Victoria University of Wellington

Te Whare Wānanga o te Ūpoko o te Ika a Maui

Wormholes and non-trivial topology

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Why is topology even an issue?

Observation:

- The Einstein equations are local:

$$G^{\mu\nu} = 8\pi\, G_{\text{Newton}}\, T^{\mu\nu}.$$

- Even at the semi-classical level they are "quasi-local":

$$G^{\mu\nu} = 8\pi\, G_{\text{Newton}}\, \langle\psi|T^{\mu\nu}|\psi\rangle.$$

- The Einstein equations do not, by themselves, constrain *any* global features —
  - topology;
    (spatial or temporal)
  - orientability;
    (spatial, temporal, or spacetime)

Consequence:

- Unless you enforce topological and/or orientability constraints "by hand", general relativity (Einstein gravity) *seems* to be absolutely infested with peculiar topological objects.

- We have no direct observational/ experimental evidence for the existence of such objects; but GR — which in many other ways is spectacularly successful — does not (by itself) preclude them.

- The observational dearth of such topologically nontrivial objects is therefore somewhat puzzling...

- Ultimately, the key issue is (in GR language) that of "prior structure";
perhaps better called "ab initio structure".

Non-traversable wormhole:

- Old-style wormholes (the Schwarzschild wormhole, *aka* the Einstein–Rosen bridge, and its cousins) are non-traversable — any attempt at crossing from one asymptotically flat region to the other leads you into the singularity. You will die.

- Even if your personal death is not a concern, the presence of an event horizon means you are not getting any messages back to the folks at home. As far as they are concerned you might as well not have sacrificed yourself.

- (Unless, of course, you have FTL drive or FTL communications; but that opens up another can of worms — see chronology protection.)

Wheeler wormholes:

- John Wheeler introduced (among many other ideas) the notion of "charge without charge"; the idea that electrons and positrons do *not* carry point charges, but that the electric flux lines go into a wormhole throat on the electron, through "hyperspace" via a wormhole, and then emerge from its positron partner.

- Net effect: objects that look similar to point charges, but with flux lines that nowhere terminate. (And no event horizons.)

- If you think of these as classical solutions of Einstein plus Maxwell, we now know all such solutions to be unstable. (Sphaleron)

- Quantum Wheeler wormholes $\Rightarrow$ "spacetime foam".

Warning on spacetime foam:

- Wheeler's argument in favour of "spacetime foam" has its limitations.

- If you linearize gravity around Minkowski space, and quantize the linearized theory, then in that model you can certainly calculate the metric-metric 2-point function:

$$\langle 0|\Delta g(x)\,\Delta g(y)|0\rangle = k\,\frac{\ell_P^2}{(x-y)^2}$$

- This certainly implies that metric fluctuations are of order unity for distances of order the Planck length.

- This guarantees that quantized gravity will be strongly interacting at the Planck scale.

- (But what about the light cone?)

Warning on spacetime foam:

- Despite "common wisdom", strongly interacting does *not* guarantee that topology is fluctuating at the Planck scale.

- There is in fact no *guarantee* that topology fluctuates at the Planck scale.

- There is a lot of *speculation* that topology fluctuates at the Planck scale.

- There is certainly no empirical or theoretical *need* for topology fluctuation at the Planck scale.

- There is a big difference between strongly interacting (crumpled) and the creation/ destruction of links between distinct points.

Traversable wormhole:

- Mike Morris and Kip Thorne asked:

  "What sort of matter distribution would be needed to generate a traversable wormhole, defined to be a classical geometry with two asymptotically flat regions connected by a throat, and with no event horizon between them?"

- You can ask this question in classical GR or semiclassical GR; it is not clear how to formulate the question once the geometry is fluctuating significantly.

- Answer: You need violations of the Averaged Null Energy Condition (ANEC);
  and therefore violations of all the standard GR energy conditions.
  (At this stage classical physicists tend to choke and have difficulty in breathing.)

Energy conditions:

- Violating the ANEC is not as big a deal as you might naively think.

- Classically, ANEC violations are relatively difficult to achieve — though non-minimally coupled scalar fields can cause havoc in this regard.

- Quantum mechanically, two significant examples of semiclassical ANEC violation are known:

  - Casimir effect — observationally verified (with caveats).

  - Hawking radiation — ANEC violations needed to get around the classical area increase theorem.

Hawking radiation:

- The ANEC violation in Hawking radiation is particularly important.

- It demonstrates that while quantum violations of ANEC are typically small (order $\hbar$) there are situations in which their effect can be secular; leading to massive changes in the classical picture.

- Indeed in Hawking radiation, a tiny semi-classical quantum effect completely reverses the inference you would have drawn from the classical area increase theorem.

- Semiclassical quantum effects, though small, can have enormous implications.

Topological censorship:

- The best general theorem we currently have is the "topological censorship theorem".

- (Roughly) If you have an asymptotically flat globally hyperbolic spacetime, with multiple disjoint past or future null infinities, then there is at least one inextendible null geodesic from past null infinity to future null infinity along which ANEC is violated.

- The phrase "disjoint past or future null infinities" implies there are null curves from past null infinity to future null infinity that are not homotopic to the trivial curve.

- The theorem relates the occurrence of "visible" nontrivial topology (those that can be probed by "optical means") to the occurrence of ANEC violations.

SSS wormholes:

- Instead of mucking around with global analysis on Lorentzian manifolds, you can get the essence of the result by looking at spherically symmetric static (SSS) traversable wormholes — as did Morris and Thorne.

- Write down some model geometry, note that if a traversable wormhole throat exists then a pencil of radially moving null rays will be defocussed by the wormhole throat.

- Apply the focussing theorem, in reverse, to deduce ANEC violations.

- Alternatively; simply calculate the Einstein tensor, and integrate it along a radial null geodesic through the wormhole throat.

SSS wormholes: Calculation

- Without loss of generality

$$\mathrm{d}s^2 = -\exp[2\phi(\ell)]\mathrm{d}t^2 + \mathrm{d}\ell^2 + r(\ell)^2 \left[\mathrm{d}\theta^2 + \sin^2\theta \, \mathrm{d}\varphi^2\right]$$

Here ℓ is a proper radial coordinate.

- Einstein components in orthonormal frame:

$$G_{\hat{t}\hat{t}} = -\frac{2r'' - 1 + (r')^2}{r^2}$$

$$G_{\hat{r}\hat{r}} = \frac{-1 + (r')^2 + 2r\phi' r'}{r^2}$$

$$G_{\hat{\theta}\hat{\theta}} = \frac{r'' + \phi' r' + r\phi'' + r(\phi')^2}{r}$$

SSS wormholes: Calculation

- Look along the radial null direction:

$$G_{\widehat{t}\widehat{t}} + G_{\widehat{r}\widehat{r}} = -2\frac{r'' - \phi' r'}{r}$$

- Integrate by parts along a radial null geodesic:

$$\oint G_{ab}\, k^a k^b \,\mathrm{d}\lambda = -\oint \exp[-\phi(\ell)]\frac{(r')^2}{r^2}\mathrm{d}\ell < 0.$$

  Here λ is the null affine parameter.

- Contributions from asymptotic limits vanish by assumed asymptotic flatness.

SSS wormholes: Calculation

- Reminder:

$$\oint G_{ab}\, k^a k^b \, \mathrm{d}\lambda = - \oint \exp[-\phi(\ell)] \frac{(r')^2}{r^2} \mathrm{d}\ell < 0.$$

- This is at this stage a purely geometrical statement; no information (yet) about energy conditions.

- Ditto for the full topological censorship theorem: You construct a purely geometrical statement that

$$\oint G_{ab}\, k^a k^b \, \mathrm{d}\lambda < 0$$

along at least one inextendible null geodesic.

Energy condition violations:

- It is only after you apply the Einstein equations in the form

$$G^{\mu\nu} = 8\pi \, G_{\text{Newton}} \, T^{\mu\nu}_{\text{effective}}$$

that you get an ANEC violating theorem:

$$\oint T^{\text{effective}}_{ab} \, k^a k^b \, \mathrm{d}\lambda < 0$$

- Warning: The ANEC violations are for the $T^{\text{effective}}_{ab}$ defined in exactly this way; not any other definition of stress energy.

- You can cause endless confusion (and, unfortunately, published papers) by playing linguistic games and “slicing and dicing” $T^{\text{effective}}_{ab}$ in peculiar ways.

- Warning: Similar confusion arises in time-dependent wormholes if you do not follow the null geodesic all the way through.

Local definition of a wormhole:

- If you want (as above) a local definition of wormhole that side-steps issues of null infinity and homotopic inequivalence, then the only known characterization is in terms of the wormhole throat and its extremality properties.

- Extremality properties are geometric;
  in particular they are metric,
  and depend on conformal frame.

- While a (nonsingular) conformal transformation does not change the null geodesics, and does not change the topology, it can (and typically will) change the location (and number) of wormhole throats.

Local definition of a wormhole:

- Similarly, (nonsingular) conformal transformations (with suitable fall-off at infinity) can affect the value (but not the sign) of the ANEC integral.

- Warning: If you use a local definition of wormhole in terms of the extremality properties of the throat; then make sure that when you define the wormhole you also calculate the effective stress-energy in the same conformal frame.

- Warning: Singular conformal transformations will lead to no end of confusion. (Especially if the conformal transformation goes singular where the throat used to be in the original manifold...)

Euclidean wormholes:

- There are related but distinct results for Euclidean wormholes.
  Try to avoid confusing the two.

- The null and dominant energy conditions (NEC and DEC) make no sense in Euclidean signature.

- You can define Euclidean versions of the weak and strong energy conditions (WEC and SEC), but they are now mutually exclusive. (This is very different from the situation in Lorentzian signature.)

- Euclidean WEC seems the most sensible...

- Warning — Do not try jumping from Euclidean to Lorentzian signature in the middle of the calculation.

Prior structure:

- GR by itself does not seem to place particularly strong constraints on topology.

- Nevertheless we have not directly observed any nontrivial topology.

- Maybe, to get an accurate representation of empirical reality, we should be using Einstein equations *plus some extra conditions*?

  - Prior geometry?

  - Prior topology?

  - Prior time-ordering?

  - Prior whatever?

  (At this stage general relativists tend to choke and have difficulty in breathing.)

Prior structure:

- To the general relativity community, "prior structure" is an anathema.

- Remember that GR cones in two segments:
  - Manifold picture/ Einstein Equivalence principle/ UFF.
  - Einstein field equations.

- UFF (and by implication EEP and the usefulness of the manifold picture) are tested to one part in 10^{13} via Eötvös-type experiments.

- Specific tests of the Einstein field equations are much less stringent.

Aside — curvature squared:

- As a particular example, consider:

$$S = \int \sqrt{-g} \left\{ \kappa R + \lambda R^2 \right\} \mathrm{d}^4 x$$

  Here λ is a dimensionless number.

- The direct experimental bounds are pitiful

$$|\lambda| < 10^{64}$$

- The bounds are so poor because:

  – Schwarzschild is still an exact solution of the field equations for arbitrary λ.

  – In linearized gravity there is now a "massive component" to the graviton, but with mass

$$m = \frac{m_{Planck}}{\sqrt{\lambda}}$$

Back to prior structure:

- Relativists have looked for "prior structure" both observationally and theoretically.
  - PPN formalism — no sign of prior structure.
  - All known attempts at "prior structure" are rather ugly.
  - Historically, Einstein made a big deal of his "principle of general covariance".
  - These days, relativists view "general covariance" as close to tautological.
  - What Einstein meant by his "principle of general covariance" is in modern language simply "absence of prior structure".

Absence of prior structure?

- "Absence of prior structure" is simply the assertion that the spacetime geometry is determined dynamically by the field equations — Einstein (plus distortions) — without any "ab initio" restriction on the geometry or topology.

- But as soon as you adopt this viewpoint, you also open the door to nontrivial topology (and worse: chronology violations).

- Maybe this is too high a price for maintaining the purity of one's "geometric roots"?

- Is there a middle ground?

Acceptability of prior structure?

Question: Is there an "a priori" restriction on geometry that is sufficiently weak to avoid severely constraining the field equations, but sufficiently strong to:

- Protect chronology.
- (Possibly)
  Suppress topology change.
- (Possibly)
  Supress all strange topologies?

Answer: We really don't know because we've been to busy looking under other rocks.

- This is very definitely a minority viewpoint.

Acceptability of prior structure?

Example: The "acoustic metrics" are a little too strong a constraint

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + \delta_{ij}\,(\mathrm{d}x^i - v^i\,\mathrm{d}t)\,(\mathrm{d}x^j - v^j\,\mathrm{d}t)$$

- Topology and causal structure are automatically trivial.
- Any spherically symmetric geometry (not necessarily static) can be put in this form (at least locally) — eg Schwarzschild in Painleve–Gullstrand form.
- Kerr cannot be put in this form.
- Maximal analytic extensions are a non-trivial issue.
- Inspired by acoustics in a flowing fluid.

Acceptability of prior structure?

Example: The "globally ADM metrics" are quite suitable:

$$ds^2 = -c^2 dt^2 + g_{ij}\,(dx^i - v^i\,dt)\,(dx^j - v^j\,dt)$$

- We want at least one global (or close to global) slicing where the metric takes this form with $c^2 > 0$ everywhere and $\det\{g_{ij}\} > 0$ everywhere.
- Causal structure is then automatically trivial. (Stable causality.)
- Topology (spatial) can still be nontrivial.
- Schwarzschild and Kerr can (with caveats) be put in this form.
- Maximal analytic extensions are still a nontrivial issue.

Conclusions:

- Without some ab initio prior structutre, without some constraint on spatial and temporal topologies, the Einstein are equations (by themselves) incapable of preventing serious weirdness in the physics.

- To protect chronology, and supress wierd and wonderful topologies, you need to either:

  - Blame it all on quantum gravity, and pray that when we finally have a decent theory of quantum gravity everything will become pelluidly clear.

  - Put some constraints in at the front end; before we try to quantize anything.

    For example: Lorentzian lattice quantum gravity.

Conclusions:

- Be aware that "Lorentzian lattice quantum gravity" is an anathema to relativists.

  (Prior structure.)

- Be aware that "Lorentzian lattice quantum gravity" is an anathema to string theorists.

  (For very different reasons; string theorists view the absence of supersymmetry with great fear and loathing.)

- Personal view:
  Prior structure is well worth pursuing.

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