

# Sampling Heuristics for Multi-Objective Dynamic Job Shop Scheduling Using Genetic Programming

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**Abstract.** Dynamic job shop scheduling is a complex problem in production systems. Automated design of dispatching rules for these systems, particularly with genetic programming based hyper-heuristics (GPHH) has been a promising approach in recent years. However, GPHH is a computationally intensive approach. Parallel evolutionary algorithms are one of the key approaches to tackle this drawback. Furthermore when scheduling is performed under uncertain manufacturing environments while considering multiple conflicting objectives, evolving good rules requires large and diverse training instances. This further increases the computational burden for GPHH. In order to address this issue, a method to sample those problem instances which have the potential to promote the evolution of good rules is proposed. In particular, a sampling heuristic which successively rejects clusters of problem instances in favour of those problem instances which show potential in improving the pareto front for a dynamic multi-objective scheduling problem is developed. We exploit the efficient island model-based approaches to simultaneously consider multiple training instances for GPHH. By using appropriate migration policies we deploy our sampling heuristic on the island model to evolve rules which are able to generate significantly better pareto fronts.

**Keywords:** Scheduling · Genetic programming · Parallel algorithms

## 1 Introduction

Job shop scheduling is a complex problem in manufacturing industries. It is an NP-hard problem [17] which deals with the allocation of resources to complete a set of jobs while optimizing one or more objectives, under some constraints. In dynamic job shop scheduling (DJSS) problems, new jobs continuously arrive and no prior information is known about them. In general, researchers consider deterministic scheduling problems in which once the information of a new job is obtained it remains constant. But in practice, due to events like machine breakdown, variation in raw material quality, operator availability, etc. uncertainty is ubiquitous in manufacturing environments. In fact, with increasing uncertainty, scheduling becomes more difficult [10].

Dispatching rules have been widely used for generating schedules for different objectives. Basically, a dispatching rule is a function which is used to assign a priority value to every operation queued on a machine. The machine then processes the operation with the least priority value. Dispatching rules have shown good success for DJSS problems [15]. Furthermore, for scheduling under uncertainty, dispatching rules have been shown to outperform branch-and-bound technique particularly when uncertainty is high [11].

Dispatching rules have very low computational cost and are applicable to practical scenarios. But designing them requires considerable expertise and rigorous experimentation. Genetic programming based hyper-heuristic (GPHH) approach has been very successful in automatically evolving dispatching rules [1, 6, 14]. In GPHH, a dispatching rule is a genetic program which is composed of terminals (attributes of jobs/shop) and functions (e.g. arithmetic operators). A discrete event simulation (DES) is employed to automatically evaluate the performance of the evolved GP rules. Due to the flexible representation and powerful search ability of GP, it has been possible to evolve very good rules for diverse shop scenarios [9] while handling multiple objectives for DJSS problems [13].

Since DES must be used to evaluate each individual in every generation of the evolutionary system it demands high computational cost and time. Surrogate models [5] have been proposed as an alternative but they suffer from poor accuracy among other drawbacks [13]. A key approach to solving this problem is to use parallel evolutionary algorithms [2]. In particular, the island model which uses a spatially structured network of subpopulations (on different processors) to exchange promising individuals among each other is an effective approach. For example, [18] use a specialized island model for multi-objective optimization. More recently, [7] use a similar approach for multi-objective job shop scheduling on static problems.

Furthermore, in order to improve the generalization of the evolved dispatching rules it is vital to use a large training set containing instances with diverse characteristics. This is more important when we consider uncertain manufacturing environments as they present varying shop scenarios [9, 8]. Moreover, when we consider multiple objectives the importance of using diverse training set is compounded. For example, makespan and total tardiness are two frequently considered conflicting objectives. Minimizing the makespan results in high throughput where as minimizing tardiness requires jobs to be not very late. A conflicting scenario arises when a set of jobs with long processing times but shorter deadline compete with a set of jobs with shorter processing times and longer deadlines. For higher throughput the shorter jobs must be completed first as against the longer jobs which adversely affects the tardiness. For evolving good dispatching rules it is important to present the evolutionary system with training instances which capture scenarios highlighting such conflict amongst the objectives, under different shop scenarios.

Therefore, for evolving practical dispatching rules with good generalization ability we need a large and diverse training set. But this in turn will increase the already high computational cost of GPHH. To address this problem we need an approach which selects those training instances which present potentially complex scenarios to the evolutionary system. In other words, we need a method which can effectively sample training instances to significantly improve the performance of GPHH

without exceeding the computational budget. The parallel island model has already been shown to be efficient for GPHH when dealing with multiple objectives. Furthermore, the inherent potential of migration policies [16] used in the island model approach presents an opportunity to develop a sampling technique which utilizes the parallel evolutionary system to simultaneously consider multiple training instances for evolving rules.

In this work, we consider multi-objective DJSS problems with a focus on uncertainty in processing times which affects all our objectives. Our primary goal is to develop a sampling heuristic for GPHH which selects good training instances toward evolving a significantly better pareto front. To this end, our specific objectives are: (1) Develop a feature extraction method toward clustering the training dataset into DJSS problem instances with different characteristics. (2) Develop a sampling heuristic which iteratively rejects the clusters (*successive reject heuristic*) in favor of those which have the potential to improve the pareto front. (3) Determine the migration policies of the island model toward improving the efficacy of the proposed sampling heuristic.

## 2 Background

DJSS problems are characterized by continuous arrival of jobs from a Poisson process [12].  $n_j$  operations constitute a job  $j$  with the constraint that they must be processed in a predefined route say  $(o_{j,1} \rightarrow o_{j,2} \rightarrow \dots, o_{j,n_j})$ . Each operation can be processed only on one machine in its route. Other conditions/assumptions which are followed in this work are no preemption, no recirculation of jobs, no machine failure and zero transit times.

The processing time of an operation  $p_{j,i}$  is usually different from the actual processing time ( $p'_{j,i}$ ) due to uncertainty, which is known only at the time of realization on the machine. Also  $p'_{j,i} > p_{j,i}$  is a practical assumption [9].

Total tardiness, makespan, total flow time are some of the objectives considered for DJSS problems. In this work, we consider two potentially conflicting objectives [12]: (1) total weighted tardiness (*TWT*) and (2) makespan ( $C_{max}$ ).

$$TWT = \sum w_j \times \max(C_j - d_j, 0),$$

where  $C_j$  = completion time,  $d_j$  = due date and  $w_j$  are weights.

$$C_{max} = \max(f_j),$$

where  $f_j$  is the flowtime of a job.

## 3 Proposed Method

In this section, we propose our sampling heuristic for GPHH and present our algorithm based on the island model approach. Firstly, we develop a method to extract features from the DJSS problem instances which are used for clustering. During evolution the sampling heuristic iteratively rejects the clusters in favor of those

which show promise to evolve a better pareto front. We use the fitness evaluation strategies of NSGA-II to develop the successive reject heuristic. Finally, we discuss the migration policies of the proposed island model.

### 3.1 Clustering of DJSS Problem Instances

**Table 1.** Job Features

Feature	Description
#operations	number of operations per job.
$p$	processing time of the job.
$\Delta^p$	$\frac{p'}{p}$ , $p'$ is the actual processing time with uncertainty.
due date factor	$\frac{(\delta_{due\ date} - \delta_{rel\ date})}{p'}$ ; where $\delta_{due\ date}$ is the due date and $\delta_{rel\ date}$ is the release date

We extract features from the DJSS problem instances based on the parameters: number of operations per job, processing time, due date factor and uncertainty in processing time. Firstly, the basic features for each job are extracted as described in Table 1. The estimated processing time  $p$  is the value of processing time which is used by the dispatching rule to make sequencing decisions. The realized processing time  $p'$  is the *actual* processing time obtained after the job is completed on the machines.

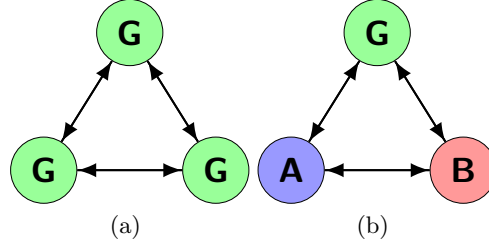
Once the feature values are aggregated for all the jobs in an instance; the first, second and third quartiles of each aggregate are calculated to form a 12-dimensional feature vector characterizing each problem instance. The objective of extraction of these features is to cluster the problem instances into classes with different characteristics. Considering our earlier example, assume we have one DJSS problem instance with high variability in number of operations per job and their processing times and another instance with low variability. It can be easily seen that our feature vectors corresponding to the two problem instances can be used to differentiate them.

$\mathcal{T}$  is the training set containing  $n$  DJSS problem instances. We extract features for all these problems and cluster them into  $\mathcal{C}_1$  and  $\mathcal{C}_2$  using K-means clustering. We apply K-means clustering again on each of these clusters to yield  $\{\mathcal{C}_{11}, \mathcal{C}_{12}\}$  and  $\{\mathcal{C}_{21}, \mathcal{C}_{22}\}$  respectively. This process can be repeated to obtain more sub-clusters  $\{\{\mathcal{C}_{111}, \mathcal{C}_{112}\}, \{\mathcal{C}_{121}, \mathcal{C}_{122}\}\}$  and  $\{\{\mathcal{C}_{211}, \mathcal{C}_{212}\}, \{\mathcal{C}_{221}, \mathcal{C}_{222}\}\}$  and so on.

### 3.2 Proposed Island model

Unlike [7] we consider a heterogeneous island model in this work. We use two classes of islands in our evolutionary system. The first class of islands, represented as  $G$  in Figures 1(a) & 1(b), sample training instances from the set  $\mathcal{T}$  throughout the evolutionary process. The second class of islands represented as  $A$  and  $B$  in

Figure 1(b) sample problem instances from different clusters at during the stages of evolution. The appropriate choice of the cluster is controlled by the *successive reject heuristic*.



**Fig. 1.** (a) Standard island model, (b) Island model for successive reject heuristic

**Input:**  $\mathcal{T}$   
**Output:**  $\{\omega_1, \omega_2, \dots, \omega_p\}$

- 1 **for**  $g \leftarrow 1 : N_G$  **do**
- 2     Sample an instance  $\mathcal{I} \in \mathcal{T}$ .
- 3     Run  $g^{th}$  iteration of NSGA-II using  $\mathcal{I}$ .
- 4     Receive/Send individuals using migration policies.
- 5     Wait for all the communication to complete.(synchronous)
- 6 **end**
- 7 Collect the genetic programs corresponding to the Pareto front :  $\{\omega_1, \omega_2, \dots, \omega_p\}$ .

**Algorithm 1:** Island  $G$

**Input:**  $\mathcal{C}_Z, \mathcal{N}_{SRH}$   
**Output:**  $\{\omega_1^z, \omega_2^z, \dots, \omega_p^z\}$

- 1 **for**  $g \leftarrow 1 : N_G$  **do**
- 2     Sample an instance  $\mathcal{I} \in \mathcal{C}_Z$ .
- 3     Run  $g^{th}$  iteration of NSGA-II using  $\mathcal{I}$ .
- 4     Receive/Send individuals using migration policies.
- 5     Wait for all the communication to complete.(synchronous)
- 6     **if**  $g \in \mathcal{N}_{SRH}$  **then**
- 7          $\mathcal{C}_Z \leftarrow SRH(P_k^A, P_k^B, \mathcal{C}_A, \mathcal{C}_B)$
- 8     **end**
- 9 Collect the genetic programs corresponding to the Pareto front :  $\{\omega_1^z, \omega_2^z, \dots, \omega_p^z\}$ .

**Algorithm 2:** Island  $Z$  ( $Z \in \{A, B\}$ )

We first describe the evolutionary process of island  $G$  in Algorithm 1. In every generation, a new training instance is sampled from  $\mathcal{T}$ . Due to our familiarity with

NSGA-II [3] and the fact that we consider only two objectives in this work, we chose NSGA-II as our underlying evolutionary algorithm. In line 3, an iteration of NSGA-II is performed. After each generation, the migration policy determines (line 4) if there will be an exchange of individuals among the islands. Since we consider synchronous communication protocol in the model there is a wait period until all the islands complete the migration process. The output of the algorithm is a set of dispatching rules which can generate a pareto front.

In Algorithm 2, we describe the evolutionary process of the islands  $A$  and  $B$  (Figure 1(b)). Both islands are similar, except that they sample their training instances from different clusters. Their migration policies are same. The cluster  $\mathcal{C}_Z$  is changed at discrete stages during the evolutionary process. The set  $\mathcal{N}_{SRH}$  contains the generations at which the successive reject hypothesis is invoked to change  $\mathcal{C}_Z$  (line 6). Rest of the procedure is same as in Algorithm 1.

**Successive Reject Heuristic** The successive reject heuristic (SRH) is described in the Algorithm 3. When the SRH is invoked by islands  $A$  and  $B$  at generation  $g \in \mathcal{N}_{SRH}$ , the top- $k$  individuals from each island,  $P_k^A$  and  $P_k^B$  respectively, are sent to the island  $G$  on which the SRH algorithm is run. Note that it is possible to run SRH on any island. The two sets of individuals are then combined to get  $P_{2k}$  (line 3). A set of DJSS problem instances is sampled from  $\mathcal{T}$  with the purpose of assigning new fitness values to each individual in  $P_{2k}$  (lines 4–11).

After the fitness assignment, we use the NSGA-II fitness strategies to sort the individuals. NSGA-II first ranks the individuals based on non-dominance and then the individuals with same rank are ordered using crowding distance operator [3]. We use the same approach to sort the individuals in  $P_{2k}$  (line 12). After sorting, the best  $k$  individuals are extracted from  $P_{2k}$  into the list  $TOP_k$  (line 13). Then we count the number of individuals corresponding to each island in the list  $TOP_k$  (lines 14–15). The cluster corresponding to the island with the lower number of individuals in  $TOP_k$  is rejected (line 17 or 20). The  $\mathcal{C}_Z$  of the winning island is further clustered into two sub-clusters (lines 18 or 21). The new clusters which are the output of SRH algorithm are then randomly assigned to the islands. Till the next invocation of SRH, the evolution in the islands is continued using DJSS problem instances sampled from the new clusters.

**Migration Policies** Migration policies play a major role in the performance of the island models [2, 16]. A migration policy states the number individuals to be sent to the destination island, frequency of migration and the generation from which the migration starts. For the standard island model shown in Figure 1(a) designing a policy is straightforward. Due to symmetry, a single policy for all the islands will suffice [7]. Since we consider heterogeneous islands which are working together for a common goal, different migration policies must be designed for participating islands.

Formally, a policy  $\mathcal{M}_{I_1, I_2}^{\rightarrow}$  from island  $I_1$  to  $I_2$  is defined by a triplet  $\langle \text{start generation, frequency, \#individuals to send} \rangle$ . We consider the migration policy  $\mathcal{M}_{I_1, I_2}^{\rightarrow}$  to be different from  $\mathcal{M}_{I_2, I_1}^{\rightarrow}$ .

**Input:**  $P_k^A, P_k^B, \mathcal{C}_A, \mathcal{C}_B$   
**Output:**  $\{\mathcal{C}_A^{new}\}, \{\mathcal{C}_B^{new}\}$  to respective islands.

- 1  $\mathcal{T} \leftarrow$  set of all DJSS training instances.
- 2  $\mathbb{I} \leftarrow$  sample from  $\mathcal{T}$
- 3  $P_{2k} \leftarrow \{P_k^A, P_k^B\}$
- 4 **foreach**  $p \in P_{2k}$  **do**
- 5      $tot.fit. \leftarrow \vec{0}$
- 6     **foreach**  $\mathcal{I} \in \mathbb{I}$  **do**
- 7          $obj.values \leftarrow$  Simulation for  $(p, \mathcal{I})$ .
- 8          $tot.fit. \leftarrow tot.fit. + obj.values$
- 9     **end**
- 10     $fit(p) \leftarrow tot.fit.$
- 11 **end**
- 12 Sort  $P_{2k}$  using NSGA-II fitness strategies.
- 13  $TOP_k \leftarrow$  Extract top-k individuals from  $P_{2k}$ .
- 14  $TOP_k^A \leftarrow |P_k^A \cap TOP_k|$
- 15  $TOP_k^B \leftarrow |P_k^B \cap TOP_k|$
- 16 **if**  $TOP_k^A \geq TOP_k^B$  **then**
- 17     **Reject**  $\mathcal{C}_B$ .
- 18      $\{\mathcal{C}_A^{new}\}, \{\mathcal{C}_B^{new}\} \leftarrow$  K-means cluster( $\mathcal{C}_A$ )
- 19 **else**
- 20     **Reject**  $\mathcal{C}_A$ .
- 21      $\{\mathcal{C}_A^{new}\}, \{\mathcal{C}_B^{new}\} \leftarrow$  K-means cluster( $\mathcal{C}_B$ )
- 22 **end**

**Algorithm 3:** Successive Reject Heuristic**Table 2.** Notation

Notation	Description
$\mathcal{T}$	set of all DJSS problem instances for training.
$N_G$	total number of generations for evolutionary process.
$\mathcal{C}_Z$	cluster corresponding to island $Z \in \{A, B\}$ .
$P_k^Z$	top $k$ individuals from island $Z \in \{A, B\}$ .
$M_{\overrightarrow{X, Y}}$	migration policy from island $X$ to $Y$ .
$P_{2k}$	combined list of top $k$ individuals from islands $A$ and $B$ .
$TOP_k$	list of top $k$ individuals across island $A$ and $B$ .
$TOP_k^{Z_k}$	#individuals which are present both in $P_k^Z$ and $TOP_k$ , $Z \in \{A, B\}$
$\mathcal{N}_{SRH}$	set of generations at which SRH is invoked.

For island  $G$ , it is more productive to receive individuals from  $A$  and  $B$  frequently as it will improve its diversity. This is because the evolved rules in  $A$  and  $B$  are exposed to training instances which are different from  $G$ . On the other hand a high frequency of migration between  $A$  and  $B$  will homogenize the islands, making SRH less effective. Moreover, the frequency of migration in  $M_{AG}^{\rightarrow}$  is much higher than  $M_{GA}^{\leftarrow}$  (similar for island  $B$ ) for the same reasons. The same analysis holds for determining the number of individuals to be migrated between the two.

The migration policy  $M_{AB}^{\rightarrow}$  is restricted to exchanging individuals only and immediately after invocation of SRH. This is necessary to maintain the desired heterogeneity of the system.  $M_{AB}^{\rightarrow}$  and  $M_{BA}^{\leftarrow}$  are symmetric and defined by  $\mathcal{N}_{SRH}$ . Immediately after SRH, large number of individuals are exchanged among the two islands before commencing the sampling of instances from the new clusters.

## 4 Experiment Design

### 4.1 Simulation Model

We use a DES system [4] to generate DJSS problem instances. The job arrival follows a Poisson process with  $\lambda = 0.85$  [9]. For every run of the simulation, the first 500 jobs are considered as warm-up and the objective values are calculated for the next 2000 jobs.

The uncertainty in processing times is simulated using the model considered in [9]. Basically for an operation  $o_{j,i}$  the relationship between the processing time with uncertainty  $p'_{j,i}$  and processing time without uncertainty  $p_{j,i}$  is:

$$p'_{j,i} = (1 + \theta_{j,i})p_{j,i}, \theta_{j,i} \geq 0.$$

$\theta$  follows exponential distribution [9]. In Table 3, the parameter  $\beta$  corresponds to the scale parameter of the exponential distribution.

In order to create problem instances with varying characteristics, DJSS problem instances are generated with many combinations of the simulation parameters shown in Table 3. The combination of these four pairs of parameters can simulate 16 types of jobs. For composing a training DJSS problem instance, 3 job types are considered at a time. This results in approximately 680 possible configurations. Consequently, 20 DJSS problems are created for each configuration and the training set  $\mathcal{T}$  is created.

For testing, we create a new set (say  $\mathcal{Y}$ ) of DJSS problems using the 680 possible configuration mentioned above. We randomly sample 30 DJSS problem instances from  $\mathcal{Y}$  to obtain our first test set. Furthermore, *four* more test sets are created by clustering  $\mathcal{Y}$  and sampling 30 problem instances from each. These test sets are denoted by 3- $\mathcal{Y}$ , 3-I, 3-II, 3-III and 3-IV, where 3 stands for number of job types.

We also create more test sets by considering 4 job types. the number of unique configurations possible in this case as high as 3080. Performing the same procedure described above generates the following test sets: 4- $\mathcal{Y}$ , 4-I, 4-II, 4-III and 4-IV.



**Table 3.** DJSS simulation parameters

Simulation paramter	Values
Processing time range	[0,49],[20,69]
Uncertainty scale parameter ( $\beta$ )	{0.2, 0.4}
Due date tightness	{1.5, 2.5}
# operations per job	{8, 10}

## 4.2 Genetic Programming System

The terminal set for genetic programming is listed in Table ???. For all our islands we use a population size of 800 each. We also compare our method with the standard NSGA-II for which the population size is set at 3000. With a tree depth of 6, the crossover and mutation are 0.854 and 0.1 respectively. Each evolutionary algorithm is run for 150 generations.

**Table 4.** Migration Policies

Island-pairs Policies	
$\overrightarrow{GG}$	< 20, 20, 30 >
$\overrightarrow{AB}$	< 50, 50, 60 >
$\overrightarrow{BA}$	< 50, 50, 60 >
$\overrightarrow{AG}$	< 20, 20, 30 >
$\overrightarrow{BG}$	< 20, 20, 30 >
$\overrightarrow{GB}$	< 50, 25, 10 >
$\overrightarrow{GA}$	< 50, 25, 10 >

**Table 5.** Terminal Sets for GP.

Terminal Set Meaning	
PT	Processing time of operation
RO	Remaining operations for job
RJ	Ready time of job
RT	Remaining processing time of job
RM	Ready time of machine
DD	Due date
W	Job weight
ERC	Ephemeral Random constant

## 4.3 Island model

The migration policies are presented in Table 4. We use the SRH algorithm at generations 49 and 99, i.e.  $\mathcal{N}_{SRH} = \{49, 99\}$ . While deciding the frequency parameter of the migration policies involving islands  $A$  and  $B$ ,  $\mathcal{N}_{SRH}$  has been taken into account. The exchange of individuals starts after a delay as the evolved rules in the early generations are not good. For the  $TOP_k$  individuals the value  $k = 30$  was chosen, after experimental evaluation.

## 5 Results & Discussion

In this Section, we present the results from our experiments. We compare the performance our method with standard NSGA-II algorithm and standard island model approach. The hypervolume ratio, inverted generational distance and spread indicators are considered for comparison as they are frequently used in the literature [14]

to compare the generated pareto fronts. In order to approximate the true pareto front the the individuals from all the methods across all runs are combined. For each method the solutions are compared over 30 problem instances from a test set. 30 independent runs produce 30 sets of dispatching rules for each method. The Wilcoxon-rank-sum test is used to compare the performance. We consider a significance level of 0.05.

The results are summarized in Tables 6-8. Each cell in the tables consists of a triplet which represents  $[win - draw - lose]$ . For example, in Table 6 the comparison between standard island model and NSGA-II approach is summarized. For the training set 3 –  $\mathcal{Y}$ , if we consider hypervolume indicator, then island model has significantly outperformed NSGA-II in 18 problem instances and there is no significant difference observed for 12 problem instances.

**Table 6.** NSGA-II versus Island-Model

	3- $\mathcal{Y}$	3-I	3-II	3-III	3-IV	4- $\mathcal{Y}$	4-I	4-II	4-III	4-IV
HV	[18-12-0]	[11-19-0]	[18-12-0]	[19-11-0]	[17-13-0]	[14-16-0]	[15-15-0]	[18-12-0]	[16-13-0]	[12-18-0]
IGD	[24-6-0]	[21-9-0]	[25-5-0]	[29-1-0]	[27-3-0]	[24-6-0]	[21-9-0]	[22-8-0]	[22-8-0]	[19-11-0]
SPREAD	[3-22-5]	[0-18-12]	[4-23-0]	[5-25-0]	[2-28-0]	[3-21-6]	[6-22-2]	[4-23-3]	[5-20-5]	[2-24-2]

In Table 6, we compare NSGA-II with standard island model. As expected, the performance of island model is much better, which is line with the observations made in [7]. For HV and IGD performance indicators, the performance is very good, but for SPREAD indicator there is no clear winner. This significant difference in performance is consistent across all the test sets including 4-job type configurations.

**Table 7.** NSGA-II versus SRH-Island Model

	3- $\mathcal{Y}$	3-I	3-II	3-III	3-IV	4- $\mathcal{Y}$	4-I	4-II	4-III	4-IV
HV	[23-7-0]	[17-3-0]	[23-7-0]	[25-5-0]	[24-6-0]	[20-10-0]	[24-6-0]	[22-8-0]	[24-6-0]	[21-9-0]
IGD	[30-0-0]	[30-0-0]	[27-3-0]	[29-1-0]	[30-0-0]	[30-0-0]	[27-3-0]	[30-0-0]	[28-2-0]	[29-1-0]
SPREAD	[1-23-6]	[0-25-5]	[1-27-2]	[3-26-1]	[0-28-2]	[4-21-5]	[1-27-2]	[1-27-2]	[2-22-6]	[0-25-5]

In Table 7, we compare the performance of NSGA-II and our proposed method based on SRH. Across all the test sets the proposed method has done well. Particularly for hypervolume performance indicator, the SRH method has significantly done better in more than 20 problem instances for almost every test set. Similar performance is observed for IGD as well. Though once gain, with respect to SPREAD, there is no consistency. This is because the obtained pareto fronts are sparse for all the algorithms.

Finally we compare, the SRH approach with the standard island model. Once again the SRH approach performs significantly better on a lot of problem instances

**Table 8.** Island model versus SRH-Island Model

	3- $\mathcal{Y}$	3-I	3-II	3-III	3-IV	4- $\mathcal{Y}$	4-I	4-II	4-III	4-IV
HV	[10-20-0]	[5-24-1]	[6-22-2]	[9-21-0]	[14-16-0]	[10-20-0]	[10-20-0]	[8-21-1]	[9-21-0]	[11-19-0]
IGD	[18-12-0]	[20-10-0]	[19-11-0]	[19-11-0]	[20-10-0]	[15-15-0]	[14-15-1]	[13-17-0]	[15-15-0]	[18-20-0]
SPREAD	[5-18-7]	[10-20-0]	[3-24-3]	[1-25-4]	[0-23-7]	[5-20-5]	[1-22-7]	[2-20-8]	[4-17-9]	[3-24-3]

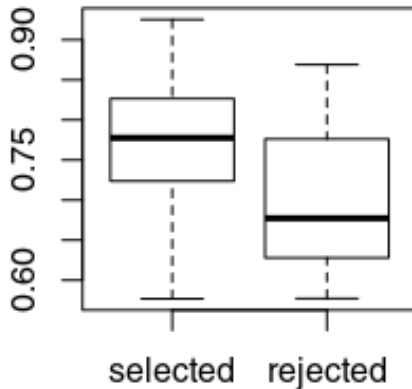
from each test set. This confirms that SRH approach was able to associate useful training instances through the successive rejection of clusters of training set.

### 5.1 Analysis

A typical path taken by the successive reject heuristic is shown below.

$$\mathcal{T} \rightarrow \{\mathcal{C}_1, \mathcal{C}_2\} \rightarrow \{\mathcal{C}_{21}, \mathcal{C}_{22}\} \rightarrow \{\mathcal{C}_{211}, \mathcal{C}_{212}\}$$

In **retrospect**, we analyze the clusters which showed potential to guide the GPHH toward evolving better rules. In order to further validate the ability of SRH, we took the clusters represented by  $\mathcal{C}_{21}$  and  $\mathcal{C}_{22}$  as training sets and applied NSGA-II algorithm to evolve dispatching rules. We observed that the cluster rejected by SRH did perform significantly poor. A boxplot for comparing the performance on one of the problem instances is shown in Figure 2.



**Fig. 2.** HV comparison between NSGA-II over: cluster rejected vs. cluster accepted by SRH.

Furthermore, we also analyzed the problem configurations associated with cluster  $\mathcal{C}_{21}$ . One of the reasons for analyzing  $\mathcal{C}_{21}$  and not  $\mathcal{C}_2$  is its smaller size and also

the fact that out of 30 independent runs, this path was chosen by SRH for 20 of the runs. We observed that the DJSS instances whose job types were pertaining to equal proportion of high and low level of uncertainty were in high numbers. Also, DJSS instances comprising jobs with low and high number of operations per job were found in large numbers. In other words, SRH is biased towards instances with high variability in their jobs.

## 6 Conclusions

Most of the research works in evolutionary scheduling focus on improving only the different aspects of algorithms. But it is also important to develop methods to select appropriate training instances for the evolutionary algorithms to produce desired outcome. We have successfully taken a step in this direction by demonstrating that a simple sampling heuristic using basic features extracted from the problem instances could promote the evolutionary process. By exploiting the potential of island model approach we obtained significantly better results with no additional computational cost. We demonstrated the efficacy of our approach using just two objectives and in future, we would extend our work to tackle many-objective scheduling problems.

## References

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