VICTORIA UNIVERSITY OF WELLINGTON
Te Whare Wananga o te Upoko o te Ika a Maui

School of Engineering and Computer Science

COMP 307 — Lecture 13

Uncertainty and Probability 1

Reasoning under Uncertainty Basics

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Outline

- Introduction
- Product Rule
- Sum Rule
- Normalisation
- Independence
- Summary

Uncertainty

- Many algorithms are designed as if knowledge is perfect, but it rarely is.
- There are almost always things that are unknown, or not precisely known.

- Fundamental role of uncertainty in AI
- Probability theory can be applied to many problems

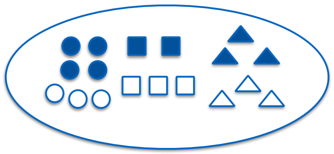
Basics

- Unconditional/prior probability
  - P (X): the probability of X occurring

- Conditional/posterior probability
  - P (X|Y): the probability of X occurring given Y has occurred.

- Joint probability
  - P (X, Y): probability of X and Y occurring

General Example



Y(shape)

X (fill ?)	A	B	C	
T				9
¬T				9
	7	5	6	18

The Product Rule

X \ Y	A	B	C	
T	4	2	3	9
¬T	3	3	3	9
	7	5	6	18

- P(A)=7/18
- P(X=T) = 9/18
- P(X=T, Y=A) = 4/18
- P(X=T/Y=A) = 4/7
- P(Y=A/X=T) = 4/9

- P(X=T, Y=A) = P(X=T)*P(Y=A/X=T)

- ***The Product Rule:***
  P(X,Y)=P(X)*P(Y/X)

The Sum Rule

X \ Y	A	B	C	
T	4	2	3	9
¬T	3	3	3	9
	7	5	6	18

- P(X=T, Y=A) = 4/18
- P(X=T, Y=B) = 2/18
- P(X=T, Y=C) = 3/18
- P(X=T) = 9/18

- P(X=T) = P(X=T, Y=A) +P(X=T, Y=B) +P(X=T, Y=C)

- ***The Product Rule***:

$$\mathrm{P(X)}=\textstyle\sum_y \mathrm{P(X, Y)}$$

Normalisation

X \ Y	A	B	C	
T	4	2	3	9
¬T	3	3	3	9
	7	5	6	18

- P(X=T) = 9/18
- P(X=¬T) = 9/18
- P(Y=A/X=T) = 4/9
- P(Y=B/X=T) = 2/9
- P(Y=C/X=T) = 3/9

- P(X=T) +P(X=¬T) = 1
- P(Y=A/X=T) +P(Y=B/X=T) +P(Y=C/X=T) = 1
- ***The Normalisation Rule***:

$$\textstyle\sum_x \mathrm{P(X)}=1$$

$$\textstyle\sum_x \mathrm{P(X/Y)}=1$$

Question

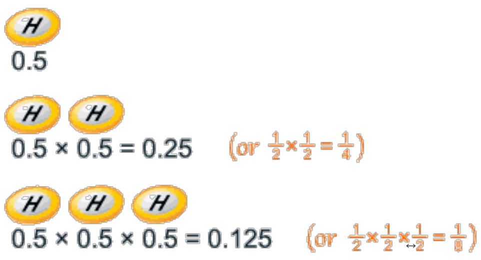
- If P(D|E) = 1/4,
- do we know
  - P(D|¬E) ?
  - P(¬D|E) ?
  - P(¬D|¬E) ?

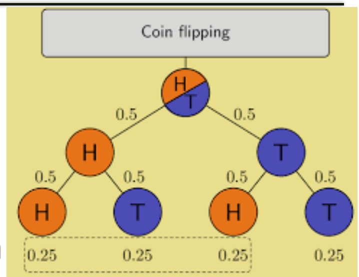
Independence

- Independence: two variables are independent when neither event can be related to the other events occurrence.
- Variable X_1: the firs flip
- Variable X_2: the second flip

- $P(X_1=H, X_2=H) = P(X_1=H) * P(X_2=H \mid X_1=H)$
- *$P(X_2=H) = P(X_2=H \mid X_1=H)$ because X_1 and X_2 are* ***independent*** *to each other*
- $P(X_1=H, X_2=H) = P(X_1=H) * P(X_2=H)$

Independence





- Independence $X \perp\!\!\!\perp Y$
- ↔ P(X|Y) = P(Y)
- ↔ P(X, Y) = P(X) * P(Y)

Example: Rolling a Die

- What is the probability to get a “1” ?
- What is the probability to get a “6” ?

- If rolling twice, what is the probability of get a “2” at the first time, then get a “3” the second time ?
- Further:
  - If rolling twice, what is the probability of get two “6”s ?
  - If rolling once, what is the probability of a “2” or a “5” ?

Example

- **W**indy or **C**alm
- **D**ay 1 ——>**D**ay 2
- P(D1=W) = 0.5
- P(D2=W|D1=W) = 0.6
- P(D2=W|D1=C) = 0.3
- P(D1=C) = 0.5
- P(D2=C|D1=W) = 0.4
- P(D2=C|D1=C) = 0.7

- Question: P(D2=W) ?

Summary

- Uncertainty is everywhere
- Different rules

- Frequentist probability VS Bayesian probability

- Next Lectures: Bayes Rules and Naive Bayes