Shape Deformation via Interior RBF

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Motivation

- Animation tool for creating natural poses
- Space deformation vs. direct deformation
  - Subspace mapping: Can embed any type of object
  - Performance
Control Structures

• Lattice [Sederberg and Parry 1986]
Cage-based Methods

• Mean Value Coordinates [Ju et al. 2005]
• Harmonic Coordinates [Joshi et al. 2007]
  – Interior locality property
• Green Coordinates [Lipman et al. 2008]  
  – Quasi-conformal mapping
• VHM [Ben-Chen et al. 2009]
  – Convenient handles to control the deformation
  – Minimizes As-Rigid-As-Possible energy
  – The cage is still a problem: A well-constructed cage usually involves a considerable amount of manual work.
IRBF Properties

• Real-time
• Convenient handles to control the deformation
• Minimizes As-Rigid-As-Possible energy
• No cage
• Interior locality property
• Simpler formulation
• Can handle touching surface
Method Overview

1. IRBF centers are sampled from the surface of the shape.
2. The shape is automatically filled with spheres.
3. Interior distances are calculated from the IRBF centers to:
   - The local rigidity structures that represent the spheres
   - The deformed points
   - The anchor points
4. Using *local/global* optimization the coefficients of the IRBF are calculated, and the shape is deformed.
RBF

• Scattered data approximation to functions on $\mathbb{R}^d$ [Franke 1982].

• An approximation at $p$:

$$F(p) = \sum_{c \in C} a_c \Phi(\| p - c \|), \quad a_c \in \mathbb{R},$$

where $C \subset \mathbb{R}^d$ is a set of centers and $\Phi$ is a real function.
IRBF

• Using interior distances $d_I$:

$$T_{\Pi}(p) = Ap + t + \sum_{c \in C} a\phi(p, c), \quad a_c \in \mathbb{R}^3$$

Reproduce affine mapping

• where

$$\phi(p, c) = \frac{1}{D_I(p)} \Phi(d_I(p, c))$$

$$D_I(p) = \sum_{c \in C} \Phi(d_I(p, c))$$

$$\Phi(r) = \frac{1}{\sqrt{r^2 + h^2}}$$

Normalization for reproducing the constant function

Inverse multiquadric — regular everywhere
The Discrete Energy

• **Energy:**

\[ E(\Pi) = \widetilde{U}(\Pi) + \lambda V(a) \]

Subject to positional constraints

- **Regularization term:**

\[ V(a) = \sum_{c \in C} \| a_c \|^2 \]
– As-Rigid-As-Possible [Sorkine and Alexa 2007; Chao et al. 2010] distortion measure for a set of spheres $B$:

$$\bar{U}(\Pi) = \sum_{b \in B} \bar{\rho}(b)$$

$$\bar{\rho}(b) = \sum_{i=1}^{3} w_{i,b} \| M_b e_{i,b} - e'_{i,b} \|^2, \; M_b \in SO(3)$$
Local Shape and Volume Control

- Using prescribed transformation $S(b)$:

$$
\bar{\rho}(b) = \sum_{i} \| M_{b} S(e_{i,b}) - e'_{i,b} \|^2
$$
Anisotropic scaling
Interior Distances

- Euclidean distances
- Fast Marching Method (FMM)
- Mean Value Coordinates (MVC)
- Geodesics in Heat [Crane et al. 2013]

Crane et al. 2013
• Curve skeleton
  – At sampled points along the skeleton, we place spheres with a radius of 90% of the distance from the sampled point to the surface.
• Sphere tree
  – Approximating a shape using spheres [Bradshaw and O’Sullivan 2004]

• Post processing: The spheres are pruned according to a threshold of maximum overlap, and minimal size.
Choosing The IRBF Centers

• Random sampling
• Farthest point sampling
• Sampling can be adaptive according to user hints (much easier than adapting a cage).
Algorithm Steps

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Results
Survey models
[Botsch and Sorkine 2008]
VHM

IRBF

VHM

IRBF