Heat Transfer During Annealing of Steel Coils

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Abstract Steel becomes brittle during the cold rolling process which is used to produce sheet metal. Heat treatment (annealing) is required to release stresses and reform the crystalline structure. The 2008 Mathematics-and Statistics-in-Industry Study Group in Wollongong (MISG08) modelled the approach used by New Zealand Steel for which steel coils are heated in a batch annealing furnace. Determining the temperature within each coil is complicated by height-dependent gaps within the coils. Deciding on suitable boundary conditions for the outside of the coils provides a further challenge. Having made reasonable assumptions, a linear model has been found to be sufficient for modelling the heating process and allows the cold point in the steel coil to be established.

1 Introduction

Manufactured steel becomes brittle during cold rolling to produce sheet metal. Annealing reforms the crystal structure. Initially, coiled metal strips are heated to a high temperature ($\sim 700^\circ$C). This temperature is then maintained for several hours. The time required for the initial heating is determined by the part of the coil that is slowest to heat. To minimise the heating time this point needs to be found and the time for it to reach the desired temperature obtained. Simple and accurate models that incorporate different coil properties will allow this process to be optimised reducing heating costs and avoiding poorly annealed steel that needs reprocessing. In Auckland, New Zealand Steel use a *Uniflow Annealing System* (UAS) furnace.

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Steel coils, typically nine in a horizontal square formation, are placed on their ends with their axes aligned vertically on top of a ventilated steel platform. The platform is moved into the furnace. The coils are heated directly by radiant burners spread across the ceiling of the furnace. Additional burners along the sides are shrouded and do not directly heat the coils. The gas within the furnace is an inert mixture of nitrogen (93% by volume) and hydrogen. Circulating this gas provides indirect heating. Experimental data for the furnace are limited because there are practical difficulties in taking measurements.

A full account of the work of MISG08 is given in [1]. Some further consideration of the problem is presented in [2]. In the following, we summarise the model for the process.

2 Modelling the Steel Coils

We model each individual coil as a continuous vertical hollow cylinder with anisotropic (position and direction dependent) thermal conductivity. Vertically, conduction is in the axial z direction, in individual coil layers, and the conductivity k_z [J/m/s/K] is taken to be that of steel k_s. Conductivity in the radial r direction will be lower due to gaps between layers of the metal. For the present we shall assume that there is an effective radial conductivity k_r. This is considered further in the next section.

Temperature $T = T(r,z,t)$ [K] is governed by

$$\frac{\partial(c_p \rho T)}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(k_r r \frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right) , \tag{1}$$

where c_p [J/kg/K] and ρ [kg/m^3] are the heat capacity and density of steel, respectively. Table 1 lists appropriate dimensions and properties. Initially $T = T_0$. It is presumed that the ventilated platform will reach furnace temperature extremely quickly. The coils heat by radiation, conduction and convection. Heating of each coil's flat surfaces (the ends $z = 0, L$) is very effective due to radiation from the heaters above and conduction from the ventilated platform below. These surfaces are expected to rapidly assume furnace temperature T_g. (The limited experimental data partially confirm this [1].) Curved surfaces $r = a, b$ are heated by convection (Newton's Law of Cooling) leading to the boundary condition $k_r \partial T/\partial r = \pm H(T - T_g)$, where H is the heat transfer coefficient. A number of approaches for estimating the value of H have been considered, however, for the units used here, they all give values in the range 3-5 [1].

The concept of mean action time [3] can be used to allow for changing thermal capacity c_p and conductivity k_s with rising steel temperatures. This has been considered [1] but is not discussed further here.

Table 1 Typical Steel, Coil and Furnace Properties

steel density	ρ	7854	kg/m^3 at 300K
steel thermal conductivity	k_s	60.5–30	W/m/K at 300K–1000K
steel thermal capacity	c_p	434–1169	J/kg/K at 300K–1000K
steel strip thickness		0.4–3	mm
steel strip width	L	700–1500	mm
coil mass		10–20	tonnes
coil inner diameter	a	508	mm
coil outer diameter	b	1.5	m
gas thermal conductivity	k_g	0.06	W/m/K
furnace circulation		800	m^3/minute
platform mass		37	tonnes
furnace dimensions		$6.5 \times 6.5 \times 4$	m^3

3 Radial Conductivity

Coils can be modelled as concentric annular cylinders of metal separated by hot gas [4, 5]. This approximates the different mechanisms of heat transport (direct contact, gaseous diffusion, and radiation) between layers of the coil [6–8]. The effective conductivity across the layers can be taken to be

$$k_{\text{eff}} \approx \frac{d_s + d_g}{\frac{d_s}{k_s} + \frac{d_g}{k_g}} \tag{2}$$

where d_s and d_g are the thicknesses of the steel and gas layers. This expression is exact for the steady state and the limit of infinite layers [9] and is a reasonable model here. A potential complication is vertical variation of the gaps, as a rolled steel strip has a crown: it is thinner at the edges than the middle. Examples at 1000 K suggest that in the central $3/4$ of the coil by height (middle of the strip), the radial conductivity k_r is about $1/2$ to $3/4$ that of steel k_s. This rapidly decreases towards the flat ends where thinning due to the crowning occurs [1]. However, as the ends are heated very effectively from above and below this has little effect. Potentially, radial conductivity k_r is also radially dependent due to coil tension and differential expansion during heating, although these effects are thought to be small.

4 Linear Solution

If the coil is assumed homogeneous, but with anisotropic heat conductance with constant conductivities k_r, k_z, then separation and Sturm-Liouville Theory lead to a solution for the temperature T [1, 2]. (The solution for the purely radial case is stated on page 530 of [10].)

First, the heat transfer equation (1) is rewritten more simply as

$$\frac{\partial T}{\partial t} = D_r \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + D_z \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right). \tag{3}$$

The diffusivities $D_r = k_r/(\rho c_p), D_z = k_z/(\rho c_p)$ are assumed constant. The system is rescaled: using the typical values $t = t_0, r = b, z = L, T = T_g$, so that $r = br^*$, $z = Lz^*$, $t = (\rho c_p L^2/k_z)t^*$, $u = (T - T_g)/(T_0 - T_g)$, where r^*, z^*, t^*, u are non-dimensional. Relative diffusivity $D = k_r L^2/k_z b^2$, the ratio of original lengths $\alpha = a/b$, and $h = Hb/k_r$. There is a choice of two obvious time scales t_0, using either D_r or D_z. For the problem of interest they are of similar magnitude and it is unclear which dominates. Hence, without loss of generality, $t_0 = L^2/D_z$ is assigned. The * notation is dropped for convenience. The new system is

$$\frac{\partial u}{\partial t} = D\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{\partial^2 u}{\partial z^2}, \tag{4}$$

$$\left[\frac{\partial u}{\partial r}\right]_{r=\alpha} = -\left[\frac{\partial u}{\partial r}\right]_{r=1} = hu\,, \quad [u]_{t=0} = 1\,, \quad [u]_{z=0,1} = 0. \tag{5}$$

This equation now represents the non-dimensional cooling of a unit cylinder from an initial temperature of unity to a surrounding temperature of zero. Separating variables as $u(r,z,t) = R(r)Z(z)T(t)$, a series solution can be found with Bessel functions

$$u = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{mn}\, e^{-(D\lambda_m^2 + (n\pi)^2)t}\, \sin(n\pi z)\, C_m(r), \tag{6}$$

where the eigenvalues λ_m satisfy

$$\begin{vmatrix} hJ_0(\alpha\lambda_m) + \lambda_m J_1(\alpha\lambda_m) & hY_0(\alpha\lambda_m) + \lambda_m Y_1(\alpha\lambda_m) \\ hJ_0(\lambda_m) - \lambda_m J_1(\lambda_m) & hY_0(\lambda_m) - \lambda_m Y_1(\lambda_m) \end{vmatrix} = 0\,, \tag{7}$$

and are found numerically. Coefficients A_{mn} and functions $C_m(r)$ are given by

$$C_m \equiv J_0(\lambda_m r) + B_m Y_0(\lambda_m r)\,,\ A_{mn} = \frac{\frac{4}{n\pi}(1-(-1)^n)\left[rC_{m1}\right]_\alpha^1}{\lambda_m\left[r^2\left(C_m^2 + C_{m1}^2\right)\right]_\alpha^1}, \tag{8}$$

$$B_m = \frac{\left(\frac{\lambda_m}{h}J_1(\lambda_m) - J_0(\lambda_m)\right)}{\left(Y_0(\lambda_m) - \frac{\lambda_m}{h}Y_1(\lambda_m)\right)}\,,\ C_{m1} \equiv J_1(\lambda_m r) + B_m Y_1(\lambda_m r). \tag{9}$$

5 Further Modelling of Radial Conductivity

Further modelling [1] has allowed for height dependent radial conductivity $k_r(z)$ due to crowning. Using the finite difference method, numerical simulations were conducted with realistic values. These confirmed the linear solution and found only very modest differences when variation in radial conductivity was introduced. The linear model seems sufficiently accurate here. The dominant heating occurs at the ends of the coils and this is unchanged.

Purely radial simulations indicate that the Newtonian boundary condition is the main restriction on radial heat flow rather than conductivity rates within the coil. If curved sides were heated more directly then the cold point's heating time would reduce [2].

The leading eigenfunction associated with eigenvalues λ_1 and π dominates the linear model. Consequently, time dependence is governed by the decay of

$$e^{-(D\lambda_1^2+\pi^2)t}. \tag{10}$$

At the cold point, in scaled units, $z = 1/2$ and $r = r_c$ where r_c is given by the extremum of the function $C_1(r)$. As illustrated in Fig. 1 the coil's cold point is closer to the inner curved surface due to its smaller surface area (and hence heat flux).

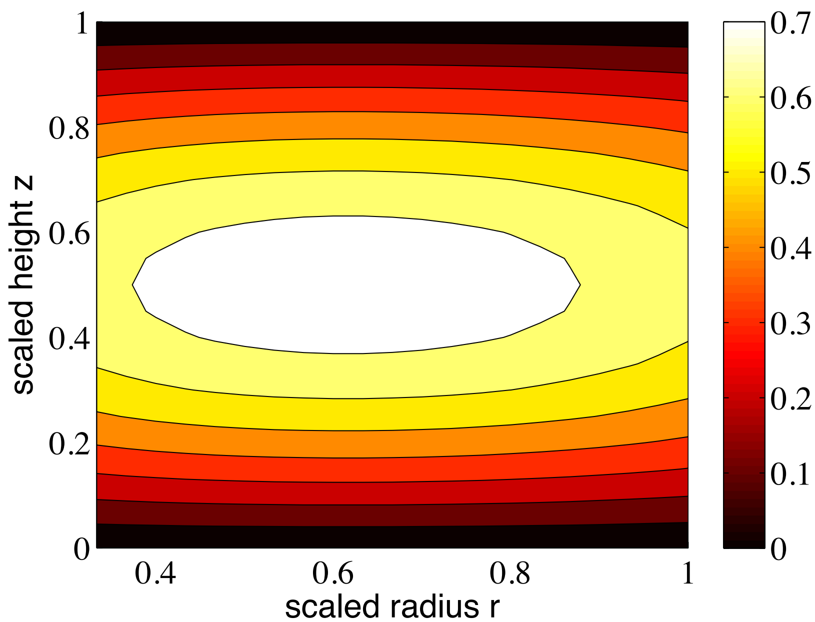

Fig. 1 A contour plot of scaled temperature for a cross-section of a model coil during heating. The cold spot (white) is nearer the inside of the coil ($r = 1/3$). Note the curved surfaces (on the sides) are still approaching the surrounding gas temperature (black).

6 Conclusions

We have presented a simple model of the heating process for a steel coil during annealing. The model illustrates that a major constriction is the slow transport of heat through the sides of coils. The linearised model seems adequate for calculations and itself tends to be dominated by a leading eigenvalue term.

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References

1. McGuinness, M., Sweatman, W.L., Baowan, D., Barry, S.I.: Annealing Steel Coils. In: T. Merchant, M. Edwards, G. Mercer (eds.), Proceedings of the 2008 Mathematics and Statistics in Industry Study Group, pp. 61–80, University of Wollongong, Australia (2009)
2. Barry, S.I., Sweatman, W.L.: Modelling heat transfer in steel coils. ANZIAM J. (E) 50, pp. C668–C681, (2009)
3. Landman, K., McGuinness, M.: Mean Action Time for Diffusive Processes. J.Appl.Math.Decis.Sci. 4 (2), pp. 125–141 (2000)
4. Stikker, U.O.: Numerical simulation of the coil annealing process. In: Mathematical Models in Metallurgical Process Development, *Iron and Steel Institute, Special Report*, 123, pp. 104–113 (1970)
5. Willms, A.R.: An exact solution of Stikker's nonlinear heat equation. SIAM J.Appl.Math., 55 (4), pp. 1059–1073 (1995)
6. Sridhar, M.R., Yovanovicht, M.M.: Review of elastic and plastic contact conductance models: Comparison with experiment. J.Thermophys.Heat Trans. 8, pp. 633–640 (1994)
7. Zuo, Y., Wu, W., Zhang, X., Lin, L., Xiang, S., Liu, T., Niu, L., Huang, X.: A study of heat transfer in high-performance hydrogen Bell-type annealing furnaces. Heat Transfer — Asian Research, 30 (8) 615–623 (2001)
8. Zhang, X., Yu, F., Wu, W., Zuo, Y.: Application of radial effective thermal conductivity for heat transfer model of steel coils in HPH furnace Int.J.Thermophysics 24 (5), pp. 1395–1405 (2003)
9. Hickson, R., Barry, S., Mercer, G.: Exact and numerical solutions for effective diffusivity and time lag through multiple layers. ANZIAM J. (E) 50, pp. C682–C695. (2009)
10. Budak, B.M., Samarskii, A.A., Tikhonov, A.N.: A collection of problems in mathematical physics. Dover, New York (1964)