Our Mathematical Age

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My apologies in advance

- I teach only university students, some school students for enrichment classes (particularly in Singapore), and Scottish Country dancing for which I actually have a teaching certificate!
- Indeed having taught SCD to a group of around 18, 9-10 years olds in Johnsonville School with a teacher I gained immense respect for her.
- However, I do have a bit of insight into modern mathematics so I hope the following might be of some use.
My own educational experience

- Primary School 6 (SIX) schools in 7 years, moving from Toowoomba to parts of Brisbane and the Sunshine Coast.
- The final little school (Rainworth) had a clever old headmaster/teacher who tried to inspire with maths (and the cane!).
- Then Brisbane State High School, insisting to my parents that I would stay there even though we again moved to somewhere which gave 2 hours travel each way.
- Typical reports: “Could try harder”, “Has a bad attitude”, “Can be disruptive” (but the occasional “has a distinct aptitude for the subject” and “results speak for themselves”)
- I think I was bloody minded; but they did allow me to me the only student who did logic with the science students.
- University; I got all the Gough Whitlam support for poor people... These things really do work.
And then

- My parents both working class; first Downey to ever go past 10th grade.
- Dad a storeman and packer, mum made people pay debts.
- Really the family had no idea why someone would even go to University; one of my main motivations being told not to do things.
- Actually, I had no idea why someone would go to University unless you wanted to be a doctor or a much-hated lawyer.
- Parents only really happy when I got paid.
- My wife from a professional family.
- After a wee bit abroad came to New Zealand for “a couple of years” ...
- 2 Boys, one a designer at Xero and one works in Google’s Waymo.
- Newlands College....
Notwithstanding some conflicts here and there, boys thrived in Newlands.

Great vertical support structures; good science and math teachers and good leadership.

Carlton, the oldest was given extension mathematics.

With the best of intentions, this consisted of being moved into the next year of maths.

This was not good because stuff was left out and also for social development.

We refused such things for the other boy.

What might be good extension? What resources might be good for lateral extension?
I think mathematicians build symbolic models of the world.
This might be the physical world, but could be e.g. thought, logic.
Then manipulating them allows them to understand/predict/explore.
The Egyptians/Babylonians/Greeks/Chinese/Incas invented geometry to help building and the motions of the cosmos etc.
They and others invented methods of calculating interest rates etc to make money.
Later from physics we invented differential equations which can be used to describe rates of change.
Logicians built models of reasoning and this led to computers and are the basis of modern society.
It is the reasoning and modelling which is the core of maths, not the procedures.
In my own experience I stay at this college in Singapore and give little lectures to arts student. 
http://homepages.ecs.vuw.ac.nz/~downey/

I try to show them a wee bit of math/history and point at forefronts of modern mathematics.

Each of the presentations has a lot of references I am sure a keen student could follow up on.
Some names

- Graph Colouring as a Mathematical Minature (graph theory and combinatorics, leading to complexity theory)
- Codebreaking at Bletchley (about Turing etc, study in statistics) also a public lecture in my talks
- The life of Π (development of analysis and calculus—also Fascination of Science)
- I will look at these not because I think they are especially good, but they have good pointers for projects in them, and I know them.
Can find these talks to download at
http://homepages.ecs.vuw.ac.nz/~downey/talks.html

Also an advanced class at NTU: (Order, Chaos and Algorithms-
Unpublished Notes for Singapore Lectures at Nanyang for Gifted
Undergraduates and Junior College Students) in
http://homepages.ecs.vuw.ac.nz/~downey/publications/
Some ideas 1

- My own background is in **discrete mathematics**.
- Theoretical computer science, algorithms, computation.
- **Graph theory and algorithmics** is an excellent vehicle.
- **How not to be Wrong** (about $NZ17) has a wealth of material about particularly statistics, and mathematical reasoning for the layman. Stresses **The Mathematical Way of Thought**.
- I am sure you are aware of videos like those on **Mathsreach**. [https://mathsreach.org/Welcome](https://mathsreach.org/Welcome)
- Kid Crypto: Type this into Google with Fellows after it.
Graph theory and algorithmics is an excellent vehicle.

Euler’s Formula: For $E \geq 1$, and $G$ connected. $V + E - F = 2$ “The Raging Sea Proof”

Then the 5 Colour Theorem.
Your plane graph is sitting inside a raging sea, breaking on the structure from the outside.

You remove an outer edge and the sea floods in. $F := F - 1$, $E := E - 1$, $V := V$.

Continue till only a jetty remains. Then remove outer edges one at a time: $E := E - 1$, $V := V - 1$, $F = 1$ still.

At the end $V + E - F$ has not changed, but you have a single edge with 2 vertices, and one face. so $V + E - F = 2 + 1 - 1 = 2$.

Then show this means there must be a vertex of degree 5. (not too hard)
Heawood, 1890’s-can talk about Kempp’s proof and point them at the 4CT.

I begin with the smallest planar graph that is not 5-colourable.

If I remove a vertex $\nu$ of degree $\leq 5$ I can 5-colour the rest.

So do this and 5-colour the smaller graph.

Can I put $\nu$ back in?
Can I recolor as $1 \rightarrow 4$?

Smallest counterexample.
Figure: A Kempp Chain.

IF NO!  Kemp Chain
I don’t think standards have been falling. Certainly not since Euclid.

There is a gap left by the loss of Euclidean Geometry, the only achievement standard which addresses this is 2.6 Apply algebraic methods in solving problems.

I think that the students are a wee bit different now. (With no evidence except observations)

It is hard to convince them about effort to get to deep knowledge.

Adept at finding stuff out on the Internet. ("research")
But I believe that are a bit more easily demoralized by harder things that will come with effort but are not instantly understandable. For me, for example, proofs by induction.

The current breadth in the school curriculum and choice of topics available, does make it more difficult for the student entering University.

We find many students make bad choices, and need remedial work. How can kids know what they want so young and then make subject choices accordingly?

We have had to upgrade the requirements for entry into core maths courses at VUW, such as 161, to ask for NCEA merits.

I lament at the loss of geometry, because the heart and soul of maths is deductive reasoning. We need some kind of replacement in the new curriculum, discrete maths would do.
I hope this gives you some pointers as to what might be useful materials for your classes.

It hope this gives you a wee bit of insight into things to say, why maths?

Thanks for Listening!