by GA and PSO respectively. After that, these sub-populations are merged to exchange their achievement before being randomly splitted again in the next iterations.

In this chapter, a new PSO based feature selection algorithm, which intergrates two genetic opreators: crossover and mutation in the updating process, is developed. The proposed approach is examined and compared with the original PSO and another PSO based feature selection proposed by Lane [20] to investigate whether the new approach can better explore the search space to avoid the premature convergence problem to improve the performance.

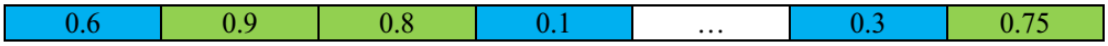


Figure 5.1: Standard PSO Representation

5.1 Continuous PSO for Feature Selection

5.1.1 Particle's Representation

In this chapter, a continuous PSO is applied to solve feature selection problems, where each position value in the position vectore is a real number and corresponds to a feature from the original feature set. In particular, each particle is represented by a vetor of real numbers, $x_i = (x_{i1}, x_{i2}, \ldots, x_{iN})$ where N is the total number of features. $0 \leq x_{in} \leq 1$ shows the probability of the n^{th} feature being selected. A threshold θ is introdued to determine whether or not a feature is selected. Particularly, if $x_{in} \geq \theta$, the n^{th} feature is selected. Otherwise, the n^{th} feature is not selected. So θ is an important parameter which can be used to control the number of selected features. The higher θ is, the less chance a feture being selected. According to initia experience, 0.7 is a good value for θ, which balances between the number of selected fetures and the classification accuracy. The visualisation of particle's representation can be shown in Figure 5.1, where the green entries indicate the selected features and the blue ones correspond to the ones which are not selected.

5.1.2 Fitness function

In most of PSO based feature selection algorithms, the fitness function is set to the classification performance, which guides particles to achieve high classification accuracy. This fitness function's formula is given in Equation 5.1

$$Fitness_1 = ErrorRate \tag{5.1}$$

where

$$ErrorRate = \frac{FP + FN}{TP + TN + FP + FN} \tag{5.2}$$

where TP, TN, FP and FN are short for true positive, true negetive, false positive and false negative, respectively.

However, feature selection is a multi-objective problem, which aims to minimize the number of selected features while maximize the classification accuracy. In order to address this problem, a new fitness function is proposed in [38], which is shown in Equation 5.3

$$Fitness_2 = \alpha * \frac{\#Features}{\#AllFeatures} + (1 - \alpha) * \frac{ErrorRate}{Error_o} \tag{5.3}$$

where $\alpha \in [0, 1]$ shows the relative importance between the number of selected features and the classification error rate. $ErrorRate$ is the classification error rate obtained by the selected feature subset. $Error_o$ is the error rate obtained by using all original features. However, because $Error_o$ is not changed, introducing this value causes more computation. In addition, the fraction $\frac{ErrorRate}{Error_o}$ might be greater than 1, which is not fair because $\frac{\#Features}{\#AllFeatures} \in [0, 1]$. Therefore, a similar fitness function (see Equation 5.4) is used in this work, which eliminates the $Error_o$ component.

$$Fitness_3 = \alpha * \frac{\#Features}{\#AllFeatures} + (1 - \alpha) * ErrorRate \tag{5.4}$$

5.2 Intergrating Crossover and Mutation into PSO

5.2.1 Crossover

In order to avoid premature convergence, the crossover operation is performs between a number of particle pairs in each iteration. A roulette wheel selection based on the fitness value is used to select particles, which then are used as the parents of the crossover operation. A uniform crossover is applied to the selected particles to derive a new particle, called a child. This child's fitness value is calculated and compared with the *pbest*. If its fitness is better than the parent's current fitness value, this child will replace the parent in the swarm. If the child's fitness is even better than the parent's personal best fitness, the parent's personal best position is then replaced by the child's position. The improvement of those parents is then propagated through the swarm during the sharing process.

Since the particles within the swarm will be more similar near the end of the run, the crossover will have more impact at the beginning, when the population is just randomly initialized. Therefore, the number of particles , which are used to apply the crossover operation, is reduced with respect to the increase of the number of iterations (see Equation 5.5). By doing so, the computation cost will be reduced, while the crossover performance is maintained.

$$P_i = \lfloor N - i * \frac{N-2}{I} \rfloor \tag{5.5}$$

where P_i is the number of selected particles for crossover at i^{th} iteration, I is the total number of iterations.

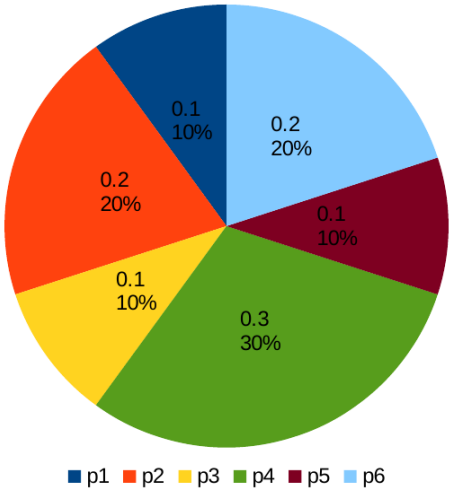
The pseudocode of crossover in each iteration is shown in Algorithm 3

Algorithm 3 : Pseudo-code of Crossver in iteration i^{th}

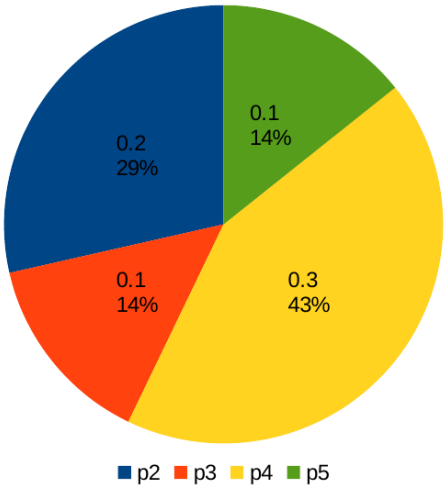
```
1: begin
2: calculate P_i according to Equation 5.5
3: noCrossover = ⌊P_i / 2⌋
4: while noCrossover is not reached do
5:     select a pair of particles as parents;
6:     perform uniform crossover between these parents to get a child;
7:     calculate child's fitness value;
8:     if child's fitness is better than parent's current fitness then
9:         parent's current position ← child's position
10:        parent's current fitness ← child's fitness
11:    end if
12:    if child's fitness is better than parent's personal fitness then
13:        parent's personal best position ← child's position
14:        parent's presonal best fitness ← child's fitness
15:    end if
16:    remove the two particles from candidate parents list
17: end while
18: end
```

An exmple of typical run-through of selecting particles for crossover operation is outlined below. In this example, suppose the swarm contains 6 particles $\{p_1, p_2, p_3, p_4, p_5, p_6\}$ with these respective fitness values $\{0.1, 0.2, 0.1, 0.3, 0.1, 0.2\}$. We are going to do the crossover operation 2 times, which means 4 particles being selected. So accroding to Algorithm 3, $P_i = 4$ and $noCrossover = 2$.

- Step 1: The probabilities of selecting each particle is shown in Figure 5.2a. Two features are chosen via a roulette wheel selection. Since we aim to minimise the fitness value (error rate), the lower a partice's fitness value is, the better that particle is. As can be seen from the figure, the particle, which has higher fitness value than the others, has more chance to be chosen. Therefore, the worse particle will be more likely to be improved via the crossover operation. In this example, we assume that the roulette wheel selection determines that the particle 1 and particle 6 are selected. This step is the first iteration in Algorithm 3. After that, the process inside the loop of Algorithm 3 is applied to do crossover operation between feature 1 and 6.
- Step 2: After being chosen, particle 1 and 6 are removed from the wheel, so other particles will have chance to improve its fitness via crossover operation. This step corresponds to the second iteration of Algorithm 3. In this iteration, another two particles are chosen as parents of the crossover operation.



(a) The probability of a particle being selected to be a parent of the crossover operation



(b) The probability of a particle being selected to be a parent of the crossvoer operation after feature p_1 and p_6 being selected

Figure 5.2: Selecting parents for the crossover opreation basing on fitness value

5.2.2 Mutation

Although the crossover operation can be used to better explore the search space, it has less impact near the end of run because particles in the swarm become very similar. Oppsite to crossover, mutation operation has less impact at the beginning of a run, and more near the end [8]. Therefore, intergrating a mutation operation into the PSO algorithm would also improve the exploration ability.

Each particle within the swarm records a global best position, which is the best position so far being discovered by the particle and its neighbours. If the *gbest*'s fitness value is not improved after a number of iteration, the particle is probably trapped in local optima. This is when a mutation operation needs to be applied on the *gbest* postion.

Belief in a feature

Similar to particle, the global best is also represented by a vetor of real numbers, which are continuous values in the interval [0,1]. Suppose the gbest's position is encoded as

$(g_{i1}, g_{i2}, \ldots, g_{iN})$. These values not only indicate which features are slected but also show the "belief" of that feature. For example, $g_{i1} = 0.8$ and $g_{i2} = 0.9$ shows that both feature 1 and feature 2 are selected, since both values are greater than the threshold $\theta = 0.7$. However, it is believed that feature 2 is more deserved to be selected than feature 1, because g_{i2} is far from θ than g_{i1}. Similarly, if $g_{i2} < g_{i1} < \theta$ then the second feature is more likely not being selected than the first one. In other word, the further distance between the feature's value and the threshold, the more belief that feature is selected or not selected.

Mutation on *gbest*

Taking the idea about belief in a feature, a new mutation method is proposed for *gbest* position, where the less confident feature is more likely to be mutated. Firstly, the confident rate (CR) of each feature is calculated basing on the distance from its value to the threshold θ, which is shown in Equation 5.6

$$CR_i = \begin{cases} \frac{f_i - \theta}{1 - \theta} & \text{if } f_i > \theta \\ \frac{\theta - f_i}{\theta} & \text{if } f_i \leq \theta \end{cases} \tag{5.6}$$

where CR_i is the confident rate of i^{th} feature, f_i is the i^{th} feature's value in *gbest*.

A temporary position, called a child, is generated by applying mutation on the gbest position. After calculating the confident rate for a feature, a random number r is generated . If $r < CR_i$, then the i_{th} child's position entry is set to the corresponding entry in *gbest*. Otherwise, the i_{th} child's position entry is a mutated value of gbest's correspoding entry, which is determined by Equation 5.7.

$$child_i = 1 - gbest_i \tag{5.7}$$

where $child_i$ is the i^{th} position entry value of the *child*, $gbest_i$ is the corresponding entry value in *gbest*.

It can be seen that if a feature is selected, it will not be selected after being mutated and vie versa. More improtant, the confident rate CR plays a role in the mutation rate. The higher the CR is, the lower chance that its correspoding feature being mutated.

The pseudocode of mutation in each iteration is shown in Algorithm 4

Mutation example

Suppose *gbest* fitness value of a particle p is not changed over 3 iterations, we are going to mutate this *gbest*. Assume that:

- Threshold θ = 0.7
- The total number of features are 4
- The *gbest* position is [0.14, 0.76, 0.91, 0.21]

According to the Equation 5.7, the confident rate of each features can be calculated as below:

- $CR_1 = \frac{0.7 - 0.14}{0.7} = 0.8$
- $CR_2 = \frac{0.76 - 0.7}{1 - 0.7} = 0.2$
- $CR_3 = \frac{0.91 - 0.7}{1 - 0.7} = 0.7$
- $CR_4 = \frac{0.7 - 0.21}{0.7} = 0.7$

Suppose the first random number is generated, $r_1 = 0.45$, which is smaller than CR_1. So the child's first position entry, which corresponds to the first feature, is set to $gbest_1 = 0.14$. Similarly, another random numbers are generated for the other features to fully build the child, which is shown as below:

- $r_2 = 0.3 > CR_2$, so $child_2 = 1 - gbest_2 = 1 - 0.76 = 0.24$
- $r_3 = 0.65 < CR_3$, so $child_3 = gbest_3 = 0.91$
- $r_4 = 0.75 > CR_4$, so $child_4 = 1 - gbes_4 = 1 - 0.21 = 0.78$

The mutated child's position is 0.14, 0.24, 0.91, 0.78

The pseudocode of crossover in each iteration is shown in Algorithm 4

Algorithm 4 : Pseudo-code of Mutation

```
1: begin
2: for each particle p_i in the swarm do
3:     if p_i's global fitness is not improved in 3 iterations then
4:         child ← p_i's gbest
5:         for i = 1 to gbest size (number of features) do
6:             calculate CR_i of the i^th feature according to Equation 5.6;
7:             generate a random number r
8:             if r ≥ CR_i then child_i = 1 − gbest_i
9:             end if
10:            if child's fitness is better than p_i's current fitness then
11:                p_i's current position ← child's position
12:                p_i's current fitness ← child's fitness
13:            end if
14:            if child's fitness is better than p_i's personal fitness then
15:                p_i's personal best position ← child's position
16:                p_i's presonal best fitness ← child's fitness
17:            end if
18:            if child's fitness is better than p_i's global fitness then
19:                p_i's global best position ← child's position
20:                p_i's global best fitness ← child's fitness
21:            end if
22:        end for
23:    end if
24: end for
25: end
```

The pseudo-codeof the proposed algorithm, Crossover-Mutation PSO (CMPSO), is given in Algorithm 5

5.3 Experimental Design

5.3.1 Benchmark Technques

To examine the performance of the proposed algorithm (CMPSO), a binary PSO based feature selection algorithms ,GPSO1[20]), and the PSOCC algorithm (Chapter 3) are used as

benchmark techniques in the experiment. In addition, to further analyis the evolutionary process, CMPSO is also compared with the original continuous PSO (PSO).

Dataset	Number of features	Number of clusters	Number of classes	No of instances
Wine	13	6	3	178
Vehicle	18	6	4	846
Ionosphere	34	11	2	351
Sonar	60	12	2	208
Musk1	166	14	2	476
Arrhythmia	279	15	16	452
Madelon	500	11	2	4400
Multiple Features	649	15	10	2000

Table 5.1: Datasets

Algorithm 5 : Pesudo-code of CMPSO

```
1:  begin
2:  randomly initialise the position and velocity of each particle;
3:  while Maximum iteration is not reached do
4:      evaluate the fitness of each particle;
5:      for i = 1 to PopulationSize do
6:          update the pbest of particle i;
7:      end for
8:      perform crossover operation on the swarm;
9:      update the gbest /*Ring topology*/;
10:     perform mutation operation on the swarm;
11:     for i = 1 to PopulationSize do
12:         Update velocity of particle i;
13:         Update position of particle i;
14:     end for
15: end while
16: end
```

5.3.2 Datasets and Parameter Settings

Eight datasets (Table 5.1) chosen from the UCI machine learning repository are used in thr experiments. Those datasets are the same ones, which are used in GPSO1 and PSOCC to ensure fair comparisons.

The parameters of CMPSO and OPSO are set as follow: $w = 0.7298, c_1 = c_2 = 1.49618$, $v_{max} = 0.2$, the population size is 30, the maximum iteration is 100 and the threshold θ is set as 0.7. The same paraemters are set for GPSO1, except the maximum velocity v_{max} is set to 6.0.

5.4 Experimental Results

This section firstly discusses the performance of CMPSO and the other three PSO based feature selection algorithms (Table 5.2), then compares the search ability between CMPSO and PSO.

5.4.1 CMPSO versus GPSO1 and CPSO

Dataset	Method	Ave-Size	Ave-Train ± Std-Train	Ave-Test ± Std-Test	T
Wine	All	13		76.54	+
	GPSO1	5.4	96.71 ± 7.77E-14	96.59 ± 2.76	+
	GPSOCC	4.60	97.37 ± 0.42	97.70 ± 2.52	=
	PSOCC	4.75	95.05 ± 0.58	96.70 ± 3.10	+
	CMPSO	4.70	97.28 ± 0.34	97.24 ± 2.89	
Vehicle	All	18		83.86	+
	GPSO1	8.94	86.11 ± 0.20	84.30 ± 0.62	=
	GPSOCC	7.30	90.10 ± 0.40	84.74 ± 0.49	-
	PSOCC	5.87	84.61 ± 0.56	84.72 ± 0.87	=
	CMPSO	7.57	90.25 ± 0.43	84.49 ± 0.44	
Ionosphere	All	34		83.81	+
	GPSO1	7.66	91.59 ± 0.47	89.50 ± 1.68	-
	GPSOCC	3.17	93.90 ± 0.67	86.89 ± 1.80	+
	PSOCC	9.7	90.04 ± 0.99	88.63 ± 1.68	=
	CMPSO	3.77	93.75 ± 0.86	87.94 ± 2.00	
Sonar	All	60		76.19	+
	GPSO1	17.64	86.74 ± 0.94	78.19 ± 4.14	+
	GPSOCC	10.17	90.67 ± 1.60	78.25 ± 2.95	+
	PSOCC	14.33	87.01 ± 2.00	78.94 ± 4.02	+
	CMPSO	11.60	91.59 ± 1.74	79.42 ± 2.48	
Musk1	All	166		83.92	+
	GPSO1	39.64	90.02 ± 0.60	84.95 ± 2.73	=
	GPSOCC	38.93	93.22 ± 1.40	83.29 ± 2.48	+
	PSOCC	35.03	89.78 ± 1.25	83.12 ± 3.41	+
	CMPSO	39.93	93.47 ± 1.12	85.06 ± 2.49	
Arrhythmia	All	279		94.46	+
	GPSO1	45.5	94.87 ± 0.91	94.85 ± 0.34	=
	GPSOCC	42.03	95.75 ± 0.18	95.12 ± 0.34	=
	PSOCC	44.17	95.11 ± 0.20	94.96 ± 0.38	=
	CMPSO	44.97	95.75 ± 0.19	95.07 ± 0.42	
Madelon	All	500		70.9	+
	GPSO1	36.08	85.45 ± 0.73	85.68 ± 1.10	-
	GPSOCC	51.13	89.20 ± 1.41	84.06 ± 1.64	-
	PSOCC	54.39	83.73 ± 1.74	83.40 ± 2.00	-
	CMPSO	107.2	89.4 ± 0.73	81.57 ± 1.46	
Multiple features	All	649		98.63	+
	GPSO1	91.4	99.38 ± 0.38	99.01 ± 0.13	=
	GPSOCC	51	99.36 ± 0.07	98.86 ± 0.17	+
	PSOCC	51.07	99.17 ± 0.09	98.84 ± 0.18	+
	CMPSO	110.77	99.53 ± 0.05	99.05 ± 0.01	

Table 5.2: Experimental Results of CMPSO

Table 5.2 shows the experemental results of the CMPSO algorithms, where "All" means that all the available features are used for classification. "Ave-size" shows the average number of selected features over the 30 runs. "Ave-Train", "Std-Train", "Ave-Test", "Std-Test" illustrate the average and standard deviation of the training and testing accuracies over the 30 independent runs. T shows the results of the statistical significant tests between the accuracy of CMPSO and other algorithms. "+" or "-" means that the algorithm CMPSO achieved significantly better or worse classification performance than other algorithms, "=" means there is no significant difference between them.

As can be seen from Table 5.2, on all datasets, CMPS successfully selects feature subsets,

which achieve significantly higher classification accuracy than using all feattures. In addition, on each dataset, the number of selected features is always less than a third of the total number of features.

Compared to PSOCC algorithm, CMPSO also achieves similar or higher classification acuracy, while the number of selected features remains similar. It is remarkable that on all datasets, CMPSO achieves much higher training accuracy than PSOCC. Especially, on Vehicle dataset, CMPSO's training accuracy is about 5.27% higher than PSOCC's one.

Comparing CMPSO with GPSOCC, in terms of testing accuracy, CMPSO outperforms GPSOCC on 3 datasets and achieves similar accuracy on 3 of the remaining datasets. In addition, on all datasets, CMPSO also achieves higher or similar training accuracy than GPSOCC.

Compared to GPSO1, CMPSO achieves higher testing accuracy on 3 out of the datasets and similar accuracy on 4 of the remaining datasets. However, GPSO1 performs better than CMPSO on the Ionosphere dataset (about 2%). The reason is that CMPSO selects ony 3.77 features, which is 2 times less than the number of selected features by GPSO1. Furthermore, on each dataset, CMPSO always achieve higher training accuracy than GPSO1. For example, on Sonar dataset, the training accuracy of CMPSO is 90.62%, which is 4% higher, while CMPSO selects even smaller number of features than both GPSO1.

The results suggest that intergrating crossover and mutation operations into PSO improves the search ability of a PSO algorithm, which is clearly shown by the higher training accuracy on all datasets. However, someone might argue that this improvement is from the usage of continuous PSO, which usually has better exploration ablity than binary PSO (GPSO1). In the next section, a comparison between standard PSO (PSO) and CMPSO is done to further analysis the effect of crossover and mutation operations.

5.4.2 Comparison with standard PSO (PSO)

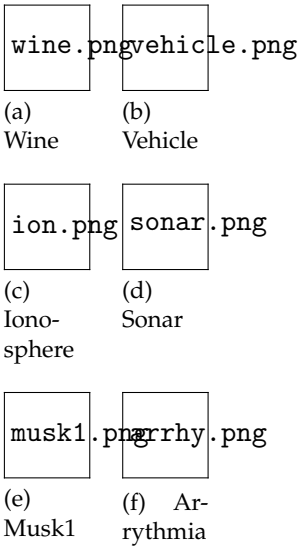
For each dataset, CMPSO and PSO are independently run 30 times. The fitness value of PSO is calculated by using the Equation 5.4. Each run contains 100 iterations, in which the fitness value is improved after each iteration. The average of fitness value at each iteration is calcualted by using 30 indepent runs' results. Those average values are used to evaluate the fitness evolution of the above algorithms. The smaller fitness an algorithm achieves, the better the algorithm is. Figure 5.3 illustrates the fitness values in each iteration on 6 datasets, in which X-axis represents the iteration index and Y-axis represents the average fitness value at that iteration. CMPSO's fitness values are represented by blue lines, while red lines are fitness values of PSO.

As can be seen from Figure 5.3, on all datasets, the blue lines are always under the red lines, which indicates that the positions discoverd by CMPSO always have better fitness values then the ones discovered by PSO. On the small dataset like Wine, Vehilce, Ionosphere or Sonar, the diffenrece between those 2 approaches are clearly shown in the middle iterations. Although at the end, the blue and red lines approach each other, the blue lines are still a little bit lower than the red lines, which idicates CMPSO still can better explores the search space than PSO.

The more significant difference is shown in a bigger dataset, Musk1. As can be seen in Figure 5.3e, despite of starting at the same position (same fitness value) the gap between the red and blue line is maintained or even getting bigger with respect to the iteration order. At the end of the run, the difference of fitness values between these two approaches is still about 0.3%. On Arrhythmia, another dataset with a big number of features, the difference is not as signigicant as Musk1. The reason is that the starting position is quite good. At that position the fitness value is already 0.02, which is quite smaller, compared to the starting

fitness of Musk1 (0.053). However, in Figure 5.3f, we still can regconise that the blue line is still under the red line.

The results clearly show that crossover and mutation operation would help PSO to better explore the search space, which is proved by the better fitness values that CMPSO can achieve during an entire run.



(a) Wine (b) Vehicle

(c) Iono-sphere (d) Sonar

(e) Musk1 (f) Ar-rythmia

Figure 5.3: Fitness evolution figures

5.5 Summary

The goal of this chapter was to develop a PSO-based feature selection approach that can better explore the search space. This goal was achieved by intergrating two genetic operators: crossover and mutation into the PSO. The results show that, on almost datasets, the proposed algorithm (CMPSO) can achieve similar or better testing accuracy than the other three methods PSOCC, GPSOCC and GPSO1. In terms of training accuracy and fitness evolution, CMPSO outperforms these three approaches as well as the standard PSO.

In CMPSO, although the training accuracy is significantly improved, the testing accuracy is not improved much or even getting worse than other PSO based approach. There might be an overfitting problem in this proposed algorithm. It is worth to develop a mechanism to balance between the training accuracy and testing accuracy. In addition, the Objective 1 shows that statistical clusering information can help PSO achieve better accuracy. It is promissing to combine both CMPSO and statistical clusterting information to achieve better classification performace.

Chapter 6

Conclusions and Future Work

The conclusions are presented in this Chapter.

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