The Effects of Limited Channel Knowledge on Cognitive Radio System Capacity

Peter J. Smith, *Senior Member, IEEE*, Pawel A. Dmochowski, *Senior Member, IEEE*, Himal A. Suraweera, *Member, IEEE*, and Mansoor Shafi, *Fellow, IEEE*

***Abstract*—We examine the impact of limited channel knowledge on the secondary user (SU) in a cognitive radio system. Under a minimum signal-to-interference-and-noise ratio (SINR) constraint for the primary user (PU) receiver, we determine SU capacity under five channel knowledge scenarios. We derive analytical expressions for the capacity cumulative distribution functions and the probability of SU blocking as a function of allowable interference. We show that imperfect knowledge of the PU-PU link gain by the SU-Tx often prohibits SU transmission or necessitates a high interference level at the PU. We also show that errored knowledge of the PU-PU channel is more beneficial than statistical channel knowledge and that imperfect knowledge of the SU-Tx to PU-Rx link has limited impact on SU capacity.**

I. Introduction

A large body of work is now available on various aspects of CR systems, including fundamental information theoretic capacity limits and performance analysis, which often assumes perfect SU-Tx to PU-Rx channel state information (CSI) [1]–[7]. In practice, there is expected to be limited (or no) collaboration between PU and SU systems. Hence, an important question is the impact of the nature of channel knowledge on CR capacity. Several recent contributions have considered imperfect CSI [8]–[14]. In [8], mean and outage capacities along with optimum power allocation policies have been investigated for a CR system in a fading environment with imperfect CSI. Here, probabilistic constraints were employed to maintain an acceptably low probability that interference exceeded some target. In our work, we also use probabilistic constraints, but apply them to a signal-to-interference noise ratio (SINR) target.

This paper differs from the existing literature in several ways. There are four link gains in a two user PU/SU channel to consider and each of them may or may not be perfectly known at the SU transmitter. Previous studies [8]–[10], [12], [14] have only assumed imperfect knowledge of the SU-Tx to PU-Rx link. Additionally, in previous work, the effect of the interference from the PU-Tx on SU capacity is ignored. Also, we employ the SINR at the PU-Rx to impose probabilistic constraints to protect the PU-Rx, while prior works, with the exception of [10], have considered an interference outage constraint. Finally, we consider several cases where the imperfect CSI manifests itself in the form of statistical channel knowledge (i.e., knowledge of the mean link gains). Such a form of imperfect CSI is attractive from a practical stand point, since obtaining accurate knowledge is almost impossible for some links, such as the PU-Tx to PU-Rx link. Moreover, the mean value does not impose a large system burden as it only requires infrequent updates. Note that the inclusion of PU-Tx to SU-Rx interference and probabilistic constraints enables a rigorous evaluation of the benefits of various types of CSI. In this paper, we establish the following key observations and results:

- In four of the five scenarios considered, we derive analytical expressions for the cumulative distribution function (cdf) of the SU SINR and use it to evaluate the SU capacity cdf.
- For all scenarios, we derive the probability of SU blocking as a function of the permissible interference at the PU-Rx.
- By evaluating our results for a range of system parameters, we demonstrate the importance of accurate knowledge of the PU-Tx to PU-Rx link at the SU-Tx.
- We demonstrate the very high sensitivity of SU performance to the error in the estimation of the PU-Tx to PU-Rx and SU-Tx to PU-Rx links.
- We show that errored knowledge of the PU-Tx to PU-Rx link and SU-Tx to PU-Rx link (if available) is better for SU capacity than a knowledge of the mean link gains.
- By considering a single probabilistic SINR constraint, a unified framework is presented which enables fair comparisons between different types of channel knowledge.

II. System Model

Consider a CR system (shown in Fig. 1) with the SU-Tx and PU-Tx transmitting simultaneously to their respective receivers. Independent point-to-point flat Rayleigh fading channels are assumed for all links. Let $g_{\mathrm{p}} = |h_{\mathrm{p}}|^2$, $g_{\mathrm{s}} = |h_{\mathrm{s}}|^2$, $g_{\mathrm{ps}} = |h_{\mathrm{ps}}|^2$ and $g_{\mathrm{sp}} = |h_{\mathrm{sp}}|^2$ denote the exponentially

P. J. Smith is with the Department of Electrical and Computer Engineering, University of Canterbury, Christchurch, New Zealand (email: peter.smith@canterbury.ac.nz).

P. A. Dmochowski is with the School of Engineering and Computer Science, Victoria University of Wellington, PO Box 600, Wellington, New Zealand (email: pdmochowski@ieee.org).

H. A. Suraweera is with the Singapore University of Technology and Design, 20 Dover Drive, Singapore 138682, Singapore (email: himalsuraweera@sutd.edu.sg).

M.Shafi is with Telecom New Zealand, PO Box 293, Wellington, New Zealand (email: mansoor.shafi@telecom.co.nz).

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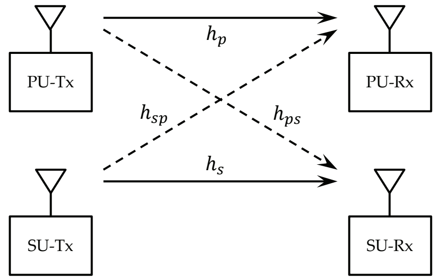

Fig. 1. System Diagram.

distributed instantaneous link gains of the PU-Tx to PU-Rx, SU-Tx to SU-Rx, PU-Tx to SU-Rx and SU-Tx to PU-Rx links, respectively, with $\Omega_{\rm p} = \mathbb{E}(g_{\rm p}), \Omega_{\rm s} = \mathbb{E}(g_{\rm s})$, $\Omega_{\rm ps} = \mathbb{E}(g_{\rm ps})$ and $\Omega_{\rm sp} = \mathbb{E}(g_{\rm sp})$, where $\mathbb{E}(\cdot)$ denotes the expectation operator.

As described further in this Section, the SU transmission under the SINR constraint is governed solely by the state of the $g_{\rm p}$ and $g_{\rm sp}$ links[1]. Thus, in this paper we consider the following five scenarios for the knowledge of $g_{\rm p}$ and $g_{\rm sp}$ by the SU-Tx.

Scenario 1: The link gains, $g_{\rm p}$ and $g_{\rm sp}$, are perfectly known. This clearly unrealistic scenario serves as a benchmark for comparison.

Scenario 2: The link gain, $g_{\rm p}$, is perfectly known while only the mean, $\Omega_{\rm sp}$, of $g_{\rm sp}$ is known.

Scenario 3: The mean, $\Omega_{\rm p}$, and the exact link gain, $g_{\rm sp}$, are known. In contrast to *Scenario 2*, this case is considered mainly for completeness.

Scenario 4: Only the means, $\Omega_{\rm p}$ and $\Omega_{\rm sp}$, are known. This scenario arises when only statistical information about the channels is available to the SU-Tx as a result of limited feedback resources.

Scenario 5: Only estimates of $g_{\rm p}$ and $g_{\rm sp}$ are available. This may arise due to channel estimation, feedback quantisation and delay.

Where possible, we impose a constraint, $\gamma_{\rm T}$, on the PU-Rx SINR, denoted by $\gamma_{\rm p}$. Hence,

$$\gamma_{\rm p} = \frac{P_{\rm p} g_{\rm p}}{P_{\rm s} g_{\rm sp} + \sigma_{\rm p}^2}, \text{ and } \gamma_{\rm p} \geq \gamma_{\rm T}, \tag{1}$$

where $\gamma_{\rm T}$ is an SINR threshold, $P_{\rm p}$ is the PU transmit power (assumed to be fixed and known to the SU-Tx in all scenarios), and $\sigma_{\rm p}^2$ is the additive white Gaussian noise (AWGN) variance at the PU-Rx. $P_{\rm s}$ is a provisional maximum value for the SU transmit power chosen to satisfy the relevant criteria, (1) in this case. The actual SU transmit power, $P_{\rm t}$ is a function of $P_{\rm s}$. For example, if the PU-Rx SNR lies in the region, $P_{\rm p} g_{\rm p}/\sigma_{\rm p}^2 < \gamma_{\rm T}$, then (1) cannot be satisfied unless $P_{\rm s} < 0$ and as a result, $P_{\rm t} = 0$. If the PU SNR is above the SINR threshold $\gamma_{\rm T}$, the SU-Tx will adapt $P_{\rm s}$ to a maximum level satisfying (1) as determined under the five scenarios, regardless of the gain $g_{\rm s}$.

[1]The link gains $g_{\rm s}$ and $g_{\rm ps}$ have an impact on achievable SU capacity, however the level of their knowledge by the SU-Tx does not impact the transmit power.

We also impose a maximum SU transmit power constraint, $P_{\rm m}$. Thus, in *Scenario* 1, where the SU-Tx knows $g_{\rm p}$, $P_{\rm t}$ is given by

$$P_{\rm t} \;=\; \begin{cases} 0 & \frac{P_{\rm p} g_{\rm p}}{\gamma_{\rm T}} < \sigma_{\rm p}^2 \\ \min\left(P_{\rm s}, P_{\rm m}\right) & \text{otherwise}, \end{cases} \tag{2}$$

where $P_{\rm s}$ is obtained from (1) by solving $\gamma_{\rm T} = \gamma_{\rm p}$. Furthermore, the constraints described above can only be guaranteed if the SU-Tx has perfect knowledge of the gains $g_{\rm p}$ and $g_{\rm sp}$, i.e., under *Scenario* 1. In analysing *Scenarios* 2-5, we use probabilistic constraints. Hence, we require the SINR constraint to hold with an acceptably high probability, $1-\alpha$, where α is small.

In analysing the SU capacity, we first consider the SINR at the SU-Rx, denoted by $\gamma_{\rm I}$,

$$\gamma_{\rm I} = \frac{P_{\rm t} g_{\rm s}}{P_{\rm p} g_{\rm ps} + \sigma_{\rm s}^2}, \tag{3}$$

where $\sigma_{\rm s}^2$ is the AWGN variance at the SU-Rx and $P_{\rm p} g_{\rm ps}$ is the interference from PU-Tx, treated as noise in the capacity calculations. We denote the pdf and cdf of $\gamma_{\rm I}$ by $f_{\gamma_{\rm I}}(x)$ and $F_{\gamma_{\rm I}}(x)$, respectively. The instantaneous SU capacity is given by $C = \log_2(1+\gamma_{\rm I})$, where the mean, $\bar{C}$, can be derived using $f_{\gamma_{\rm I}}(x)$ as

$$\bar{C} = \mathbb{E}(C) = \int_0^\infty \log_2(1+x)\, f_{\gamma_{\rm I}}(x)\; dx. \tag{4}$$

The cdf of C can be obtained from $F_{\gamma_{\rm I}}(x)$ by noting that

$$F_C(y) = \Pr(\gamma_{\rm I} \leq 2^y - 1) = F_{\gamma_{\rm I}}(\tilde{y}), \tag{5}$$

where $\Pr(\cdot)$ denotes probability and $\tilde{y} = 2^y - 1$. Using (3), we can express (5) as

$$\begin{aligned} F_{\gamma_{\rm I}}(\tilde{y}) &= \mathbb{E}_{g_{\rm ps}}\left\{ \Pr\left(P_{\rm t} g_{\rm s} < \tilde{y}(\sigma_{\rm s}^2 + P_{\rm p} g_{\rm ps})\right) \middle| g_{\rm ps} \right\} \\ &= \int_0^\infty F_\gamma\left(\tilde{y}(\sigma_{\rm s}^2 + P_{\rm p} v)\right) \frac{e^{-v/\Omega_{\rm ps}}}{\Omega_{\rm ps}}\; dv, \end{aligned} \tag{6}$$

where we have defined $\gamma = P_{\rm t} g_{\rm s}$ with a cdf $F_\gamma(x)$. In what follows, we derive expressions for $F_\gamma(x)$ which, using (5) and (6), allows us to compute the capacity cdf.

We parameterize the main system variables by two key parameters. The first, $c_1 = \Omega_{\rm sp}/\Omega_{\rm s}$, represents the ratio of the mean interference at the PU-Rx to the mean of the desired channel strength for the SU. The second, $c_2 = \gamma_{\rm T}\sigma_{\rm p}^2/P_{\rm p}\Omega_{\rm p}$, is the ratio of the minimum target SINR to the mean SNR at the PU-Rx. Hence, increasing c_2 corresponds to reducing the allowable interference (with $c_2 = 1$ corresponding to zero average allowable interference).

III. SU Capacity

The capacity mean in (4) and the cdf in (6) require a knowledge of the distributions of $\gamma = P_{\rm t} g_{\rm s}$ and $\gamma_{\rm I}$. Hence, in this section we derive the cdfs for γ and $\gamma_{\rm I}$ for *Scenarios* 1-4. For *Scenario* 5, an alternative approach is required (see Section III-E).

A. Scenario 1

In this scenario, $P_{\rm s}$ can be obtained directly from (1), giving

$$P_{\rm s} = \frac{\frac{P_{\rm p} g_{\rm p}}{\gamma_{\rm T}} - \sigma_{\rm p}^2}{g_{\rm sp}}. \tag{7}$$

We note that while we ignore the $P_{\rm t} = 0$ case in (2), the following derivation is valid since $\Pr(\gamma > 0) = 0$ for $P_{\rm t} \leq 0$. In finding $F_\gamma(x)$, we solve for the complementary cdf given by

$$\begin{aligned}\Pr(\gamma > x) &= \Pr\left(g_{\rm s} \min(P_{\rm m}, P_{\rm s}) > x\right) \\ &= \Pr\left(g_{\rm s} > \frac{x}{P_{\rm m}}, \left(\frac{P_{\rm p} g_{\rm p}}{\gamma_{\rm T}} - \sigma_{\rm p}^2\right) g_{\rm s} > x g_{\rm sp}\right). \end{aligned} \tag{8}$$

Noting that $g_{\rm p}$ is an exponentially distributed RV, we can rewrite (8) by taking the conditional probability over $g_{\rm p}$ and then averaging over $g_{\rm s}$ and $g_{\rm sp}$. This gives,

$$\Pr(\gamma > x) = \frac{e^{-\frac{\gamma_{\rm T}\sigma_{\rm p}^2}{P_{\rm p}\Omega_{\rm p}}}}{\Omega_{\rm sp}\Omega_{\rm s}} \int_{\frac{x}{P_{\rm m}}}^{\infty} \frac{e^{-\frac{u}{\Omega_{\rm s}}}}{\frac{\gamma_{\rm T} x}{P_{\rm p}\Omega_{\rm p} u} + \frac{1}{\Omega_{\rm sp}}}\, du. \tag{9}$$

After simplifying (9), the cdf $F_\gamma(x) = 1 - \Pr(\gamma > x)$ can be shown to be [15, Eq. (3.351.2)]

$$\begin{aligned}F_\gamma(x) = 1 - e^{-\frac{\gamma_{\rm T}\sigma_{\rm p}^2}{P_{\rm p}\Omega_{\rm p}}} \Bigg[e^{-\frac{x}{P_{\rm m}\Omega_{\rm s}}} - \frac{\Omega_{\rm sp}\gamma_{\rm T} x}{P_{\rm p}\Omega_{\rm p}\Omega_{\rm s}} e^{\frac{\Omega_{\rm sp}\gamma_{\rm T} x}{P_{\rm p}\Omega_{\rm p}\Omega_{\rm s}}} \\ \times \Gamma\left(0, \frac{\Omega_{\rm sp}\gamma_{\rm T} x}{P_{\rm p}\Omega_{\rm p}\Omega_{\rm s}} + \frac{x}{P_{\rm m}\Omega_{\rm s}}\right)\Bigg],\end{aligned} \tag{10}$$

where $\Gamma(\cdot,\cdot)$ is the upper incomplete gamma function. Substituting (10) into (6) results in

$$\begin{aligned}F_{\gamma_I}(\tilde{y}) = 1 - \frac{P_{\rm m}\Omega_{\rm s} e^{-\left(\frac{\gamma_{\rm T}\sigma_{\rm p}^2}{P_{\rm p}\Omega_{\rm p}} + \frac{\tilde{y}\sigma_{\rm s}^2}{P_{\rm m}\Omega_{\rm s}}\right)}}{P_{\rm m}\Omega_{\rm s} + \tilde{y} P_{\rm p}\Omega_{\rm ps}} \\ + \frac{\Omega_{\rm sp}\gamma_{\rm T}\tilde{y}}{\Omega_{\rm ps}\Omega_{\rm p}\Omega_{\rm s} P_{\rm p}} \exp\left\{\frac{\Omega_{\rm sp}\gamma_{\rm T}\sigma_{\rm p}^2}{\Omega_{\rm p}\Omega_{\rm s} P_{\rm p}}\left(\tilde{y} - \frac{\Omega_{\rm s}}{\Omega_{\rm sp}}\right)\right\} \\ \times \int_0^\infty (\sigma_{\rm p}^2 + P_{\rm p} v) \exp\left\{\left(\frac{\Omega_{\rm sp}\gamma_{\rm T}}{\Omega_{\rm p}\Omega_{\rm s}}\tilde{y} - \frac{1}{\Omega_{\rm ps}}\right) v\right\} \\ \times \Gamma\left(0, \frac{\Omega_{\rm sp}\gamma_{\rm T} P_{\rm m} + P_{\rm p}\Omega_{\rm p}}{P_{\rm p} P_{\rm m}\Omega_{\rm p}\Omega_{\rm s}}(\sigma_{\rm p}^2 + P_{\rm p} v)\tilde{y}\right)\, dv.\end{aligned} \tag{11}$$

To the best of the authors' knowledge, the integral in (11) does not have a closed-form solution. In Section IV, the capacity cdf results are obtained by numerical integration.

In order to obtain the expression for mean capacity, we can derive the pdf, $f_{\gamma_{\rm I}}(x)$, by differentiating (11) with respect to $\tilde{y}$. Alternatively, using (6) we have

$$f_{\gamma_{\rm I}}(\tilde{y}) = \int_0^\infty (\sigma_{\rm p}^2 + P_{\rm p} v) f_\gamma(\tilde{y}(\sigma_{\rm p}^2 + P_{\rm p} v)) \frac{e^{-v/\Omega_{\rm ps}}}{\Omega_{\rm ps}}\, dv, \tag{12}$$

where $f_\gamma(x)$ was computed in [16] as,

$$\begin{aligned}f_\gamma(x) = e^{-\frac{\gamma_{\rm T}\sigma_{\rm p}^2}{P_{\rm p}\Omega_{\rm p}}} \Bigg[\left(\frac{1}{P_{\rm m}\Omega_{\rm s}} - \frac{\Omega_{\rm sp}\gamma_{\rm T}}{P_{\rm p}\Omega_{\rm p}\Omega_{\rm s}}\right) e^{-\frac{x}{P_{\rm m}\Omega_{\rm s}}} \\ + e^{\frac{\Omega_{\rm sp}\gamma_{\rm T} x}{P_{\rm p}\Omega_{\rm p}\Omega_{\rm s}}} \left(\frac{(\Omega_{\rm sp}\gamma_{\rm T})^2 x}{(P_{\rm p}\Omega_{\rm p}\Omega_{\rm s})^2} + \frac{\Omega_{\rm sp}\gamma_{\rm T}}{P_{\rm p}\Omega_{\rm p}\Omega_{\rm s}}\right) \\ \times \Gamma\left(0, \frac{\Omega_{\rm sp}\gamma_{\rm T} x}{P_{\rm p}\Omega_{\rm p}\Omega_{\rm s}} + \frac{x}{P_{\rm m}\Omega_{\rm s}}\right)\Bigg].\end{aligned} \tag{13}$$

The expression resulting from substituting (13) into (12) cannot be written in closed-form. Thus, the mean capacity, $\bar{C}$, must be calculated numerically by substituting (13) into (12) and (4).

B. Scenario 2

In *Scenarios* 2-5, with imperfect channel knowledge, the SU cannot guarantee that (1) is satisfied. Thus, we constrain the SU to satisfy (1) with an acceptably high probability, $1 - \alpha$.

Hence, for *Scenario* 2, where the SU knows only the mean, $\Omega_{\rm sp}$, of $g_{\rm sp}$, we consider the probability of satisfying the SINR constraint with a probability of $1 - \alpha$. This gives

$$\Pr\left(\frac{P_{\rm p} g_{\rm p}}{P_{\rm s} g_{\rm sp} + \sigma_{\rm p}^2} \geq \gamma_{\rm T} \middle| g_{\rm p}, \Omega_{\rm sp}\right) = 1 - \alpha. \tag{14}$$

Rewriting in terms of the cdf of $g_{\rm sp}$, we derive $P_{\rm s}$ as

$$P_{\rm s} = -\frac{P_{\rm p} g_{\rm p} - \gamma_{\rm T}\sigma_{\rm p}^2}{\ln(\alpha)\gamma_{\rm T}\Omega_{\rm sp}}. \tag{15}$$

Using (15), the complementary cdf of γ can be shown to be

$$\Pr(\gamma > x) = \mathbb{E}\left[\Pr\left(P_{\rm m} g_{\rm s} > x, P_{\rm s} g_{\rm s} > x \middle| g_{\rm p}\right)\right], \tag{16}$$

which can be expressed as

$$\begin{aligned}\Pr(\gamma > x) = \int_{\psi_0}^{\psi} \Pr\left(g_{\rm s} > \frac{x}{P_{\rm s}}\right) f_{g_{\rm p}}(y)\, dy \\ + \int_{\psi}^{\infty} \Pr\left(g_{\rm s} > \frac{x}{P_{\rm m}}\right) f_{g_{\rm p}}(y)\, dy,\end{aligned} \tag{17}$$

where $\psi_0 = \frac{\gamma_{\rm T}\sigma_{\rm p}^2}{P_{\rm p}}$ and $\psi = \frac{\gamma_{\rm T}(\sigma_{\rm p}^2 - P_{\rm m}\ln(\alpha)\Omega_{\rm sp})}{P_{\rm p}}$. The lower integration limit in the first term of (17) takes into account the $P_{\rm t} = 0$ condition in (2). After some manipulation, we can simplify (17) to obtain $F_\gamma(x) = 1 - \Pr(\gamma > x)$ as,

$$\begin{aligned}F_\gamma(x) = 1 - \exp\left\{-\frac{x}{P_{\rm m}\Omega_{\rm s}} - \frac{\psi}{\Omega_{\rm p}}\right\} \\ - \frac{1}{\Omega_{\rm p}} \int_{\psi_0}^{\psi} e^{-\frac{\ln(\alpha)\gamma_{\rm T}\Omega_{\rm sp} x}{(\gamma_{\rm T}\sigma_{\rm p}^2 - P_{\rm p} y)\Omega_{\rm s}}} e^{-\frac{y}{\Omega_{\rm p}}}\, dy.\end{aligned} \tag{18}$$

Once again, there exists no closed-form solution to the integral in (18). Following the same approach as in *Scenario 1*, we use

(18) and (6) to find $F_{\gamma_{\rm I}}(\tilde{y})$ as

$$\begin{aligned} F_{\gamma_{\rm I}}(\tilde{y}) = & 1 - \frac{P_{\rm m}\Omega_{\rm s} e^{-\left(\frac{\tilde{y}\sigma_{\rm p}^2}{P_{\rm m}\Omega_{\rm s}}+\frac{\psi}{\Omega_{\rm p}}\right)}}{\Omega_{\rm ps}P_{\rm p}\tilde{y}+P_{\rm m}\Omega_{\rm s}} \\ & + \frac{1}{\Omega_{\rm s}}\int_{\psi_0}^{\psi} e^{-\left(\frac{\Omega_{\rm sp}\gamma_{\rm T}\sigma_{\rm p}^2 \ln \alpha\tilde{y}}{\gamma_{\rm T}\sigma_{\rm p}^2\Omega_{\rm s}-P_{\rm p}\Omega_{\rm s}z}+\frac{z}{\Omega_{\rm s}}\right)} \\ & \times \frac{\gamma_{\rm T}\sigma_{\rm p}^2\Omega_{\rm s}-P_{\rm p}\Omega_{\rm s}z}{\gamma_{\rm T}\sigma_{\rm p}^2\Omega_{\rm s}+\Omega_{\rm sp}\gamma_{\rm T}P_{\rm p}\Omega_{\rm ps}\ln(\alpha)\tilde{y}-P_{\rm p}\Omega_{\rm s}z}\ dz. \end{aligned} \tag{19}$$

Here, again, the capacity cdf is obtained using (5) and numerically integrating (19).

To compute the SU mean capacity, we differentiate (19) with respect to $\tilde{y}$ to find the pdf

$$\begin{aligned} f_{\gamma_{\rm I}}(x) = & -\sigma_{\rm p}^2 e^{-\left(\frac{x\sigma_{\rm p}^2}{P_{\rm m}\Omega_{\rm s}}+\frac{\psi}{\Omega_{\rm p}}\right)} \\ & + \frac{\Omega_{\rm sp}\gamma_{\rm T}\sigma_{\rm p}^2\ln(\alpha)}{\Omega_{\rm s}}\int_{\psi_0}^{\psi} e^{-\left(\frac{\Omega_{\rm sp}\gamma_{\rm T}\sigma_{\rm p}^2\ln(\alpha)x}{\gamma_{\rm T}\sigma_{\rm p}^2\Omega_{\rm s}-P_{\rm p}\Omega_{\rm s}z}+\frac{z}{\Omega_{\rm s}}\right)} \\ & \times (\gamma_{\rm T}\sigma_{\rm p}^2\Omega_{\rm s}+\Omega_{\rm sp}\gamma_{\rm T}P_{\rm p}\Omega_{\rm ps}\ln(\alpha)x-P_{\rm p}\Omega_{\rm s}z)^{-2} \\ & \times \big((\gamma_{\rm T}\sigma_{\rm p}^2\Omega_{\rm s}-P_{\rm p}\Omega_{\rm s}z)(\Omega_{\rm sp}\gamma_{\rm T}P_{\rm p}\Omega_{\rm ps}\ln\alpha-1) \\ & + \Omega_{\rm sp}\gamma_{\rm T}P_{\rm p}\Omega_{\rm ps}\ln(\alpha)x\big)\ \ dz. \end{aligned} \tag{20}$$

The mean capacity is then computed by substituting (20) into (4) and numerically integrating.

C. Scenario 3

In *Scenario* 3, where the SU has exact knowledge of $g_{\rm sp}$ and knows only the mean $\Omega_{\rm p}$, we once again satisfy the SINR constraint with a probability of $1-\alpha$. Hence,

$$\Pr\left(\frac{P_{\rm p}g_{\rm p}}{P_{\rm s}g_{\rm sp}+\sigma_{\rm p}^2} \geq \gamma_{\rm T}\middle|\Omega_{\rm p}, g_{\rm sp}\right) = 1-\alpha. \tag{21}$$

Following the same approach as for *Scenario* 2 , one can show that

$$P_{\rm s} = -\left(\frac{\ln(1-\alpha)P_{\rm p}\Omega_{\rm p}}{\gamma_{\rm T}}+\sigma_{\rm p}^2\right)\frac{1}{g_{\rm sp}}. \tag{22}$$

From (22), the SU SINR cdf, $F_{\gamma_{\rm I}}(\tilde{y})$, can be derived as (see [17, Appendix A] for details)

$$F_{\gamma_I}(\tilde{y}) = 1 - s(\tilde{y}) - h(\tilde{y})\mathrm{E}_1(r(\tilde{y})), \tag{23}$$

where $s(\tilde{y})$, $h(\tilde{y})$ and $r(\tilde{y})$ are given by

$$\begin{aligned} s(\tilde{y}) &= \frac{K_1 e^{-b\tilde{y}}}{1+a\tilde{y}}, \quad h(\tilde{y}) = \frac{K_2 e^{-b\tilde{y}+r(\tilde{y})}}{\tilde{y}}, \\ r(\tilde{y}) &= \frac{(P_{\rm p}\Omega_{\rm ps}\tilde{y}+P_{\rm m}\Omega_{\rm s})(\sigma_{\rm s}^2\Omega_{\rm sp}\tilde{y}+Q\Omega_{\rm s})}{P_{\rm m}P_{\rm p}\Omega_{\rm s}\Omega_{\rm ps}\Omega_{\rm sp}\tilde{y}}, \end{aligned} \tag{24}$$

with constants, $K_1 = 1 - e^{Q/P_{\rm m}\Omega_{\rm sp}}$, $K_2 = \frac{Q\Omega_{\rm s}e^{Q/P_{\rm m}\Omega_{\rm sp}}}{P_{\rm p}\Omega_{\rm ps}\Omega_{\rm sp}}$, $a = \frac{Pp\Omega_{\rm ps}}{P_{\rm m}\Omega_{\rm s}}$ and $b = \sigma_{\rm s}^2/P_{\rm m}\Omega_{\rm s}$. Hence,

$$f_{\gamma_I}(\tilde{y}) = -s'(\tilde{y}) - h'(\tilde{y})\mathrm{E}_1(r(\tilde{y})) + h(\tilde{y})r'(\tilde{y})\frac{e^{-r(\tilde{y})}}{r(\tilde{y})}, \tag{25}$$

where $s'(\tilde{y})$, $h'(\tilde{y})$ and $r'(\tilde{y})$ are the derivatives of (24).

D. Scenario 4

When the SU-Tx has knowledge of only $\Omega_{\rm p}$ and $\Omega_{\rm sp}$, then we have

$$\Pr\left(\frac{P_{\rm p}g_{\rm p}}{P_{\rm s}g_{\rm sp}+\sigma_{\rm p}^2} \geq \gamma_{\rm T}\middle|\Omega_{\rm p}, \Omega_{\rm sp}\right) = 1-\alpha. \tag{26}$$

Using conditioning and after some manipulation, $P_{\rm s}$ can be derived as

$$P_{\rm s} = \frac{P_{\rm p}\Omega_{\rm p}}{\gamma_{\rm T}\Omega_{\rm sp}}\left(\frac{e^{-\frac{\gamma_{\rm T}\sigma_{\rm p}^2}{P_{\rm p}\Omega_{\rm p}}}}{1-\alpha}-1\right). \tag{27}$$

Here, $P_{\rm s}$ and $P_{\rm t}$ are deterministic, depending on the system parameters. The latter is given by

$$P_{\rm t} \;=\; \begin{cases} 0 & P_{\rm s} < 0 \\ P_{\rm s} & 0 < P_{\rm s} < P_{\rm m} \\ P_{\rm m} & P_{\rm s} > P_{\rm m}. \end{cases} \tag{28}$$

Using (28), we obtain the cdf of $\gamma = P_{\rm t}g_{\rm s}$, which, when substituted into (6) and (5), results in

$$F_C(y) = 1 - \frac{P_{\rm t}\Omega_{\rm s}}{\tilde{y}P_{\rm p}\Omega_{\rm ps}+P_{\rm t}\Omega_{\rm s}}e^{-\frac{\tilde{y}\sigma_{\rm s}^2}{P_{\rm t}\Omega_{\rm s}}}. \tag{29}$$

In order to compute the mean capacity, $\bar{C}$, we note that $F_{\gamma_I}(x)$ can be trivially derived from (29) and (5). Differentiating to obtain $f_{\gamma_I}(x)$ and substituting into (4), one obtains

$$\begin{aligned} \bar{C} = & \frac{1}{\ln(2)}\int_1^{\infty}\left(\frac{\sigma_{\rm s}^2}{P_{\rm p}\Omega_{\rm ps}t+P_{\rm t}\Omega_{\rm s}}+\frac{P_{\rm t}P_{\rm p}\Omega_{\rm s}\Omega_{\rm ps}}{(P_{\rm p}\Omega_{\rm ps}t+P_{\rm t}\Omega_{\rm s})^2}\right) \\ & \times \ln(t)e^{\frac{-t\sigma_{\rm s}^2}{P_{\rm t}\Omega_{\rm s}}}\ dt, \end{aligned} \tag{30}$$

where we have used the change of variable, $t = 1 + x$.

E. Scenario 5

Here, the SU-Tx operates on estimates of $g_{\rm p}$ and $g_{\rm sp}$. In such a case, we aim to satisfy:

$$\Pr\left(P_{\rm p}g_{\rm p} \geq \gamma_{\rm T}P_{\rm s}g_{\rm sp}+\gamma_{\rm T}\sigma_{\rm s}^2\middle|\hat{g}_{\rm p}, \hat{g}_{\rm sp}\right) = 1-\alpha, \tag{31}$$

which must be solved for $P_{\rm s}$. We use the classic model for imperfect CSI [9] given by $\hat{h} = \rho h + \sqrt{1-\rho^2}\epsilon$, where h is a generic channel coefficient, ρ controls the level of CSI, ϵ is statistically identical to h and $\hat{g} = |\hat{h}|^2$. The complexity of (31) makes further capacity analysis infeasible. Instead, (31) is derived in [17, Appendix B] and is shown to be equivalent to

$$\begin{aligned} & \sum_{j=0}^{\infty}\frac{(\lambda_1/2)^j}{j!}e^{-\lambda_1/2}\left(1-e^{-\frac{\lambda_2+\beta}{2}}e^{\frac{\lambda_2}{4(\delta+1)}}\sqrt{\frac{8}{\lambda_2(\delta+1)}}\right. \\ & \left.\times\sum_{r=0}^{j}\sum_{s=0}^{r}\left(\frac{\beta}{2}\right)^r\frac{\left(\frac{2\delta}{\beta(\delta+1)}\right)^s}{(r-s)!}M_{-s-1/2,0}\left(\frac{\lambda_2}{2(\delta+1)}\right)\right) = \alpha, \end{aligned} \tag{32}$$

where $\lambda_1 = 2\rho^2\hat{g}_{\rm p}/(\Omega_{\rm p}(1-\rho^2))$, $\lambda_2 = 2\rho^2\hat{g}_{\rm sp}/(\Omega_{\rm sp}(1-\rho^2))$, $\beta = 2\sigma_{\rm s}^2\gamma_{\rm T}/(\Omega_{\rm p}(1-\rho^2)P_{\rm p})$, $\delta = \gamma_{\rm T}P_{\rm s}\Omega_{\rm sp}/(\Omega_{\rm p}P_{\rm p})$ and $M_{\mu,\nu}(\cdot)$ is a Whittaker function. $P_{\rm s}$ is then computed using a numerical root finder to solve (32) and the resulting value is used in capacity simulations.

F. SU Blocking

Using the results in Sections III-A - III-D, we derive the SU blocking conditions, that is the probability or condition under which the SU cannot transmit due to the constraint (1).

In the case of *Scenarios* 1 and 2, where P_{s} is dependent on the instantaneous value of g_{p}, via (7) and (15), respectively, we can compute the probability of SU blocking, by solving for $\Pr(P_{\mathrm{t}} \leq 0)$ or equivalently $\Pr(P_{\mathrm{s}} \leq 0)$. It is easily shown that for *Scenarios* 1 and 2

$$\Pr(P_{\mathrm{t}} \leq 0) = 1 - e^{-\frac{\gamma_{\mathrm{T}}\sigma_{\mathrm{p}}^2}{\Omega_{\mathrm{p}}P_{\mathrm{p}}}} = 1 - e^{-c_2}. \tag{33}$$

For *Scenarios* 3 and 4, the SU blocking condition is determined purely from the system parameters, and can be obtained by setting $P_{\mathrm{s}} \leq 0$ in (22) and (27), respectively. Here, the SU blocking condition is related to α and c_2 by

$$P_{\mathrm{t}} = 0 \quad \text{if} \quad \alpha \leq 1 - e^{-\frac{\gamma_{\mathrm{T}}\Omega_{\mathrm{p}}}{\sigma_{\mathrm{p}}^2 P_{\mathrm{p}}}} = 1 - e^{-c_2}. \tag{34}$$

Using (34), we note that for small values of α, that is where we guarantee the PU SINR constraint with high probability, the SU blocking condition is approximated by $\alpha \leq c_2$.

For *Scenario* 5, blocking occurs when (31) can not be satisfied, even for $P_{\mathrm{s}} = 0$. Hence, the boundary of the blocking region is equivalent to

$$\Pr\left\{\Pr\left(g_{\mathrm{p}} \geq \frac{\gamma_{\mathrm{T}}\sigma_{\mathrm{p}}^2}{P_{\mathrm{p}}}\middle|\hat{g}_{\mathrm{p}}\right) = 1 - \alpha\right\}. \tag{35}$$

In [17], the probability in (35) is rewritten as

$$\Pr\left(X \geq \frac{2c_2}{1-\rho^2}\middle|\hat{g}_{\mathrm{p}}\right) = 1 - \alpha, \tag{36}$$

where X is a non-central Chi-squared variable with 2 degrees of freedom and non-centrality parameter, $2\rho^2\hat{g}_{\mathrm{p}}/(\Omega_{\mathrm{p}}(1-\rho^2))$. We solve (36) by a simple root-finder, to find the threshold value, $\hat{g}_{\mathrm{p}} = g^*$, which satisfies (36). Then, the blocking probability is simply

$$\Pr\left(\hat{g}_{\mathrm{p}} < g^*\right) = 1 - e^{-g^*/\Omega_{\mathrm{p}}}. \tag{37}$$

IV. Simulation Results and Discussion

In all simulations, we have set $P_{\mathrm{p}}/\sigma_{\mathrm{p}}^2 = P_{\mathrm{m}}/\sigma_{\mathrm{s}}^2 = 0$ dB and $\Omega_{\mathrm{p}}/\sigma_{\mathrm{p}}^2 = \Omega_{\mathrm{s}}/\sigma_{\mathrm{s}}^2 = 5$ dB, where we assume $\sigma_{\mathrm{p}}^2 = \sigma_{\mathrm{s}}^2$. In *Scenarios* 2-5 we set $\alpha = 0.1$, and $\rho = 0.9$ is used in *Scenario* 5, unless otherwise indicated in the figures. Figures 2, 3 and 5 show the SU capacity cdfs for various scenarios and a range of c_1, c_2 values. Figure 5, with $c_1 = 0.01$, represents very favourable SU operating conditions. Figure 3 ($c_1 = 0.1$, $c_1 = 0.9$) represents increasingly difficult conditions for the SU.

From these results, we observe that *Scenarios* 1 and 2 result in similar performance, even in the case of $c_1 = 0.9$ (Fig. 3), that is where the SU interference is very prominent. Furthermore, lack of knowledge of the PU-PU link (knowing only the mean Ω_{p}) greatly reduces the achievable capacity of the SU. This is shown in Figs. 2 and 3 where *Scenarios* 3 and 4 suffer a considerable loss in comparison to *Scenarios* 1 and 2. Hence, knowledge of g_{p} is more important than g_{sp}.

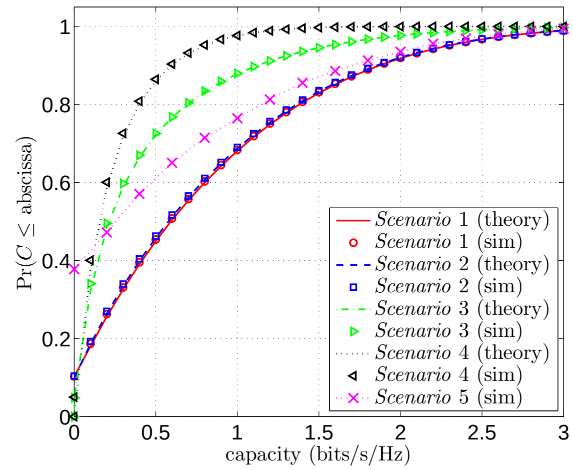

Fig. 2. SU capacity cdf for *Scenarios* 1-5 ($c_1 = c_2 = 0.1$).

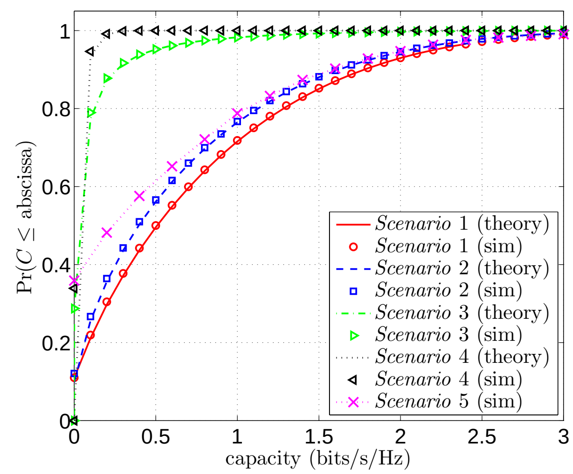

Fig. 3. SU capacity cdf for *Scenarios* 1-5 ($c_1 = 0.9, c_2 = 0.1$).

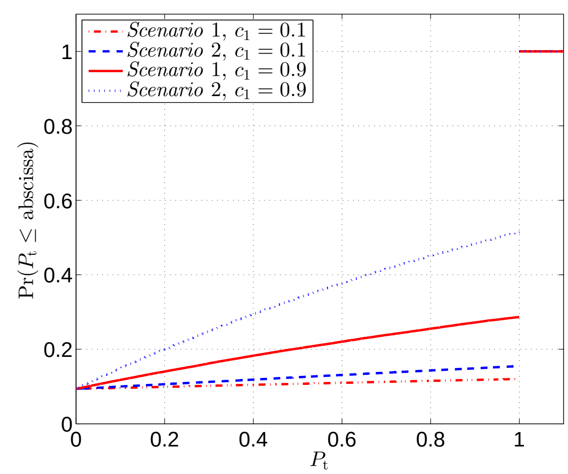

Fig. 4. SU transmit power cdf for *Scenarios* 1 and 2 ($c_1 = 0.1$ and $c_1 = 0.9$).

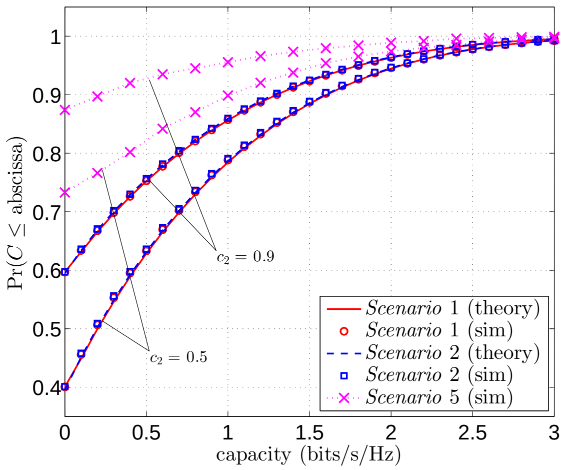

Fig. 5. SU capacity cdf for *Scenarios* 1, 2 and 5 ($c_1 = 0.01$; $c_2 = 0.5$, $c_2 = 0.9$)

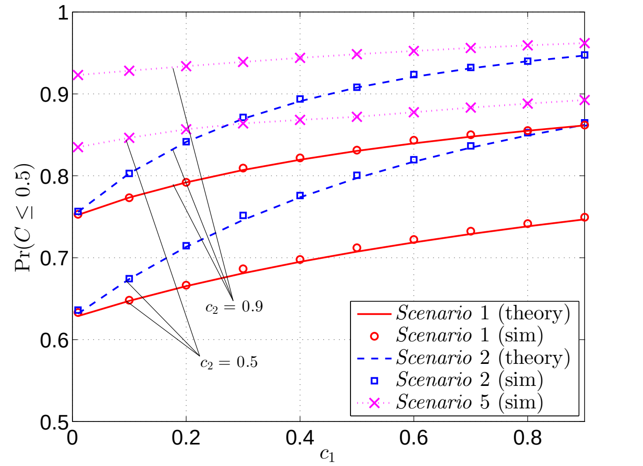

Fig. 6. SU capacity outage values for *Scenarios* 1, 2 and 5 vs c_1 ($c_2 = 0.5$, $c_2 = 0.9$).

The dependence on c_1 can be observed by comparing Figs. 2 and 3. Under very favourable conditions, $c_1 = 0.01$, *Scenarios* 3 and 4 slightly outperform *Scenarios* 1 and 2. This seemingly counterintuitive result is due to the flexibility afforded by the probabilistic SINR constraint. This is confirmed by the additional cdfs for *Scenarios* 3 and 4 in Fig. 3, with $\alpha = 0.096$, where the protection of PU SINR with higher degree of certainty causes degradation of performance for *Scenarios* 3 and 4 below that for *Scenarios* 1 and 2.

From Fig. 3, we observe that placing the SU in a demanding environment, $c_1 = 0.9$, results in very poor performance under *Scenarios* 3 and 4. Furthermore, the performance of *Scenario* 2 is noticeably degraded from that of *Scenario* 1. Further insight into this is provided by Fig. 4, which shows the cdf of the SU transmit power, P_{t}, for $c_1 = 0.1$ and $c_1 = 0.9$. We observe that in the latter case, the SU-Tx under *Scenario* 1 operates at maximum power, $P_{\mathrm{t}} = 1$, with a likelihood of 70 %, compared to approximately 50 % for *Scenario* 2. This difference is much less pronounced for the less challenging case of $c_1 = 0.1$. Finally, based on Figs. 2 and 3, we observe that the performance under *Scenario* 5 is not highly dependent on the value of c_1.

Comparing the curves for *Scenarios* 3 and 4 with those for *Scenario* 5 in Fig. 3, we note that, for the most part, imperfect knowledge of the link gains is more beneficial than a knowledge of their means. Only in the low capacity regime we observe that *Scenarios* 3 and 4 outperform *Scenario* 5, which has a relatively high blocking probability for the parameters considered. It should be noted, however, that blocking in *Scenarios* 3 and 4 is dictated by the parameter c_2 and thus, unless (34) is satisfied, the capacity cdfs for these scenarios originate at zero. Consequently, at higher capacity values there exists a crossover point with *Scenario* 5.

Figures 2 and 3 compare the scenarios using $c_2 = 0.1$, which is very generous to the SU. From (34), we see that SU transmission in *Scenarios* 3 and 4 occurs only for large values of α or for small values of c_2. That is, without the knowledge of g_{p}, the SU can only operate if the PU is willing to accept

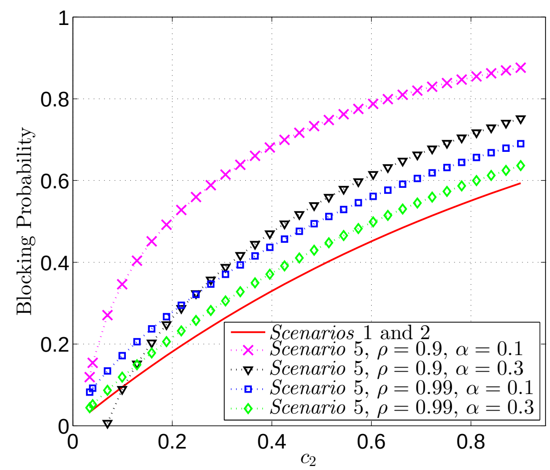

Fig. 7. Blocking probability for *Scenarios* 1, 2 and 5 for vs c_2.

large amounts of interference. Figure 5 presents the capacity results for *Scenarios* 1 and 2 with the more realistic values of $c_2 = 0.5$ and $c_2 = 0.9$, where (34) prevents SU transmission under *Scenarios* 3 and 4. While SU transmission is possible under *Scenario* 5, we observe a high blocking probability of 0.73 and 0.88 for $c_2 = 0.5$ and $c_2 = 0.9$, respectively.

Figure 6 shows the probability $\Pr(C \leq 0.5)$ as a function of c_1. As expected, for a constant c_2, the performance under *Scenario* 2 diverges from the baseline *Scenario* 1 with increasing c_1, that is as the amount of interference to the PU increases.

Finally, Fig. 7 shows the blocking probability for *Scenarios* 1, 2 and 5. We recall that the SU ability to transmit in *Scenarios* 3 and 4 is deterministic and governed by the blocking condition of (34). The results for *Scenario* 5 were obtained numerically via (36). We observe that as the channel knowledge error decreases ($\rho \to 1$), the blocking probability approaches that of *Scenarios* 1 and 2. Specifically, referring back to Fig. 5, we note from Fig. 7 where $\rho = 0.9$ that improving the channel estimate to $\rho = 0.99$ will reduce the blocking probability at $c_2 = 0.5$ and $c_2 = 0.9$ to 0.5 and 0.7, respectively, thus bringing the performance of *Scenario* 5 closer to that of *Scenario* 1. Similarly, relaxing the probabilistic SINR constraint by increasing α to 0.3 results in a significant reduction in blocking probability, as fully expected.

V. Conclusions

We have examined the effects of limited channel knowledge on the SU capacity. Considering five scenarios, we derived (in four cases) analytical expressions for the SU capacity cdf under a PU-Rx SINR constraint. We determined the SU blocking probability and blocking conditions as a function of the allowable interference at the PU-Rx. The results demonstrate the importance of the PU-PU CSI, which was shown to be much greater than that of the SU-Tx to PU-Rx link. Furthermore, we have shown that in challenging situations or in the presence of CSI error there can be extremely large blocking probabilities for the SUs.

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