

## Back Propagation Learning for Neural Networks

### (1) Feed Forward Pass

$$I_1, \quad O_1 = I_1;$$

$$I_2, \quad O_2 = I_2;$$

$$I_3 = O_1 * W_{13} + O_2 * W_{23} + b_3, \quad O_3 = f(I_3) = \frac{1}{1+e^{-I_3}};$$

$$I_4 = O_1 * W_{14} + O_2 * W_{24} + b_4, \quad O_4 = f(I_4) = \frac{1}{1+e^{-I_4}};$$

$$I_5 = O_3 * W_{35} + O_4 * W_{45} + b_5, \quad O_5 = f(I_5) = \frac{1}{1+e^{-I_5}};$$

$$I_6 = O_3 * W_{36} + O_4 * W_{46} + b_6, \quad O_6 = f(I_6) = \frac{1}{1+e^{-I_6}};$$

### (2) Back Propagation:

$$\beta_5 = d_5 - O_5;$$

$$\beta_6 = d_6 - O_6;$$

$$\beta_3 = W_{35} * O_5 * (1 - O_5) * \beta_5 + W_{36} * O_6 * (1 - O_6) * \beta_6;$$

$$\beta_4 = W_{45} * O_5 * (1 - O_5) * \beta_5 + W_{46} * O_6 * (1 - O_6) * \beta_6;$$

$$\Delta W_{35} = \eta * O_3 * O_5 * (1 - O_5) * \beta_5;$$

$$\Delta W_{36} = \eta * O_3 * O_6 * (1 - O_6) * \beta_6;$$

$$\Delta W_{45} = \eta * O_4 * O_5 * (1 - O_5) * \beta_5;$$

$$\Delta W_{46} = \eta * O_4 * O_6 * (1 - O_6) * \beta_6;$$

$$\Delta W_{13} = \eta * O_1 * O_3 * (1 - O_3) * \beta_3;$$

$$\Delta W_{14} = \eta * O_1 * O_4 * (1 - O_4) * \beta_4;$$

$$\Delta W_{23} = \eta * O_2 * O_3 * (1 - O_3) * \beta_3;$$

$$\Delta W_{24} = \eta * O_2 * O_4 * (1 - O_4) * \beta_4;$$

$$\Delta b_5 = \eta * O_5 * (1 - O_5) * \beta_5;$$

$$\Delta b_6 = \eta * O_6 * (1 - O_6) * \beta_6;$$

$$\Delta b_3 = \eta * O_3 * (1 - O_3) * \beta_3;$$

$$\Delta b_4 = \eta * O_4 * (1 - O_4) * \beta_4;$$

### (3) Update Weights

$$W_{13} = W_{13} + \Delta W_{13};$$

$$W_{14} = W_{14} + \Delta W_{14};$$

$$W_{23} = W_{23} + \Delta W_{23};$$

$$W_{24} = W_{24} + \Delta W_{24};$$

$$W_{35} = W_{35} + \Delta W_{35};$$

$$W_{36} = W_{36} + \Delta W_{36};$$

$$W_{45} = W_{45} + \Delta W_{45};$$

$$W_{46} = W_{46} + \Delta W_{46};$$

$$b_3 = b_3 + \Delta b_3;$$

$$b_4 = b_4 + \Delta b_4;$$

$$b_5 = b_5 + \Delta b_5;$$

$$b_6 = b_6 + \Delta b_6;$$