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Analogue spacetimes: Toy models for "quantum gravity".

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From quantum to emergent gravity:
Theory and phenomenology
SISSA/ISAS
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Abstract:

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Why are "analogue spacetimes" interesting?

For the purposes of this workshop the answer is simple:

Analogue spacetimes provide one with solid physically well-defined and well-understood concrete models of many of the phenomena that seem to be part of the yet incomplete theory of "quantum gravity", or more accessibly, "quantum gravity phenomenology".

Abstract:

For example "analogue spacetimes" provide concrete models of "emergence" (the effective low-energy theory can be radically different from the high-energy microphysics).

Provide controlled models of "Lorentz symmetry breaking", extensions of the usual notions of Lorentzian geometry: "rainbow spacetimes", pseudo-Finsler geometries, and more...

I will provide an overview of the key items of "unusual physics" that arise in analogue spacetimes, and argue that they provide us with hints of what we should be looking for in any putative theory of "quantum gravity".

The usual suspects:

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Silke Weinfurtner:	Victoria University of Wellington New Zealand
Stefano Liberati:	SISSA / ISAS, Trieste, Italy
Carlos Barcelo:	Instituto Astrofisica de Andalusia Granada, Spain
Angela White:	ANU, Canberra
Piyush Jain:	VUW, NZ
Crispin Gardiner:	Otago University, NZ

Emergence:

The word "emergence" is being tossed around an awful lot lately.....

But what does it really mean?

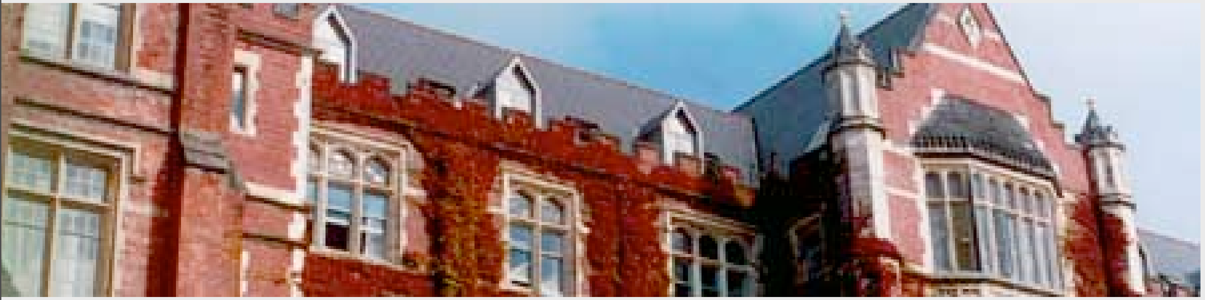
--- "More is different"?

--- The sum is greater than its parts?

--- Universality?

--- Mean field?

Short distance physics is often radically different from long distance physics...



Emergence:

Prime example: Fluid dynamics

Long distance physics: Euler equation (generic)

Continuity equation (generic)

Equation of state (specific)

Short distance physics: Quantum molecular dynamics

Note: You cannot hope to derive quantum molecular dynamics by quantizing fluid dynamics...

Emergence:

Could Einstein gravity be "emergent"?

1) Can we get an "analogue spacetime"? (generic)

2) Can we get Einstein's equations? (specific)

IF Einstein gravity is "emergent",
THEN it makes absolutely no sense
to "quantize gravity"...

The best one could then hope for is some uber-theory that approximately reduces to Einstein gravity in some limit.

Emergence:

The uber-theory would not necessarily be quantum...

It must have as approximate limits: ['t Hooft]

--- Classical Einstein gravity...

--- Quantum field theory (Minkowski)...

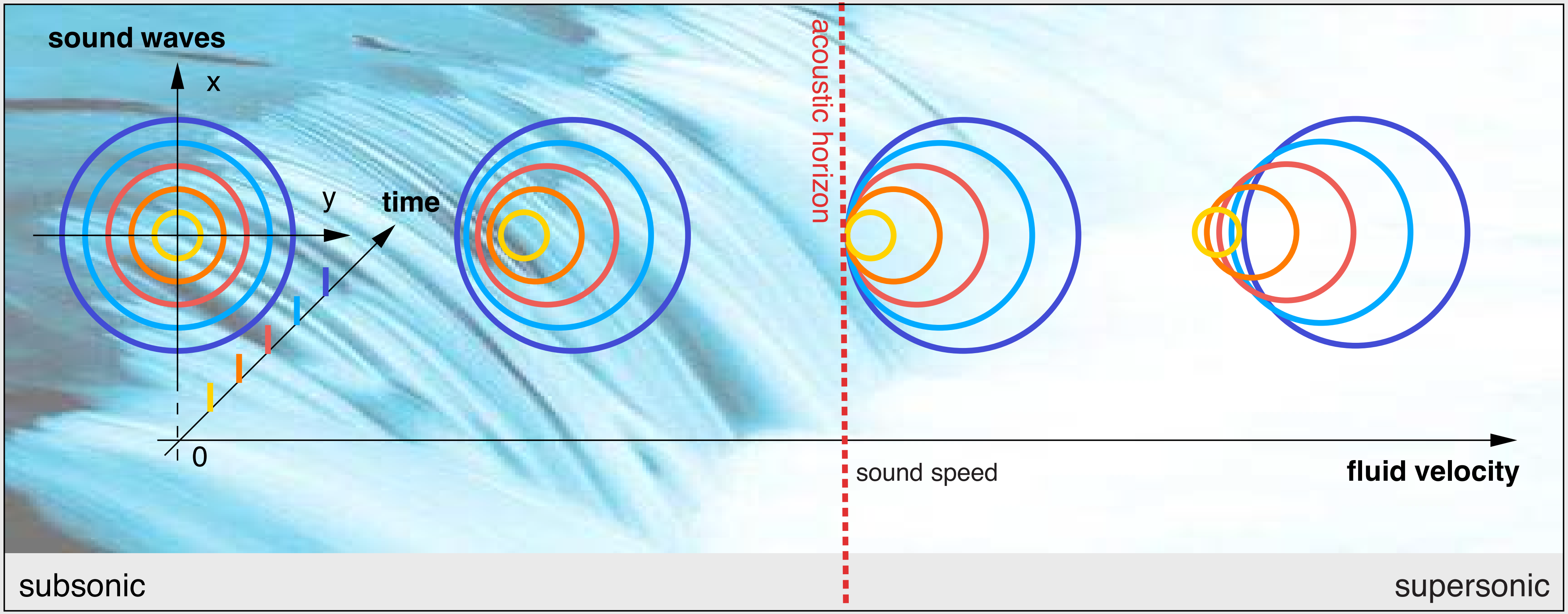
--- Curved space QFT...

--- Semiclassical quantum gravity...

Analogue spacetimes are (among other things) baby steps in this direction...

Acoustic spacetime:

The simplest "analogue spacetimes" are the "acoustic spacetimes"...



Consider sound waves in a moving fluid... [Unruh]

Acoustic spacetime:

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Theorem: Consider an irrotational, inviscid, barotropic perfect fluid, governed by the Euler equation, continuity equation, and an equation of state.

The dynamics of the linearized perturbations (sound, phonons) is governed by a D'Alembertian equation

$$\Delta_g \Phi = \frac{1}{\sqrt{g}} \partial_a \left(\sqrt{g}\; g^{ab} \partial_b\, \Phi \right) = 0$$

involving an "acoustic metric".

[Algebraic function of the background fields.]

Acoustic spacetime:

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Theorem: (3+1 dimensions)

$$g^{\mu\nu}(t,\vec{x}) \equiv \frac{1}{\rho_0 c} \left[\begin{array}{ccc} -1 & \vdots & -v_0^j \\ \cdots\cdots & \cdot & \cdots\cdots\cdots\cdots \\ -v_0^i & \vdots & (c^2\,\delta^{ij} - v_0^i\, v_0^j) \end{array} \right].$$

$$g_{\mu\nu}(t,\vec{x}) \equiv \frac{\rho_0}{c} \left[\begin{array}{ccc} -(c^2 - v_0^2) & \vdots & -v_0^j \\ \cdots\cdots\cdots\cdots & \cdot & \cdots\cdots \\ -v_0^i & \vdots & \delta_{ij} \end{array} \right].$$

$$\mathrm{d}s^2 \equiv g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = \frac{\rho_0}{c}\left[-c^2\,\mathrm{d}t^2 + (\mathrm{d}x^i - v_0^i\,\mathrm{d}t)\,\delta_{ij}\,(\mathrm{d}x^j - v_0^j\,\mathrm{d}t)\right].$$

Acoustic spacetime:

There is by now a quite sizable literature on acoustic, and other more general analogue spacetimes

Unruh: Experimental black hole evaporation,
Phys Rev Lett 46 (1981) 1351-1353.

Barcelo, Liberati, Visser: Analogue gravity,
Living Reviews in Relativity, 8:12, 2005.

Main message: Finding an effective low-energy metric is not all that difficult....

Acoustic spacetime:

Examples of exotic physics:

Controlled signature change [White, Weinfurtner]

Bose-nova [Hu, Calzetta]

c^2 propto (scattering length)

Can be controlled by using a Feschbach resonance.

Rainbow spacetime:

There is no general widely accepted precise mathematical definition of what is meant by a "rainbow geometry"...

The physicist's definition is rather imprecise:
"energy dependent metric"?
"momentum dependent metric"?
"4-momentum dependent metric"?

Q: 4-momentum of what? The observer? The object being observed?

Rainbow spacetime:

To capture the essence of "energy dependence" need a metric that depends also on the magnitude of the tangent vector....

Consider a fluid at rest, in very many cases the dispersion relation can be written in the form:

$$\omega^2 = F(k)$$

for some possibly nonlinear function F(k)...

(2nd-order in time; arbitrary order in space...)

[Unruh, Jacobson]

Rainbow spacetime:

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Phase velocity: $$c_k^2 = \frac{\omega^2}{k^2} = \frac{F(k)}{k^2}$$

Dispersion relation: $$\omega^2 = c_k^2 \; k^2$$

Fluid in motion: Doppler shift the frequency...

$$\omega \longrightarrow \omega - \vec{v} \cdot \vec{k}$$

$$\left(\omega - \vec{v} \cdot \vec{k}\right)^2 - c_k^2 \; k^2 = 0$$

Rainbow spacetime:

Rewrite as: $$g_k^{ab}\; k_a\, k_b = 0.$$

Pick off components:

$$g_k^{ab} \propto \left[\begin{array}{c|c} -\,1 & -v^j \\ \hline -\,v^i & c_k^2\;\delta^{ij} - v^i v^j \end{array}\right].$$

$$g^k_{ab} \propto \left[\begin{array}{c|c} -(c_k^2 - v^2) & -v^j \\ \hline -v^i & \delta^{ij} \end{array}\right].$$

Momentum dependent metric depending on phase velocity.

Rainbow spacetime:

Dispersion relation approach is physically transparent...

Only weakness: Conformal factor left unspecified...

(This is a standard side-effect of the geometrical quasi-particle approximation,
cf geometrical acoustics,
cf geometrical optics.)

[PDE is better]
[Weinfurtner]

The momentum in question is now the momentum of an individual "mode" of the field ---
hence phase velocity + dispersion relation.

Rainbow spacetime:

Similar (but distinct) steps can be taken to develop a rainbow metric based on group velocity.

Consider a wave packet centered on momentum k.

That packet will propagate with the group velocity.

$$(\mathrm{d}\vec{x} - \vec{v}\,\mathrm{d}t)^2 = c_k^2\,\mathrm{d}t^2$$

Group velocity.

Rainbow spacetime:

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Rewrite as: $$\mathrm{d}s^2 = 0 = g_{ab}\, dx^a\, \mathrm{d}x^b$$

Pick off components:

$$g_k^{ab} \propto \left[\begin{array}{c|c} -1 & -v^j \\ \hline -v^i & c_k^2\, \delta^{ij} - v^i v^j \end{array}\right].$$

$$g^k_{ab} \propto \left[\begin{array}{c|c} -(c_k^2 - v^2) & -v^j \\ \hline -v^i & \delta^{ij} \end{array}\right].$$

Momentum dependent metric depending on group velocity.

Rainbow spacetime:

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There are at least two distinct very different notions of "Rainbow metric" in an analogue setting.

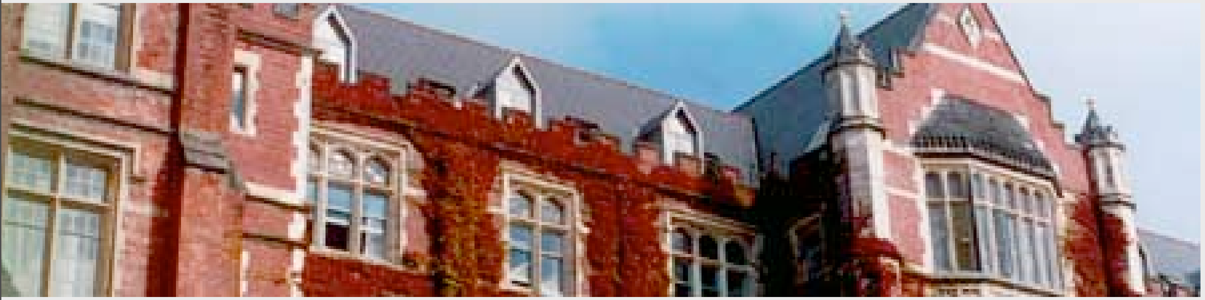
They answer different questions:

* What is the dispersion relation of a pure mode?
* How do wave packets propagate?

If you are lucky there is a "hydrodynamic" limit:

$$\lim_{k\to 0} c^2_{\text{phase}}(k) = c^2_{\text{hydrodynamic}} = \lim_{k\to 0} c^2_{\text{group}}(k)$$

$$\neq 0!$$



Rainbow spacetime:

In general: Rainbow ==> multi-metric

$$g^{ab}_k \propto \left[\begin{array}{c|c} -1 & -v^j \\ \hline -v^i & c_k^2\,\delta^{ij} - v^i v^j \end{array} \right].$$

$$g_{ab}^k \propto \left[\begin{array}{c|c} -(c_k^2 - v^2) & -v^j \\ \hline -v^i & \delta^{ij} \end{array} \right].$$

With: $$c_k \to \begin{cases} c_{\text{phase}} \\ c_{\text{group}} \\ c_{\text{hydrodynamic}} \end{cases}.$$

Signal speed?

c ==> infinity?

Rainbow spacetime:

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Bogoliubov dispersion relation (eg, BECs):

$$\omega^2 = c_0^2\, k^2 + \left(\frac{\hbar}{2m}\right)^2 k^4$$

$$c^2 = c_0^2 + \left(\frac{\hbar}{2m}\right)^2 k^2 \qquad \text{(supersonic)}$$

Controlled breaking of Lorentz invariance...

See "quantum gravity phenomenology"... [Liberati...]

See "cosmological particle production" [Weinfurtner]

Rainbow spacetime:

Surface waves in finite depth of liquid: [Lamb]

$$\omega^2 = g\,k\,\tanh(k\,d) = c_0^2\,k^2\,\frac{\tanh(k\,d)}{k\,d} \qquad c_0^2 = g\,d.$$

$$c^2 = c_0^2\,k^2\,\frac{\tanh(k\,d)}{k\,d}$$

(subsonic)

$$\omega^2 = c_0^2\,k^2\,\left\{1 - \frac{(k\,d)^2}{3} + \frac{2(k\,d)^2}{15} + \dots\right\}$$

So analogue models provide concrete examples for both supersonic an subsonic dispersion, and more...

Rainbow spacetime:

Surface waves in infinite depth of liquid:

$$\omega = \sqrt{g\, k}; \qquad c_{\text{phase}} = \sqrt{g/k}.$$

$$c_{\text{group}} = \frac{\partial \omega}{\partial k} = \frac{\sqrt{g/k}}{2} = \frac{c_{\text{phase}}}{2}.$$

No hydrodynamic limit...

No well-defined low-momentum spacetime...

[You could argue that this is an unphysical limit...]

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Surface waves in finite depth of liquid + surface tension:

$$\omega^2 = c_0^2\, k^2 \, \left\{ 1 + \frac{\sigma}{\rho\, c_0^2\, d} (kd)^2 \right\} \, \frac{\tanh(kd)}{kd}.$$

$$c^2 = c_0^2 \, \left\{ 1 + \frac{\sigma}{\rho\, c_0^2\, d} (kd)^2 \right\} \, \frac{\tanh(kd)}{kd}.$$

$$c_0^2 = g\, d.$$

Asymptotically supersonic, though it can be adjusted to have a subsonic dip.

Water: $\epsilon = \dfrac{\sigma}{\rho c_0^2 d} = \dfrac{\sigma}{\rho g d^2} = \dfrac{(0.27 \text{ cm})^2}{d^2}.$

Rainbow spacetime:

$$c^2 = c_0^2 \; \left\{1 + \epsilon \, (kd)^2\right\} \; \frac{\tanh(kd)}{kd}.$$

$$c^2 = c_0^2 \; \left\{1 + \frac{3\epsilon - 1}{3} \, (kd)^2 - \frac{5\epsilon - 2}{15} \, (kd)^4 + \mathcal{O}[(kd)^6]\right\}.$$

Can tune away the lowest order Lorentz violation...

(Water at 0.47 cm depth)

These are just some examples of the types of dispersion relation you can arrange...

Rainbow spacetime:

Can also arrange for particle masses:

$$\omega^2 = \omega_0^2 + c_0^2\; k^2 + \frac{k^4}{K^2} + \mathcal{O}[(k)^6].$$

[2 interacting BECs: Weinfurtner *et al*...]

Basic message: Lots of physically well behaved and well controlled toy models for many different types of "beyond the standard model" physics...

Finsler spacetime:

1854: $$ds = \sqrt[4]{g_{abcd}\, dx^a dx^b dx^c dx^d}$$

Riemann's inaugural lecture at Goettingen

But Riemann never developed the idea...

Left to Paul Finsler in early 20'th century...

But physicists need pseudo-Finsler spacetime, not Finsler space...

Finsler spacetime:

Physical model: Birefringent crystal [Born+Wolf]

Maxwell ==> $$f^{ab}_{\mathrm{AB}}\; p_a p_b \;\epsilon^B = 0$$

Fresnel equation $$\det[f^{ab}_{\mathrm{AB}}\; p_a p_b] = 0.$$

Expand determinant:

$$\det[f^{ab}_{\mathrm{AB}}\; p_a p_b] = Q^{abcd\dots}\; p_a p_b p_c p_d \dots$$

(OK, technically co-Finsler rather than Finsler)

Finsler spacetime:

Remember: In special relativity ----

$$d_\gamma(x,y) = \int_x^y \sqrt{g_{ab}(dx^a/d\tau)(dx^b/d\tau)}\,d\tau,$$

- $d_\gamma(x,y) \in \mathbb{R}^+$ for spacelike paths;
- $d_\gamma(x,y) = 0$ for null paths;
- $d_\gamma(x,y) \in \mathbb{I}^+$ for timelike paths;

Even in SR and GR, "distances" do not have to be real numbers...

Finsler spacetime:

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Generalize this to a Finsler structure:

Start with the simple multi-metric case:

$$Q(x,p) = \Pi_{i=1}^{n} (g_i^{ab} p_a p_b),$$

$$G(x,p) = \sqrt[2n]{\Pi_{i=1}^{n} (g_i^{ab} p_a p_b)},$$

$$G(x,p) \in \exp\left(\frac{i\pi\ell}{2n}\right) \mathbb{R}^+,$$

- $\ell = 0 \to G(x,p) \in \mathbb{R}^+ \to$ outside all n signal cones;
- $\ell = n \to G(x,p) \in \mathbb{I}^+ \to$ inside all n signal cones.

Finsler spacetime:

That is:

- Spacelike $\leftrightarrow$ outside all n signal cones $\leftrightarrow$ G real;
- Null $\leftrightarrow$ on any one of the n signal cones $\leftrightarrow$ G zero;
- Timelike $\leftrightarrow$ inside all n signal cones $\leftrightarrow$ G imaginary;
- plus the various "intermediate" cases:

$$\text{“intermediate”} \leftrightarrow \text{inside } \ell \text{ of } n \text{ signal cones} \leftrightarrow G \in i^{\ell/n} \times \mathbb{R}^+.$$

This basic idea survives even if we go beyond the multi-metric special case...

Finsler spacetime:

$Q(x,p) = 0$ defines a polynomial of degree "2n"...

... and therefore defines "n" nested "conoids"...

This is Courant-Hilbert's "Monge cone"...

$$
\begin{aligned}
Q(x,p)=0 \quad &\Leftrightarrow \quad Q(x,(E,\vec{p}))=0; \\
&\Leftrightarrow \quad \text{polynomial of degree } 2n \text{ in } E \text{ for any fixed } \vec{p}; \\
&\Leftrightarrow \quad \text{in each direction } \exists\ 2n \text{ roots in } E; \\
&\Leftrightarrow \quad \text{corresponds to } n \text{ [topological] cones.}
\end{aligned}
$$

Finsler spacetime:

In short:

[Liberati *et al*]

- pseudo-co–Finsler functions arise naturally from the leading symbol of hyperbolic systems of PDEs;
- pseudo-co–Finsler geometries provide the natural "geometric" interpretation of a multi-component PDE before fine tuning;
- In particular the natural geometric interpretation of 2-BEC models (in the hydrodynamic limit, and before fine tuning) is as a 4-smooth pseudo-co–Finsler geometry.

Despite their somewhat abstract mathematical character, Finsler spacetimes are of direct physical interest...

Conclusion:

Many interesting extensions and modifications of the general relativity notion of spacetime have concrete and well controlled models within the “analogue spacetime” framework.

This tells us which rocks to start looking under...

"It is important to keep an open mind; just not so open that your brains fall out"

--- Albert Einstein