

General comment. The use of  $|G|$  for a (directed) graph  $G$  is the usual Garey-Johnson style of measuring the overall size of a graph, and is therefore  $O(|V(G)|^2)$ . (This differs from the usage in some graph theory books like Diestel's, where  $|G|$  is taken as  $|V(G)|$ .)

Page xiv "The three basic problems referred to are:", referred becomes referred

Page 15, Definition 2.1.1 "FTP" should be "FPT" (this one is kind of embarrassing).

Page 33. Proof of Lemma 3.5.1: For a simpler presentation, Oum suggests that we would use the poly-time algorithm to find a shortest cycle (for each edge  $e$ , find a shortest path joining 2 ends.) Then the running time can be bounded by  $O^*((2k)^k)$ .

Page 34, proof of Lemma 3.2.1. Line -13 and line -1, replace  $|V|$  by  $|G|$ .

Page 36 and page 549, Question for CLOSEST STRING. This should read, "find  $s = s_j$ , for some  $j \in \{1, \dots, k\}$ ,"

Page 37, Theorem 3.6.1.  $|G|$  should be  $|\Sigma|$ .

Page 64, Line 2, Section 5.1.3 should be 5.1.5.

Page 82, Exercise 4.11.5: Bodleander  $\rightarrow$  Bodlaender

Page 99, Definition 5.1.3 (ii), should read:

... "with oracle  $L_2$ , such that  $\Phi^{L_2} = L_1$ , and on input  $\langle \sigma, k \rangle$ ,  $\Phi$  only makes queries to the oracle  $L_2$  of the form  $\langle \tau, k' \rangle$  for  $|\tau|, k' \leq g(k)$ ."

Page 110, statement of Lemma 6.2.2. 2 should say that  $Y$  contains an edge-cut instead of saying that " $Y$  is an edge cut in  $G - X$ ."

Page 110, in the proof of Lemma 6.2.2.  $G_y$  should be  $C_Y$ . It might be clearer to say edges rather than paths in the proof.

Page 119, line 1. Lemma 6.6.1 should be Lemma 6.5.1.

Page 165, Exercises 8.8.1 and 8.8.2. The sentence here means that  $\mathcal{F}$  is a family of subsets of a set  $X$  such that  $|\mathcal{F}| = r$ , *not* that the subsets have size  $r$ . (Oum suggests that probably a better name for this problem is "ANTICHAIN": to find an antichain of size  $k$  in a collection of  $r$  subsets.)

Page 185, line -4. after *Pascal*, put "are  $\leq 6$ ."

Page 202, line -7, "1966" should be "1996".

Page 275, statement of Theorem 13.4.2. Actually Theorem 13.4.2 (Courcelle and Oum) is still open as stated here. Courcelle and Oum proved only for C2MS1 logic and could only prove a weaker statement with the set predicate for the "even cardinality".

Pages 254-257.

### Myhill-Nerode Theorem for Graphs

This is a big error in the proof of this. Here is a correct (and easier) proof. Some property like parsing replacement can likely be extracted.

We prove Theorem 6.77 of [DF98]= Theorem 12.7.2 of the book.

The proof in both places, being the same, has a small error. This is fixed in this note.

The error is in the proof of (iii) $\rightarrow$ (i).

For neatness we will use the  $t$ -boundary operators given, and not worry about a general property making this work.

Now we know that a graph has pathwidth  $t$  iff it can be parsed by the above operators without using  $\oplus$ , and the analogous Parsing Theorem (6.72 of [DF98], 12.7.1 of [DF13]) holds. Now consider the parsing theorem in the context of the small universe.

It is easy to prove by induction of the length of the path, that if  $G$  is a pathwidth  $t$  graph in the small universe of treewidth  $t$  graphs, then  $G$  is isomorphic (in the universe forgetting the boundary) to a graph  $G_1$  with a parsing and the boundary vertices in the first bag and also one where the boundary is in the last bag of the parsing of  $G_1$ .

Since we can move a boundary over an  $\oplus$ , it follows that in the small universe of treewidth  $t$  graphs, if we consider a parse tree  $T$ , then  $G(T)$  is isomorphic to a parse tree  $\hat{T}$  where the boundary in the underlying tree of bags is at the root, and also one  $\hat{T}'$  where the boundary corresponds to a bag corresponding to a given leaf of  $T$ .

Now, following the proof of (iii) implies (i), we assume we are given  $T_k$  and  $T_{i,j}$  such that for all  $i, j$   $T_i \cdot_x T_{i,j} \in L$  iff  $T_j \cdot_x T_{i,j} \notin L$ , where  $L$  is the language of trees which are equivalent if the underlying graphs are isomorphic as unlabelled graphs.

The argument above says that we can regard the root of each  $T_k$  to correspond in the underlying bags given by the underlying parsing theorem to have the

boundary, and the bag corresponding to the leaf of  $T_{k,j}$  and  $x$  to have the boundary.

In that case, we see that  $G(T_i \cdot_x T_{i,j}) \cong G(T_i) \oplus G(T_{i,j})$  since with the boundaries at that placement, there is a  $G(T_i \cdot_x T_{i,j})$  corresponds simply to gluing the underlying graphs along that boundary. (Notice that, to do this it might be that we might need to make parts of the boundary corresponding to (for example)  $T_{i,j}$  disjoint. It might be, for instance, that  $T_i \cdot_x T_{i,j}$  might correspond to disjoint graphs, but this can be construed as a gluing in any case.)

Then we get a contradiction, since now  $G(T_k)$  would witness that  $\sim_{\mathcal{F}_t}$  does not have finite index.

Page 381 line 4: delete closing parenthesis.

Page 541 section 29.2 line -1: seem  $\rightarrow$  seems

Page 546: Theorem 29.5.1 The  $d$ 's and  $k$ 's are mixed up. The occurrence of 2-SAT should be  $k$ -SAT. Also 2. should read  $p \leq 2^{frac{n}{2}}$ . And the running time is  $2^{\frac{n}{2}}|\phi|^{O(1)}$ .

Page 547 line -1: add "holds" as the last word for the sentence.

Page 553 section 29.6 first sentence, delete the fullstop half way through the sentence.

Page 597, 30.10.1, line -5, (i.e. "1") delete "and"

Page 597, 30.10.1, line -3 (i.e. "2") "instance" should be "instances"