An analysis of connectivity in a MANET of autonomous cooperative mobile agents under the Rayleigh fading channel

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Abstract— **In this paper, we study the connectivity of a Mobile Ad Hoc Network (MANET) of autonomous cooperative mobile agents (e.g. mobile robots) under the Rayleigh fading channel. Connectivity is a critical performance parameter of cooperative robots deployed in real-time scenarios such as disaster and rescue scenarios. There are two major factors that affect the connectivity of the MANET. First, the mobility of the nodes causes the separation between any pair of nodes to fluctuate. Second, atmospheric condition and obstacles can cause the transmission range of the nodes to fluctuate. Based on these factors, stochastic analysis is performed to derive the connectivity probability. The connectivity probability represents the fraction of time that a node is connected to at least one other node. This probability is used to study the effect of mobility and fading on the connectivity as the transmission range or number of nodes in the network varies. Such analytical results can form the basis of performance modeling of MANET routing protocols and network optimization.**

Keywords-MANET; Ad-hoc Network; Connectivity; Rayleigh Fading; Mobility Model; Autonomous Agents; Robots

I. INTRODUCTION

In this paper, we investigate the connectivity of a team of mobile robots working cooperatively in real-time scenarios where communications is supported by a mobile ad hoc network (MANET). The connectivity is affected by the separation between two nodes and the transmission range of a node. The separation between the two nodes is affected by the mobility of the nodes whereas the transmission range of the node is affected by the channel fading process. Although there are related works [1]-[7] on the analysis of connectivity of wireless networks, they assume a stationary network with high density, which are more appropriate for sensor networks. None of them considers the fluctuation of the transmission range due to scattering obstacles and varying atmospheric conditions. Our contribution in this paper is the application of stochastic modeling to derive the connectivity probability in terms of the node mobility and channel fading processes.

The connectivity probability represents the fraction of time that a node is connected to at least one other node. As the nodes are all statistically identical, if they are connected within one hop, they are also fully connected to the network as a whole. Therefore, the connectivity probability gives a measure of how fully connected a network is. Our aim here will be to minimize either the node count or the transmission range or both so as to achieve a well connected network.

The paper is organized as follows. In Section II, we present our analytical models. The Probabilistic Mobility Model (PMM) of the nodes is described first followed by the channel fading model. In Section III, we derive the connectivity probability. In Section IV, we introduce the concept of the statistically equivalent mobility model, which allows us to obtain the approximate analytical connectivity probability of the popular simulation model known as the Random Waypoint Mobility Model (RWMM). In Section V, we present the numerical results along with the simulation results. The numerical results are used to study the effects of varying the system parameters as well as the fading parameter on the connectivity of the network. The simulation is used to verify the theoretical results. Section VI outlines the conclusions.

II. THE ANALYTICAL MODEL

A. The Mobility Model

A Probabilistic Mobility Model (PMM) [8] is used. This model is first used by Chiang in his thesis [9]. For the purpose of analysis, the state variables are defined slightly different from the original version in [8]. In this model, a node moves in discrete steps. Let Δx and Δy be the incremental changes in the x and y directions respectively. The movement is described by Markov chains for both x and y directions (Fig 1). Note that the values for the incremental changes {-1, 0, 1} are also the Markov state values.

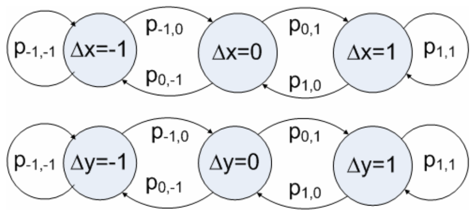

Figure 1. Markov chains for the mobility in the x and y directions

The movements in the x and y directions are independent and the Markov Chains are identically distributed in the x and y directions. At each time step, a node either moves to one of the eight neighboring positions or stays in its original position (Fig 2). The resolution of the path is increased by decreasing the step size [8].

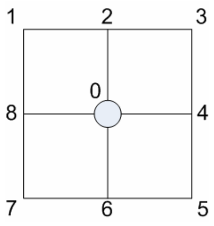

Figure 2. Possible positions that a node can move at each time step

The stationary state probabilities, $\pi_{Mob,-1}$, $\pi_{Mob,0}$ and $\pi_{Mob,1}$ of the Markov Chain are given as follows:

$$\begin{aligned}\pi_{Mob,-1} &= \frac{p_{0,-1}(1-p_{1,1})}{(1-p_{-1,-1}+p_{0,-1})(1-p_{1,1})+p_{0,1}(1-p_{-1,-1})}\\ \pi_{Mob,0} &= \frac{(1-p_{-1,-1})(1-p_{1,1})}{(1-p_{-1,-1}+p_{0,-1})(1-p_{1,1})+p_{0,1}(1-p_{-1,-1})}\\ \pi_{Mob,1} &= \frac{p_{0,1}(1-p_{-1,-1})}{(1-p_{-1,-1}+p_{0,-1})(1-p_{1,1})+p_{0,1}(1-p_{-1,-1})}\end{aligned} \quad (1)$$

The stationary state probabilities describe the fractions of time that the node is moving backward (state -1), staying in the same position (state 0) and moving forward (state 1) for either the x or y direction at steady state respectively. The mobility process is at steady state when the time, $t \to \infty$.

B. The Radio Channel Model

The radio channel is modeled using the free space propagation model with Rayleigh fading [10]. Let R_{ab} be the separation between two nodes, a and b, P_t and P_r be the powers of the transmitted and received signals and κ be the Rayleigh distributed fading gain. The free space attenuation is a function of R_{ab} and is represented by $g(R_{ab})$. The radio channel is modeled as follows:

$$P_r = \kappa^2 g(R_{ab}) P_t \quad (2a)$$

The probability density function (p.d.f.) of κ is given as follows:

$$p(\kappa) = (\kappa/\sigma^2)\exp(-\kappa^2/2\sigma^2) \quad (2b)$$

where $2\sigma^2 = E(\kappa^2)$ is the mean power gain of the received power. The fading channel is non-lossy if the mean power gain is equal to one ($2\sigma^2 = 1$). If the atmosphere contains resistive or absorbing elements which result in the dissipation of the energy, the mean power gain is less than one ($2\sigma^2 < 1$).

In a high data rate, low velocity environment, the fading is slow-varying [10]. Hence, the fading process is effectively independent from the mobility of the node. Let η_0 be the free space transmission range. The p.d.f. of the transmission range, η is derived as:

$$p(\eta) = (\eta/\sigma^2\eta_0^2)\exp(-\eta^2/2\sigma^2\eta_0^2) \quad (3)$$

III. Connectivity Probability

We are interested in the asymptotic connectivity of a node, a when $t \to \infty$. Let us assume that the nodes are independent and identically distributed (i.i.d.) and their movements are constrained within a square region of dimension ($S \times S$). S is measured in unit of steps. The position of the node in the square region is denoted as (x, y) such that $0 \le x, y \le S$ and $x, y \in$ Integer. Let N be the number of nodes in the network. Also, let c_{ab} be the one hop connectivity probability between a pair of nodes, a and b with coordinates (x_a, y_a) and (x_b, y_b) respectively when $t \to \infty$. Node a is partitioned if it is not connected to any node in the network. Let $\bar{c}_a$ and c_a be the partitioning and connectivity probabilities of node a respectively, we have,

$$c_a = 1 - \bar{c}_a = 1 - \prod_{b=1, b\neq a}^{N} (1 - c_{ab}) \quad (4a)$$

The one-hop connectivity probability, c_{ab} is the probability that the physical separation, R_{ab} between the two nodes, a and b, are less than the transmission range, η,

$$\begin{aligned} &c_{ab} = \Pr(R_{ab} \le \eta) \\ &where, R_{ab} = \sqrt{(x_b - x_a)^2 + (y_b - y_a)^2} \end{aligned} \quad (4b)$$

Also, using the total probability theorem, we have,

$$\begin{aligned} &\Pr(R_{ab} \le \eta | \eta) = \sum_{y_b=0}^{S}\sum_{x_b=0}^{S}\sum_{y_a=0}^{S}\sum_{x_a=0}^{S} p(x_a, y_a, x_b, y_b) \\ &\times \Theta(\eta, x_a, y_a, x_b, y_b) \\ &where, \Theta(\eta, x_a, y_a, x_b, y_b) = \begin{cases} 1 & R_{ab} \le \eta \\ 0 & R_{ab} > \eta \end{cases} \end{aligned} \quad (4c)$$

In other words, in order to calculate the conditional probability in equation (4c), we need to evaluate the probability of nodes, a and b jointly in all possible positions in the square region. For each pair of positions, we evaluate R_{ab} and compare with the transmission range, η. If R_{ab} is less than or equal to η, we will include the corresponding probability into the summation.

$p(x_a, y_a, x_b, y_b)$ is the asymptotic joint position probability distribution of a pair of nodes, a and b. In general, we expect the probability distributions of the positions of the nodes to be functions of time. The asymptotic position probability distribution exists only if the probability distributions of the positions of the nodes converge to stationary distributions when $t \to \infty$. To derive $p(x_a, y_a, x_b, y_b)$, we observe that the coordinate (*x*,*y*) of the node in the square region can be viewed as a 2-variable state in the Markov process because of the mobility model that we have adopted for the analysis. This view can further be simplified because *x* and *y* are independent variables. Hence, we can model the Markov process as two separate and identical single variable state Markov Chains for *x* and *y*. The state values of the Markov Chain are the same as the values that *x* and *y* can take and they are $\{k\}_{k=0}^{S}$. The Markov Chain for either one of the coordinates is shown in Fig 3. The self-transition in each state is illustrated using the dashed arrow.

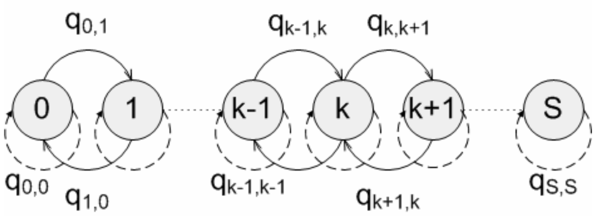

Figure 3. The Markov Chain for the position of a node

Let the transition probability of the states be $q_{i,j}$ such that $0 \le i, j \le S$ and $i, j \in$ Integer. $q_{i,j}$ can be obtained from equation (1) as follows:

$$q_{i,j} = \begin{cases} \pi_{Mob,-1} & i = j-1 \\ \pi_{Mob,0} & i = j \\ \pi_{Mob,1} & i = j+1 \\ 0 & elsewhere \end{cases} \qquad (5)$$
$$where, 0 \le i, j \le S$$

Let the stationary probability distribution of the Markov Chain be $\{\pi_k\}_{k=0}^{S}$. When we make use of the facts that the nodes are independent and identically distributed (i.d.d.) and the *x* and *y* coordinates are independent, the asymptotic joint position probability distribution can be expressed as follows:

$$p(x_a, y_a, x_b, y_b) = \pi_{x_a}\pi_{y_a}\pi_{x_b}\pi_{y_b} \qquad (6)$$

where, $\pi_{x_a}, \pi_{y_a}, \pi_{x_b}, \pi_{y_b}$ are the stationary state probabilities for the *x*, *y* coordinates of node *a* and *b* respectively.

The stationary probability distribution $\{\pi_k\}_{k=0}^{S}$ exists for the Markov Chain and is given in equation (7). We observe that the boundary conditions at $k = 0$ and $k = S$ have been taken care of in the derivation. Furthermore, it can be shown that the Markov Chain is ergodic. This implies that the Markov process always converges to the same stationary probability distribution irregardless of the initial position of the node.

$$\pi_k = \begin{cases} \left(\dfrac{\pi_{Mob,1}}{\pi_{Mob,-1}}\right)^{k-1}\left(\dfrac{1-\pi_{Mob,0}}{\pi_{Mob,-1}}\right)\pi_0 & 0 < k < S \\ \left(\dfrac{\pi_{Mob,1}}{\pi_{Mob,-1}}\right)^{S-1}\pi_0 & k = S \end{cases}$$
$$where, \pi_0 = \left[1 + \left(\frac{\pi_{Mob,1}}{\pi_{Mob,-1}}\right)^{S-1} + \sum_{k=1}^{S-1}\left(\frac{\pi_{Mob,1}}{\pi_{Mob,-1}}\right)^{k-1}\left(\frac{1-\pi_{Mob,0}}{\pi_{Mob,-1}}\right)\right]^{-1} \qquad (7)$$

Finally, we can remove the condition in the conditional probability in equation (4c) to derive the one hop connectivity probability, c_{ab} in equation (4b). The equation for the one hop connectivity probability, c_{ab} is derived as follows:

$$\begin{aligned} c_{ab} &= \int_0^\infty \Pr(R_{ab} \le \eta | \eta) p(\eta) d\eta \\ &= \sum_{y_b=0}^{S}\sum_{x_b=0}^{S}\left[\sum_{y_a=0}^{S}\sum_{x_a=0}^{S}\exp\left(-\frac{R_{ab}^2}{2\sigma^2\eta_0^2}\right)\pi_{x_a}\pi_{y_a}\right]\pi_{x_b}\pi_{y_b} \end{aligned} \qquad (8)$$

Note that in the derivation, we make use of the independence of the fading process and the mobility of the node.

IV. Statistically Equivalent Mobility Model

RWMM is a random mobility model that is popularly used in the MANET simulations. As there are a few variations, in this paper, we will discuss only one variation [8] to demonstrate the concept of the statistically equivalent mobility model. The statistical equivalent mobility model can be used to derive the approximate analytical connectivity probability of the RWMM. A RWMM is described by the following parameters: minimum and maximum speed, $[v_{min}, v_{max}]$, travel time, t_{travel} and pause time, t_{pause}. Briefly, a node in RWMM will first choose a speed and direction that are uniformly distributed between $[v_{min}, v_{max}]$ and $[0, 2\pi]$ respectively. It then travels for the duration of t_{travel} and pauses for the duration of t_{pause}, before choosing a new speed and direction and repeats the cycle again.

Since a cycle consists of one t_{travel} and t_{pause}, the equivalent pause time can be derived as follows:

$$t_{pause} = t_{travel}\pi_{mob,0}^2 / \left(1 - \pi_{mob,0}^2\right) \qquad (9a)$$

Let t_{step} and d_{step} be the transition time step in seconds and the step size in meters respectively in the PMM. In the PMM, the average time and distance that the node travels in the same direction for either x or y direction are geometrically distributed and derived as:

$$\begin{aligned} t_{travel,x} &= t_{step} / \left(\pi_{Mob,1} + 0.5\pi_{Mob,0}\right) = 2t_{step} \\ d_{travel,x} &= 2d_{step} \end{aligned} \qquad (9b)$$

Therefore, the equivalent travel time, distance and average speed when the node is traveling in the same direction are derived as,

$$
\begin{aligned}
t_{travel} &= \varepsilon \times t_{travel,x} \\
d_{travel} &= \varepsilon \times d_{travel,x} \\
\bar{v} = d_{travel}/t_{travel} &= d_{step}/t_{step} \\
where, \varepsilon &= \left(2\pi_{Mob,0} + \sqrt{2}\pi_{Mob,1}\right) / \left(2\pi_{Mob,0} + \pi_{Mob,1}\right)
\end{aligned} \tag{9c}
$$

V. NUMERICAL AND SIMULATION RESULTS

In this section, we generate a few plots to study the effects of varying the system parameters on the connectivity of the network. There are two motivations for such studies. First, usually the system parameters represent scarce resources which we will like to minimize given a reasonable quality of connectivity in the network. Second, the minimization of system resources has an additional benefit of reducing interference and channel contention at the MAC layer [11]. In addition, we will look into the effect of the channel fading on the connectivity of the network. By choosing an appropriate value for the fading parameter, we could characterize the aggregate effect of the radio environment on the connectivity of the network.

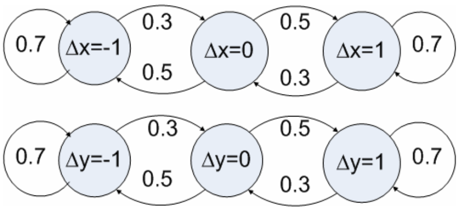

Figure 4. The transistion probabilities used for the numerical results

The following parameters are set to be the same for all plots: room size = (1000×1000) m^2, $2\sigma^2 = 0.6$. The transition probabilities for the mobility model are set to be similar to those used in [8], the values are illustrated in Fig 4.

First, we are interested in the connectivity of the nodes in an enclosed room as the transmission range of the nodes increases. One way to achieve an increase in transmission range will be to increase the transmission power of the node. Hence, by looking at the connectivity probability, we will know the amount of transmission power required to achieve a certain level of connection. In Fig 5, we show the connectivity probability over different transmission ranges for the cases with and without fading. The node count, N is fixed at 25 nodes for this plot. From the graph, the connectivity probabilities increase for both cases as the transmission range increases. When there is no fading, the connectivity probability increases at a faster rate. For example, when there is no fading, the connectivity probability is approximately one when the transmission range is 250m. The average number of hops is about (1000/250 =) 4 hops for this case. However, when there is fading, the transmission range is increased to 380m in order to achieve the same connectivity probability, with an average number of 2.6 hops.

Second, we are interested in the connectivity of the nodes in an enclosed room as the node count increases. In Fig 6, we show the connectivity probability over different node count for the cases with and without fading. The free space transmission range, η_0 is fixed at 250m for this plot. From the graph, the connectivity probabilities increase for both cases as the node count increases. Again, when there is no fading, the connectivity probability increases at a faster rate.

Simulations using PMM and RWMM for both no fading and fading cases are used to verify our analysis. The simulation code is written using C language. The common parameter values used in the simulation are shown in Table I. Using equation (9a)-(9c), we obtain the equivalent RWMM. The parameter values of the equivalent RWMM for Fig 5 and 6 are given in Table II. The simulation plots in Fig 5 and 6 show that the simulation results converge well to the theoretical results. They also show that PMM provides very good approximation to the RWMM.

TABLE I. PARAMETER VALUES USED IN THE SIMULATION

Parameter	Value
Run	100
Duration to stabilize the mobility	10,000s
Simulation duration	10,000s
Time Step, t_{step}	1s
Step Size, d_{step}	10m
Propagation Model	Free space

TABLE II. MOBILITY PARAMETER VALUES FOR FIGURE 5 AND 6

RWMM		PMM	
t_{travel}	3s	$[\pi_{Mob,-1}, \pi_{Mob,0}, \pi_{Mob,1}]$	[0.385,0.231,0.385]
t_{pause}	0s	t_{step}	1s
$[v_{min}, v_{max}]$	[0,20]	d_{step}	10m
$\bar{v}$	10m/s	$\bar{v}$	10 m/s

TABLE III. MOBILITY PARAMETER VALUES FOR FIGURE 7

RWMM		PMM	
t_{travel}	9s	$[\pi_{Mob,-1}, \pi_{Mob,0}, \pi_{Mob,1}]$	[0.296,0.408,0.296]
t_{pause}	2s	t_{step}	4s
$[v_{min}, v_{max}]$	[0,20]	d_{step}	40m
$\bar{v}$	10m/s	$\bar{v}$	10 m/s

Finally, we provide one example where parameters for the RWMM are given and those of the equivalent PMM are derived (see Table III). The result for the case with fading is plotted in Fig 7. We also plot the analytical result for the fading case from Fig 5 into Fig 7 for comparisons. Again, we see that the PMM provides very good approximation to the RWMM. We observe that there is almost no difference between the two analytical plots which are used to approximate the RWMM with different mobility parameters. Hence, connectivity is actually not very sensitive to the parameters of the mobility models. Therefore, we infer that connectivity results obtained from analysis of static node distributions is a good approximation to the connectivity of the mobile nodes.

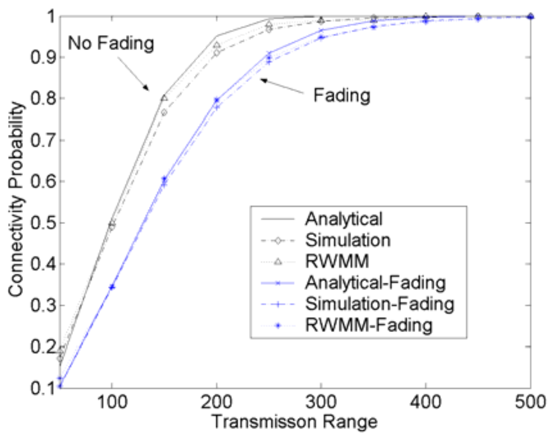

Figure 5. Theoretical vs Simulation Connectivity Probabilities when Transmission Range is varied

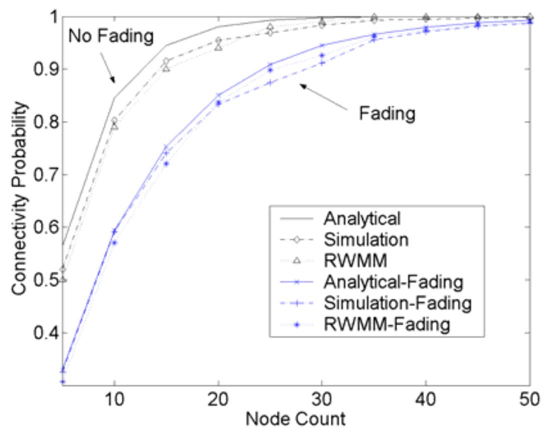

Figure 6. Theoretical vs Simulation Connectivity Probabilities when Node Count is varied

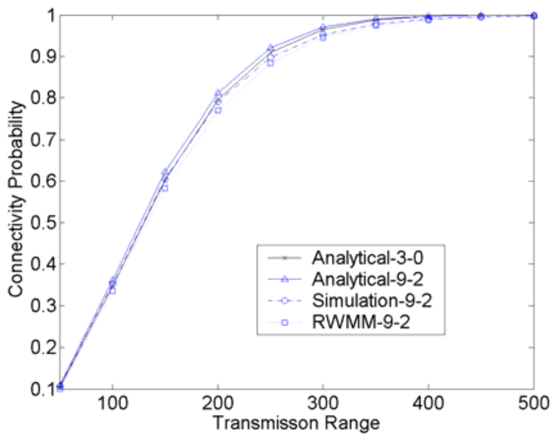

Figure 7. Theoretical vs Simulation Connectivity Probabilities when Transmission Range is varied for different mobility parameters, with fading

VI. CONCLUSION

Connectivity of MANET is affected by the mobility of the nodes and the fading of the signals in the radio channel due to varying atmospheric conditions and scattering obstacles. In this paper, we address the issues using stochastic modeling. We derive the connectivity probability and study the effects of varying the system parameters on the connectivity of the network. We hope that from the study, we could find ways to minimize the scarce resources given a reasonable quality of connectivity in the network. This minimization of the scarce resource also has an additional benefit of reducing interference and channel contention at the MAC layer. In addition, we also study the aggregate effect of the channel fading on the connectivity of the network. It is shown from the numerical result that for the case with fading, we need either a higher transmission power or node count in order to achieve the same level of connectivity compared to the case without fading. Furthermore, we observe that the connectivity of the network is actually not very sensitive to the parameters in the mobility model. In future, this model will serve as a basis to further analytical modeling of MANET routing protocols and network optimization.

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