Analysis of Clustering and Routing Overhead for Clustered Mobile Ad Hoc Networks

Mingqiang Xue[2] Inn-Inn Er[2,1] Winston K.G. Seah[1,2]

[1]*Networking Department, Institute for Infocomm Research*
21 Heng Mui Keng Terrace, Singapore 119613
[2]*Department of Computer Science, School of Computing*
National University of Singapore, Singapore 117543
xuemingq@comp.nus.edu.sg, {stuerii, winston}@i2r.a-star.edu.sg

Abstract

This paper presents an analysis of the control overhead involved in clustering and routing for one-hop clustered mobile ad hoc networks. Previous work on the analysis of control overhead incurred by clustering algorithms focused mainly on the derivation of control overhead in the Knuth big-O notation with respect to network size. However, we observe that the control overhead in a clustered network is closely related to different network parameters, e.g. node mobility, node transmission range, network size, and network density. We present an analysis that captures the effects of different network parameters on the control overhead. The results of our work can provide valuable insights into the amount of overhead that clustering algorithms may incur in different network environments. This facilitates the design of efficient clustering algorithms in order to minimize the control overhead.

1. Introduction

Mobile ad hoc networks (MANET) are autonomous systems formed by mobile nodes without any infrastructure support. Routing in MANET is challenging because of the dynamic nature of the network topology. Although numerous routing protocols have been proposed for MANETs, such as DSDV [6] and AODV [7], these routing protocols are not suitable for large MANET because the overhead for maintaining up-to-date routing information at each node quickly becomes unacceptable as network size increases. Clustering is a technique that partitions a network into different groups or clusters, creating a logical hierarchy in the network. By partitioning a network into different clusters, both storage and communication overheads for maintaining up-to-date routing information can be significantly reduced. However, clustering still incurs overhead that has yet to be investigated in depth. Control overhead is an important metric for measuring the performance of a clustering algorithm since bandwidth is a limited and valuable resource in MANETs. It is shown in [1] that the per-node capacity in a random ad hoc network with N nodes is $\Theta(1/\sqrt{N \log N})$, which is a decreasing function with network size N. Thus, as the network size increases, the utilization of bandwidth becomes a very critical factor that affects the overall performance of a network.

Over the past few years, numerous clustering schemes have been proposed [5]. However formal analysis on clustering overhead is still lacking. Most of the prior work on clustering overhead focuses on the message and time complexity of a clustering algorithm, which is a rough approximation of clustering overhead with respect to the network size. In [16], comparisons for the clustering overhead in Knuth big-O notation [2] of several clustering schemes such as MobDHop [18], Max-Min [19], Lowest-ID (LID) [12][13], Highest Connectivity Clustering (HCC) [12] and DMAC [17] is presented. In [3] and [4], the hierarchical routing overhead for a network with O(log N) level of hierarchies using LCA[15] clustering scheme is systematically analyzed, and they conclude that the messages transmissions per node in their network model is O(log N).

In contrast, our analysis considers several important network parameters that affect the amount of clustering control overhead incurred, including network size, node mobility, node transmission range, and node density. Our analysis also provides an insight on how these network parameters affect the amount of routing overhead incurred to maintain up-to-date routing information in a proactive manner within every cluster assuming a general hybrid routing protocol is in operation.

The rest of this paper is organized as follows: In section 2, we provide an overview of control messages required for clustering and proactive routing. In section 3, we present the assumptions of our network model and analyze the control overhead for general one-hop clustering algorithms based on this network model. The ratio of *cluster-heads* in a network is viewed as a variable in this analysis which may vary across different one-hop clustering algorithms and Section 4 presents simulation results to verify our analysis. In section 5, we analyze the ratio of *cluster-heads* for a well-known one-hop clustering algorithm, namely LID clustering. This ratio, when substituted into our analysis in Section 3, gives a valuable insight to the amount of clustering overhead incurred by Lowest-ID clustering. In section 6, look at the control overhead in Knuth Ω-notation, and conclude in section 7.

2. Clustering & Routing Overhead Overview

As the topology of a MANET changes, control messages are generated by nodes to 1) update routing information when route changes occur and 2) update roles in clusters when cluster changes happen. Different clustering algorithms may use different schemes and generally, three types of control messages are needed:

a) Beacon, commonly known as *HELLO* message, for nodes to learn the environment and identities of adjacencies (neighbors).
b) Cluster management (which shall refer to as *CLUSTER* message) for nodes to adapt to cluster changes and update its role.
c) Route management (which shall refer to as *ROUTE* message) for nodes to learn of route changes.

The *HELLO* message is often used. Periodically broadcasted by each node, a node adds a new node to its neighbor-list when it hears the new node's *HELLO*, and a node deletes a node from its neighbor-list when it could not hear that node for some predetermined duration. Ideally, a *HELLO* message should be sent every time a node has a new neighbor.

The *CLUSTER* message is a generalization for a sequence of messages needed by nodes to update cluster information. The execution of clustering algorithms can usually be divided into cluster formation stage and cluster maintenance stage. The *CLUSTER* message only refers to a sequence of messages needed by nodes when cluster change occurs in the cluster maintenance stage. We do not consider initial cluster formation, as we are focusing the long term performance of clustering. Such a sequence of messaging may be implemented in different ways. For example, in LID clustering, a node sends a *CLUSTER* message which indicates the cluster it belongs to when it has decided its own cluster. While in DMAC, two types of messages are sent: CH(v), sent by a node v to inform its neighbors that it is going to be a *cluster-head*, and JOIN(v,u), sent by a node v inform its neighbors that it will join the cluster whose *cluster-head* is u. Our analysis assumes reactive cluster maintenance. There are two main reasons for this. Firstly, reactive cluster maintenance usually requires less control messages; analyzing control overhead using reactive cluster maintenance can provide a lower bound. Secondly, the clustering maintenance scheme is similar for most clustering algorithms, i.e. the same set of events could trigger the *CLUSTER* message. Thus, it can be used for analyzing the control overhead of *CLUSTER* message for all clustering algorithms. The following properties for 1-HOP clustered networks should be ensured and any violation will trigger *CLUSTER* messages at relevant nodes:

P1. No two *cluster-heads* are directly connected

P2. Each node should be affiliated to one cluster; i.e. each ordinary node should have only one *cluster-head* and be at most one hop away

The Least Clusterhead Change (LCC) scheme introduced in [11] is an extension to clustering algorithms, like LID and HCC, which possess the above properties. The *ROUTE* message is periodically exchanged by neighboring nodes to update routing information, and ideally, sent every time route table information changes.

3. Theoretical Analysis

In this section, we present the clustering overhead analysis for one hop clustering algorithm taking into consideration several network parameters, viz. network size, cluster size, network density, node velocity, and transmission range.

3.1. Assumptions

There are N nodes in the network and each has a transmission range r. If two nodes are within the transmission range of each other, they form a bi-directional link between each other and become neighboring nodes.

The clustering algorithm used can be any one-hop clustering algorithm as long as the cluster structure it forms satisfies the two properties *P1* and *P2* in section 2. After the cluster formation, each node is assigned a role of either *cluster-member* or *cluster-head*. In the cluster formation stage, the clustering algorithm selects a node as the *cluster-head* with a probability P_{HEAD}. Thus, the expected number of clusters or expected number of *cluster-heads* is $N \cdot P_{HEAD}$. Here, P_{HEAD} varies for different clustering algorithms and it can be viewed as a metric of a particular clustering algorithm, which describes how *cluster-heads* are distributed over the network. In the cluster maintenance stage, each node keeps broadcasting *CLUSTER* messages when cluster changes happen due to violations of *P1* and *P2*. Upon hearing the *CLUSTER* messages from its neighbors, a node updates its role according to the rules defined by the clustering algorithm.

We assume a hybrid routing protocol which uses proactive intra-cluster routing and reactive inter-cluster routing. In our analysis, only control overhead incurred by the clustering algorithm and proactive intra-cluster routing protocol are taken into account.

3.2. Mobility Model

Different mobility models have been developed and adopted in the empirical analysis of MANETs [9]. The Random Waypoint (RWP) and Random Walk (RW) are two popular mobility models commonly used in MANET simulations. However, RWP and RW are known to be unfavorable for theoretical analysis due to the difficulty in capturing their mobility behavior as well as the uneven node distribution they generate. In this analysis, we adopt the Constant Velocity (CV) model [8]. It preserves the uniform node spatial distribution and link change rate in the network is mathematically tractable. The CV model requires an infinite number of nodes to be randomly distributed on a boundless plane. However, in our analysis, we propose a variant of CV, called Bounded Constant Velocity (BCV) model, by assuming a bounded area with a network size N:

1) Initially, at time $t = 0$, nodes are uniformly distributed on an infinitely large area with density ρ. All the nodes randomly choose directions from a uniform distribution.
2) At time $t > 0$, each node starts moving with a constant velocity v in the direction chosen.
3) A bounded square region S is selected in the boundless plane with the average number of nodes within S being N at any point of time

It is direct from the description of the mobility model that BCV produces uniform node spatial distribution.

3.3. Symbols and Notations

In this subsection, we briefly discuss the symbols and notations used in the subsequent analysis. The average cluster size is m (the average number of nodes in a cluster, including the *cluster-head*) which is given by $m = N/n$ where n is the number of clusters in the network. Therefore, the probability that a randomly selected node being a *cluster-head* is given by $\mathrm{P_{HEAD}} = n/N$. The border length of the square area, a is given by $a = \sqrt{|S|}$ and $r < a$. For each node, the link generation rate (non-neighboring nodes become neighbors) and link break rate (neighboring nodes move away from transmission range of each other) in the network are denoted by λ_{gen} and λ_{brk} respectively, with the total link change rate given by $\lambda = \lambda_{gen} + \lambda_{brk}$. The size of the *HELLO* message, *CLUSTER* message and one routing table entry (*ROUTE* message) are p_{hello}, $p_{cluster}$ and p_{route} respectively; the corresponding broadcast rates are denoted by f_{hello}, $f_{cluster}$ and $f_{routing}$ respectively. An arbitrary node in the region S has d network neighbors (neighbors outside S are not considered.) Lastly, the control overheard (bits/sec) from *HELLO*, *CLUSTER* and *ROUTE* messages are O_{hello}, $O_{cluster}$ and $O_{routing}$ respectively.

3.4. Properties of Network Model

The following two claims are made regarding the several important properties of the network model adopted in our analysis**:**

Claim 1: The expected number of network neighbors, d, of a randomly selected node in S is given by:

$$d = (N-1)\frac{r^2\rho}{N}\left(\frac{r^2}{2}\frac{\rho}{N} - \frac{8r}{3}\sqrt{\frac{\rho}{N}} + \pi\right) \tag{1}$$

Proof: It has been shown in [10] that the cumulative distribution function for the link distance between two nodes that are randomly placed in a square area with border length D is:

$$\mathrm{F}_d(\gamma = \xi D) = \xi^2\left(\frac{1}{2}\xi^2 - \frac{8}{3}\xi + \pi\right), 0 \le \xi < 1 \tag{2}$$

$\mathrm{F}_d(\xi D)$ gives the probability that two randomly selected nodes with transmission range ξD in the square of border length D are connected. Thus, in our network model where $r < a$, within an area S the expected number of neighbors of a randomly selected node is $(N\text{-}1)\mathrm{F}_d(r)$. Substituting $D = a = \sqrt{N/\rho}$, we obtain Eqn (1).

Claim 2: The link change rate at each node with other nodes in the plane is given by:

$$\lambda = \frac{16dv}{\pi^2 r} \tag{3}$$

Proof: In [8], the authors derived the link change rate for the CV model. Both the link generation and break rates at each node are $\frac{8}{\pi}\rho rv$. Hence, the total link change rate is $\frac{16}{\pi}\rho rv$. The link change rate of CV model is different from link change rate of BCV model because the link change with nodes outside the square region S is not accounted for in BCV. In BCV, the total number of connected neighbors of a node is $\pi r^2\rho$, but only d of them are within the rectangle region. We assume each established link is equally likely to break no matter whether it is inside or outside S. Thus, among the total link changes at a node in unit time, $\frac{d}{\pi r^2\rho}$ of link changes happen inside S. Thus, the link change rate for a node in S with other nodes in S is $\frac{16dv}{\pi^2 r}$. The link break rate and link generation rate is half of the link change rate.

3.5. Control Overhead Analysis

Our analysis provides the lower bound for the overheads by assuming that each cluster and route change can be detected. In the following, we estimate the rate of each type of control messages and its overhead.

3.5.1. "HELLO" Overhead. The frequency of *HELLO* message at each node should be equal at least to the rate of its neighbor changes. For a randomly selected node n_0, any link generation between it and another node n_i should be noticed by both nodes and add each other to their neighbor-lists. While any link break between n_0 and its neighboring node n_j should also be noticed by the two entities so that they remove each other from their neighbor-lists. The link generation between two neighbors can be notified by both sending *HELLO* messages, and each of the nodes can hear the *HELLO* message send by the other node. While link break between two nodes cannot be notified via sending *HELLO* messages because the two nodes cannot hear each other. Usually, the link break event is sensed via a soft timer approach, and our analysis assumes this approach to ensure minimum "HELLO" overhead. When a node cannot hear its neighbor for a pre-configured time, it removes that neighbor from its neighbor-list. Hence, in order to learn a new neighbor immediately when a new link is formed, the rate of *HELLO* message at each node should at least equal the link generation rate. Therefore,

$$f_{hello} = \lambda_{gen} \quad (4)$$

and the control overhead at each node due to *HELLO* messages is $p_{hello} \cdot f_{hello}$. We substitute $\frac{8dv}{\pi^2 r}$ for λ_{gen}, and replace d with Eqn (1) in *claim 1*, to obtain the control overhead of hello message at each node O_{hello} with respect to N, r, ρ, p_{hello} and v as

$$O_{hello} = p_{hello}(N-1)\frac{8\rho r v}{\pi^2 N}\left(\frac{r^2}{2}\frac{\rho}{N} - \frac{8r}{3}\sqrt{\frac{\rho}{N}} + \pi\right) \quad (5)$$

3.5.2. Clustering Overhead. *CLUSTER* messages are sent at relevant nodes when the two properties *P1* and *P2* in section 2 are violated. *CLUSTER* messages need to be sent at relevant nodes in order to re-adjust cluster structure and re-satisfy the two properties. Besides of *cluster-head* changes, changes of membership from one cluster to another also require sending of *CLUSTER* messages. The above cluster changes can be categorized into two link change events: 1) link break between *cluster-members* and their respective *cluster-heads*, and 2) link generation between two *cluster-heads*. All other link change events do not change the clusters and thus no *CLUSTER* message is sent. Now, we look at the two types of events respectively:

1) Link break between *cluster-members* and their *cluster-heads.* This event causes a node to change its cluster, or become a *cluster-head* when it has no neighboring *cluster-heads*. The *CLUSTER* messages due to this type of link change are sent by *cluster-members*. The ratio of such link breaks to total link breaks should be equal to the ratio of links between *cluster-members* and *cluster-heads* divided by the total number of links in the entire network. The total number of links involving *cluster-heads* (with *cluster-head* at one end of the link) should be equal to the total number of *cluster-members*, that is, $N(1\text{-}P_{HEAD})$, since each *cluster-member* forms a link with its respective *cluster-head*. In a graph, the number of edges in the graph equals half of the sum of degrees of each node. Thus, the total number of links for the entire network is half the sum of network neighbors of all nodes within *S*, which is $(Nd)/2$. Therefore, the rate of *CLUSTER* message at each *cluster-member* due to link break with *cluster-heads* should be equal to:

$$\frac{N(1-P_{HEAD})}{Nd/2}\lambda_{brk} = \frac{16v(1-P_{HEAD})}{\pi^2 r} \quad (6)$$

The number of *CLUSTER* messages sent by *cluster-members* in the network in unit time due to link break with *cluster-heads* is:

$$N(1-P_{HEAD})\frac{16v(1-P_{HEAD})}{\pi^2 r} = N\frac{16v(1-P_{HEAD})^2}{\pi^2 r} \quad (7)$$

2) Link generation between two *cluster-heads.* When a link is generated between two *cluster-heads*, one of the *cluster-heads* needs to give up its *cluster-head* role, which is to be decided by the clustering algorithm. However, the *CLUSTER* control message overhead incurred is the same. When one *cluster-head* drops its role it needs to send one *CLUSTER* message informing that it is no longer a *cluster-head* and the new cluster it belongs to. The original *cluster-members* of this former *cluster-head* lose their original cluster memberships. Thus, each of the nodes needs to send a *CLUSTER* message informing of their new clusters. We need not consider any chain reaction effects in our analysis as it does not affect our lower bound analysis. Every time a link between two *cluster-heads* appears, the number of *CLUSTER* messages generated is the same as the number of nodes in the cluster that needs to undergo re-clustering.

With the *cluster-heads* randomly distributed in the network, the total number of *cluster-heads* is NP_{HEAD} and, the density of the *cluster-heads* spatial distribution is $P_{HEAD}\rho$. Since each *cluster-head* moves in a randomly selected direction with constant velocity, the link generation rate between two *cluster-heads* follows the analysis in *claim 2*:

$$\frac{8d'v}{\pi^2 r} \quad (8)$$

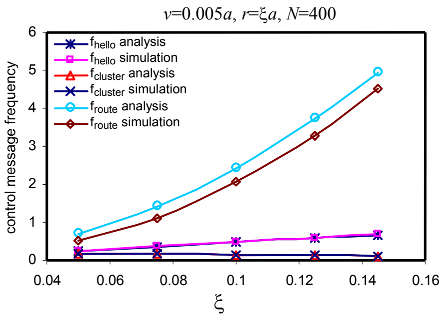


Figure 1: Control message frequencies with *r*

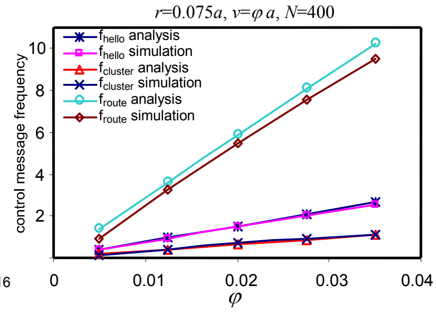


Figure 2: Control message frequencies with *v*

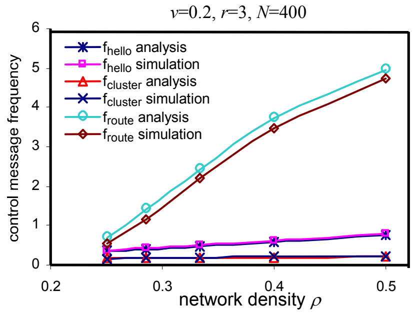


Figure 3: Control message frequencies with *ρ*

where

$$d' = (N\mathrm{P}_{\mathrm{HEAD}} - 1)\frac{r^2\rho}{N}\left(\frac{r^2}{2}\frac{\rho}{N} - \frac{8r}{3}\sqrt{\frac{\rho}{N}} + \pi\right) \qquad (9)$$

Each of such link change causes m *CLUSTER* messages. Therefore, the total number of *CLUSTER* messages sent in the network due to link generation between two *cluster-heads* in unit time is:

$$nm\frac{8d'v}{\pi^2 r} = N\mathrm{P}_{\mathrm{HEAD}}\frac{1}{\mathrm{P}_{\mathrm{HEAD}}}\frac{8d'v}{\pi^2 r} = N\frac{8d'v}{\pi^2 r} \qquad (10)$$

Combining (7) and (10), the rate of *CLUSTER* message sent at each node in unit time is:

$$f_{cluster} = \frac{16v(1-\mathrm{P}_{\mathrm{HEAD}})^2 + 8d'v}{\pi^2 r} \qquad (11)$$

Therefore, the control overhead due to *CLUSTER* messages at each node is:

$$O_{cluster} = p_{cluster}\frac{16v(1-\mathrm{P}_{\mathrm{HEAD}})^2 + 8d'v}{\pi^2 r} \qquad (12)$$

where *d'* equals the r.h.s of the equation in (9) and can be substituted in to show the relationship of clustering overhead with respect to various network parameters.

3.5.3. Routing Overhead. In steady state, a particular node in a cluster should be updated with the routes to other nodes in the cluster and the storage overhead is proportional to the size of the cluster. When a route changes due to link change within a cluster, this information should be propagated through the cluster for every node in the cluster to update their routing tables. Every link change within the cluster will initiate a round of routing information broadcasting to update the routing information at each node.

The frequency of routing information update is:

$$f_{routing} = \frac{32v((1-\mathrm{P}_{\mathrm{HEAD}})^2(-\frac{3\sqrt{3}}{8\pi}+\frac{1}{2}) + (1-\mathrm{P}_{\mathrm{HEAD}})\mathrm{P}_{\mathrm{HEAD}})}{\pi^2 r \mathrm{P}_{\mathrm{HEAD}}{}^2} \qquad (13)$$

and the control overhead due to routing update is:

$$O_{routing} = p_{route} \times \frac{32v((1-\mathrm{P}_{\mathrm{HEAD}})^2(-\frac{3\sqrt{3}}{8\pi}+\frac{1}{2}) + (1-\mathrm{P}_{\mathrm{HEAD}})\mathrm{P}_{\mathrm{HEAD}})}{\pi^2 r \mathrm{P}_{\mathrm{HEAD}}{}^3} \qquad (14)$$

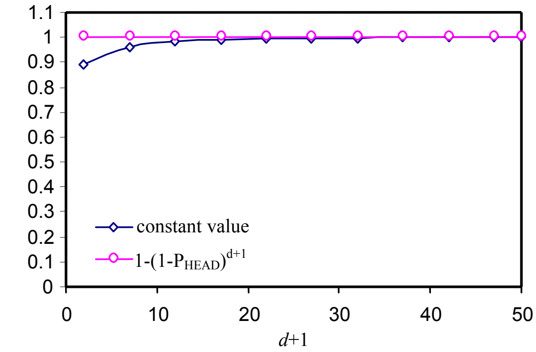
From the above analysis of overhead for the three types of control messages, the total control overhead in our network environment in bits per second is O_{hello} $+O_{cluster}$ + $O_{routing}$. In the following section, we validate these analytical results with simulations.

4. Simulation Studies

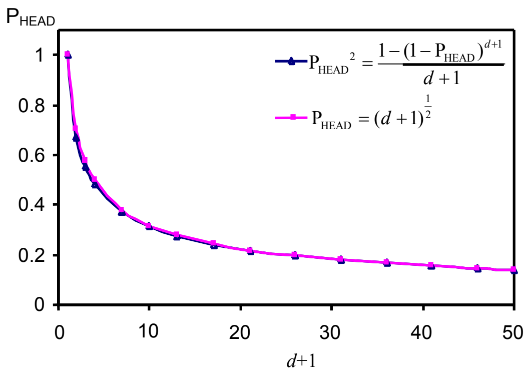
To validate our analysis, we simulated the different network scenarios and measured the frequencies of each category of control messages. Having used CV and BCV to model the random mobility model, we adopt a special case of RWP, which has similar properties as BCV in terms of link change rate and node spatial distribution. The RWP model used in the simulation has the following properties: initially N nodes are randomly uniformly distributed in an a×a square region. Then, following procedures start at time t=0:

1) At an arbitrary time epoch t, each node is static and selects a direction from a uniform distribution.
2) During time interval (t, $t+\tau$), each node moves in the selected direction with the same velocity v. τ is a configurable variable. If a node hits the border of the square region, it reappears at the same position in the opposite border and continues moving without changing its direction.
3) At time $t+\tau$, repeat step 1)

The clustering algorithm used is LID; r, ρ and v are configurable variables of the system, except $\mathrm{P}_{\mathrm{HEAD}}$ which is determined by the LID clustering algorithm. In our simulations, $\mathrm{P}_{\mathrm{HEAD}}$ for LID is measured in real time during the simulation. We compare control message frequencies measured from the simulations with the theoretical analysis. Figure 1 shows the changes of frequencies in the control messages by varying r and keeping other variables fixed. Here the transmission range r is expressed in terms of ratio ξ of border length of the square area, and the control message frequency is expressed in terms of number of messages per unit time. Figure 2 shows how control message frequencies change with node velocity. Similarly, the velocity v of a node is expressed in terms of ratio φ of border length of the square area. In figure 3, the relations between control messages and network density are presented. The density represents number of nodes in a unit area. As shown in all the figures, our analytical results for control message frequencies closely approximate in simulations results. Thus, our analysis is a good approximation of control overhead of the clustering algorithm.



(a) 1-(1-P_{HEAD})$^{d+1}$→1 as d+1 increases.

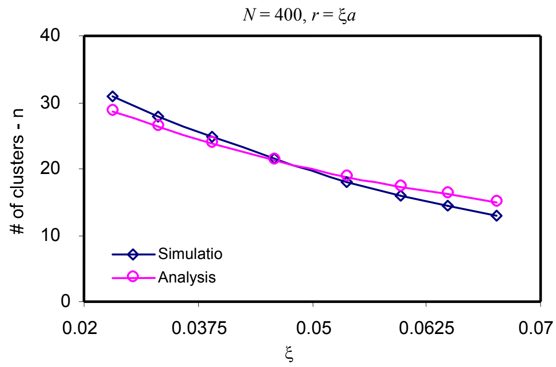


(b) P_{HEAD} as a function of d+1

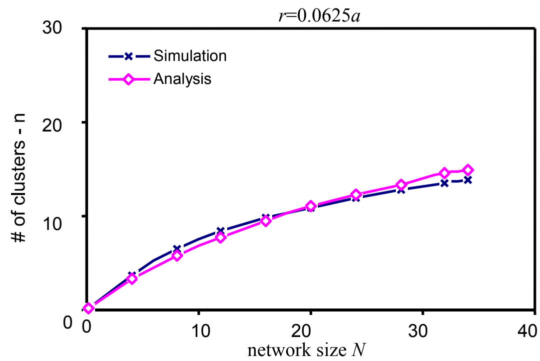
Figure 4: Validation of Eqn (16)

5. Case Study: LID Clustering Algorithm

In the previous sections, we derived and validated the control overhead for a general one-hop clustering algorithms in terms of N, ρ, v, r and P_{HEAD}. While N, ρ , v, r are configurable network parameters, P_{HEAD} depends on one-hop clustering algorithm used; different clustering algorithms form clusters of different sizes, and thus P_{HEAD} varies accordingly. P_{HEAD} for a particular clustering algorithm can either be empirically measured from simulations or derived from theoretical analysis. In this section, we analytically derive P_{HEAD} for a simple but widely used clustering algorithm, i.e. LID Clustering Algorithm.



(a) The # of clusters varies with N



(b) The # of clusters varies with r

Figure 5: Verification of cluster size analysis

LID assumes each node has a unique id and the nodes do not move during cluster formation process. A node is a *cluster-head* if and only if it has the smallest id among nodes in its immediate neighborhood that have not joined any cluster. The clustering algorithm is guaranteed to terminate, and each node is assigned a role of ordinary-node (also known as *cluster-member*) or *cluster-head*.

5.1. P_{HEAD} in LID Clustering

Since at the end of the cluster formation phase each node is assigned a role of either *cluster-member* or *cluster-head*, we could define P_{MEMBER} with respect to P_{HEAD} which is the probability that a randomly selected node being a *cluster-member*. Therefore,

$$P_{MEMBER} + P_{HEAD} = 1 \qquad (15)$$

According to the rules of LID, a node is a *cluster-head* if and only if it has smallest id among the nodes in its closed neighborhood that have not being assigned any roles. In other words, if there are nodes with smaller ids in the surroundings of our randomly selected node, these nodes should become *cluster-members* of other clusters. For analysis, we assume that a randomly selected node n_0 is the i-th smallest node in its closed neighborhood, which means there are i-1 nodes with smaller ids than n_0, where i = 1..d+1. The number of nodes in the closed neighborhood of n_0 is d+1and thus the largest possible value of i is d+1. When i =1 and i-1=0, node n_0 is the smallest node in its closed neighborhood. When node n_0 is the i-th smallest node in its closed neighborhood, the probability that the node n_0 being a *cluster-head* is thus P_{MEMBER}^{i-1} which is the probability that all the i-1 nodes that have smaller ids are members of other clusters. A randomly selected node is the i-th smallest node for any i from 1 to d+1 with probability $1/(d+1)$. Hence, by summing over all possible i and substituting P_{MEMBER} with 1-P_{HEAD}, we get:

$$P_{HEAD} = \frac{1}{d+1}\sum_{i=1}^{d+1}(1-P_{HEAD})^{i-1} \\ \Rightarrow P_{HEAD}^{\ 2} = \frac{1-(1-P_{HEAD})^{d+1}}{d+1} \quad (16)$$

$(1-P_{HEAD})^{d+1}\rightarrow 0$ as d+1 increases. Thus $1-(1-P_{HEAD})^{d+1}\rightarrow 1$ as d+1 increases, as shown in Figure 4(a). By substituting $1-(1-P_{HEAD})^{d+1}$ in Eqn (16) with 1, the expression can be rewritten as:

$$P_{HEAD} = 1/\sqrt{d+1} \quad (17)$$

In Figure 4(b), we plot Eqn (16) and its approximation derived in Eqn (17) to show that the approximation is indeed accurate.

To reflect how P_{HEAD} varies with N, ρ and r, we substitute d with $(N-1)\frac{r^2\rho}{N}(\frac{r^2}{2}\frac{\rho}{N}-\frac{8r}{3}\sqrt{\frac{\rho}{N}}+\pi)$ from *claim 1* to obtain the following equation:

$$P_{HEAD} = \frac{1}{\sqrt{(N-1)\frac{r^2\rho}{N}(\frac{r^2}{2}\frac{\rho}{N}-\frac{8r}{3}\sqrt{\frac{\rho}{N}}+\pi)+1}} \quad (18)$$

5.2. P_{HEAD} Verification

We used GloMoSim [14] to develop simulations to verify our analysis. Figure 5 compares the numerical results from the analysis against the simulation results. Figure 5(a) shows how the number of cluster changes with network size given fixed transmission range and network area while Figure 5(b) shows how the number of cluster changes with transmission range given fixed network size. We observe slight difference between the analysis and simulation plots, and also, they cross each other around the middle. This discrepancy arises due to the simplifying assumptions made in our analysis. For example, in a network with a finite number of nodes, it is not with equal probability that a randomly selected node is the i-th smallest ($1\le i\le d+1$) in a closed neighborhood, unless the number of nodes in the network is infinite. Our analysis becomes more accurate as the network size increases. Nevertheless, the analysis is sufficiently close to the simulation results, and demonstrates that our theoretical P_{HEAD} adequately models how the *cluster-head* density changes with node transmission range and network size.

6. Control Overhead in Knuth Ω-notation

As the results show, the control overhead arising from *HELLO*, and *ROUTE* messages increase with r, v, and ρ. While the volume of *CLUSTER* messages increases with v and ρ. In addition, the control overhead is also influenced by P_{HEAD}, a variable which relates to the percentage of *cluster-heads* in the network. Taking the example of LID clustering in a network with N nodes, we note that P_{HEAD} for LID is a decreasing function of ρ and r given a fixed network size. Intuitively, the more nodes there are in the transmission range of a particular node, the less likely it is to become a *cluster-head*. According to our analytical lower bound for the three types of control overhead, we could derive their lower bounds in Knuth Ω-notation with r, ρ and v as variables. On an infinitely large area, i.e. $a\rightarrow\infty$ and $N\rightarrow\infty$, all the three types of control messages are $\Omega(1)$ with N. The *HELLO* message overhead ultimately increases at the rate of $\Omega(r)$, $\Omega(\rho)$ and $\Omega(v)$ with r, ρ and v respectively, while the *CLUSTER* message overhead at each node is $\Omega(1)$ with r, $\Omega(\rho^{1/2})$ with ρ, and $\Omega(v)$ with v, and the *ROUTE* message at each node increases at $\Omega(r)$ with r, $\Omega(\rho)$ with ρ, and $\Omega(v)$ with v. *ROUTE* message overhead constitutes the main control overhead, because of its relative high broadcasting rate and large message size.

7. Conclusion

While many clustering algorithms have been proposed for mobile ad hoc networks, formal mathematical analysis of the overheads incurred by clustering has been extremely lacking. Much of the analysis is based on the big-O notation and relates to the network size only. This is very inadequate as various other network parameters affect the volume of control overhead generated. In this paper, we have analyzed the clustering overheads for a generic one-hop clustering algorithm taking into consideration node mobility, node transmission range, network size, and network density. By abstracting and representing the clustering algorithm as the probability of an arbitrarily selected node being a *cluster-head*, we also showed how to derive this value for a typical clustering algorithm, the Lowest ID algorithm. Our analysis also makes provision for three key aspects of clustering, viz., the neighbor discovery using periodic broadcast of *HELLO* messages, the clustering management and the routing information management. The work presented here provides a good basis for further analysis on the performance of clustering algorithms for mobile ad hoc networks, in aspects such as scalability and the influence of node mobility patterns.

REFERENCES

[1] P. Gupta and P. R. Kumar, "The Capacity of Wireless Networks", *IEEE Trans on Information Theory*, Vol. 46, No. 2, Mar 2000, pp. 388-404.

[2] D. E. Knuth, "Big Omicron and big Omega and big Theta", *ACM SIGACT News*, Vol. 8, No. 2, Apr-Jun 1976, pp. 18-24.

[3] J. Sucec and I. Marsic, "Clustering Overhead for Hierarchical Routing in Mobile Ad Hoc Networks", in *Proc. of INFOCOM,* New York, NY, June 2002, pp. 1698–1706.

[4] J. Sucec and I. Marsic, "Hierarchical Routing Overhead in Mobile Ad Hoc Networks", *IEEE Transactions on Mobile Computing,* Vol. 3, No. 1, Jan-Mar 2004, pp. 46–56.

[5] P. Y.-Z. Chen, A. L. Liestman and J. Liu, "Clustering Algorithms for Ad Hoc Wireless Networks", *Ad Hoc and Sensor Networks*, edited by Y. Xiao and Y. Pan, Nova Science Publisher, 2004.

[6] C. Perkins and P. Bhagwat. "Highly Dynamic Destination Sequenced Distance-Vector Routing (DSDV) for Mobile Computers", in *Proc. of ACM SIGCOMM*, October 1994.

[7] C. E. Perkins, and E. M. Royer, "Ad-Hoc On-Demand Distance Vector Routing", in *Proc. of IEEE WMCSA 1999*, New Orleans, LA, Feb. 1999.

[8] S. Cho and J.P. Hayes, "Impact of mobility on connection stability in ad hoc networks", in *Proc. of Wireless Communication and Networking Conference (WCNC),* New Orleans, LA, USA, March 2005.

[9] T. Camp, J. Boleng, and V. Davies, "Survey of mobility models for ad hoc network research", *Wireless Communication and Mobile Computing (WCMC)*, vol. 2, 2002, pp. 483-502.

[10] Lenard E. Miller, "Distribution of Link Distances in a Wireless Network", *Journal of Research of the National Institute of Standards and Technology*, Vol. 106, No.2, pp. 401-412, March-April 2001.

[11] C.-C. Chiang, "Routing in Clustered Multihop, Mobile Wireless Networks with Fading Channel", in *Proc. of IEEE SICON*, Apr 1997, pp. 197-211.

[12] M. Gerla and J.T.-C.Tsai, "Multicluster, mobile, multimedia radio network", *ACM/Baltzer Journal of Wireless Networks*, Vol. 1, No. 3, 1995, pp. 244-265.

[13] C.R. Lin and M. Gerla, "Adaptive Clustering for Mobile Wireless Networks", *IEEE Journal on Selected Areas in Communications*, Vol. 15, No. 7, Sep 1997, pp. 1265-1275.

[14] X. Zeng, R. Bagrodia and M. Gerla, "GloMoSim: a Library for Parallel Simulation of Large-scale Wireless Networks", in *Proc. of 12th Workshop on Parallel and Distributed Simulations* (PADS '98), Banff, Alberta, Canada, 1998. Available from: http://pcl.cs.ucla.edu/projects/glomosim/.

[15] D. J. Baker and A. Ephremides, "The architectural organization of a mobile radio network via a distributed algorithm", *IEEE Trans. on Communications.*, Vol. 29, No. 11, Nov 1981, pp. 1694-1701.

[16] I. I. Er and Winston K. G. Seah, 'Clustering Overhead and Convergence Time Analysis of the Mobility-based Multi-Hop Clustering Algorithm for Mobile Ad Hoc Networks', *Proceedings of the 1st International Workshop on Performance Modeling in Wired, Wireless, Mobile Networking and Computing* (PMW2MNC05), in conjunction with 11th International Conference on Parallel and Distributed Systems (ICPADS-2005), Jul 20-22, 2005.

[17] S. Basagni, "Distributed Clustering for Ad Hoc Networks", in *Proc. of the International Symposium on Parallel Architectures, Algorithms and Networks* (ISPAN'99), Jun 23–25, 1999, Washington, DC, USA.

[18] I. I. Er, and Winston K. G. Seah, "Mobility-based d-Hop Clustering Algorithm for Mobile Ad Hoc Networks", in *Proceedings of IEEE Wireless Communications and Networking Conference*, Mar 21-25, 2004, Atlanta, Georgia, USA.

[19] A. D. Amis, R. Prakash, T. H. P. Vuong, and D. T. Huynh, "Max-Min D-Cluster Formation in Wireless Ad Hoc Networks", in *Proc. of IEEE INFOCOM*, Mar 1999.