DESIGN OF TIMING ERROR DETECTORS FOR ORTHOGONAL SPACE-TIME BLOCK CODES

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ABSTRACT

We present a method for the design of low complexity timing error detectors in orthogonal space-time block coding (OSTBC) receivers. A general expression for the S-curve of timing error detectors is derived. Based on this result, we obtain sufficient conditions for a difference of threshold crossings timing estimate that is robust to channel fading. A number of timing error detectors for 3- and 4-transmit antenna codes are presented. The performance is evaluated by examining their tracking capabilities within a timing loop of an OSTBC receiver. Symbol-error-rate results are presented showing negligible loss due to timing synchronization. In addition, we study the performance as a function of the timing drift and show that the receiver is able to track up to the normalized timing drift bandwidth of 0.001.

I INTRODUCTION

As in conventional single-antenna communications, the estimation of reference parameters, such as timing epoch and channel fading samples, is critical to the performance of multiple-input multiple output (MIMO) receivers. Timing acquisition in space-time coded modems was first addressed in [1], where the receiver obtains timing information by maximizing the oversampled log-likelihood function (LLF) derived from a training sequence. A modification of the method in [1] was presented in [2] where the resulting algorithm reduces the oversampling required.

While the methods described deal with training-based timing acquisition, this paper focuses on the problem of low complexity timing tracking without the knowledge of data at the receiver. In [3] it was shown that a Mueller-and-Muller [4] (MMD)-like timing error detector (TED) can be implemented for 2-transmit antenna, pulse-shaped OSTBC. This paper provides a general framework for the design of TED for OSTBC with arbitrary number of transmit and receive antennas. We derive sufficient conditions for detectors robust to channel fading. Using these conditions we present TED’s for 3- and 4-transmit antenna codes, and evaluate their performance by examining their tracking properties within a timing loop of an OSTBC receiver.

The remainder of this paper is organized as follows. We begin with a system overview in Section II. The timing error detector design is covered in Section III. Timing correction is briefly outlined in Section IV. Simulation results are presented in Section V, where symbol-error-rate (SER) and timing bandwidth range performance is described. We conclude with a summary of findings in Section VI.

II SYSTEM OVERVIEW

We consider a communication system comprising of $N_t$ transmit and $N_r$ receive antennas employing orthogonal space-time block coding. The transmitter encodes $N_s$ information symbols and transmits them over $N_t$ antennas in $N_r$ time slots, resulting in a code rate of $R = N_s / N_c$. We denote the $i$th $N_t \times N_c$ code block by $X_i$ and its $(ik)$th entry by $x_i(lN_c + k)$. Note that $l$ is the code block index, $k = 0, \ldots, N_c - 1$ is the time slot index within the block and $i = 1, \ldots, N_t$ is the transmit antenna index. Let the $m$th information symbol used to encode block $X_i$ be $a_i^m$, where $m = 0, \ldots, N_s - 1$. Then, $X_i$ is given by the linear combination of $a_i^m$ and their conjugates [5]

$$X_i = \sum_{m=0}^{N_s-1} \mathcal{R} \{a_i^m\} A_m + i \mathcal{I} \{a_i^m\} B_m, \quad (1)$$

where the operators $\mathcal{R} \{\cdot\}$ and $\mathcal{I} \{\cdot\}$ return the real and imaginary parts of their arguments, respectively, and $A_m$ and $B_m$ are real integer code matrices of dimension $N_t \times N_c$. The pulse shaping is split between the transmitter and the receiver, each using a root raised cosine (RRC) filter denoted by $\tilde{g}(t)$. The combined Nyquist raised cosine pulse is represented by $g(t) = \tilde{g}(t) \ast \tilde{g}(t)$, where $\ast$ denotes convolution. We assume a frequency-flat Rayleigh fading channel modeled by a $N_c \times N_t$ matrix $H$. It’s components, denoted by $h_{ij}$, correspond to the state of the fading channel from $i$th transmit to $j$th receive antenna and are assumed to be independent and identically distributed (iid) for all $i$ and $j$ with a $U$-shaped power spectrum of isotropic scattering and maximum Doppler frequency of $f_D$.

The receiver diagram is given in Figure 1. The received signal at antenna $j$ is given by

$$r_j(t) = \sum_{i=1}^{N_t} h_{ij}(t) \sum_{n'} x_i(n') \tilde{g}(t - n'T) + \eta_j(t), \quad (2)$$

Figure 1: Receiver Diagram.
where \( x_i(n') \) is the encoded symbol transmitted by antenna \( i \) at time \( n' = lN_c + s \) and \( \hat{y}_i(t) \) is a zero mean complex Gaussian noise with variance \( \sigma_n^2 = N_0/2 \) per signal dimension. After matched filtering, the signal \( y_i(t) = r_i(t) * \hat{g}(t) \) is sampled at time instants \( t_n = nT + \epsilon \), where \( \epsilon \) is the unknown timing error, assumed to be equal on all branches. We express \( \epsilon \) by \( \epsilon = \tau - \hat{\tau} \) where \( \tau \) is the timing offset at the receiver and \( \hat{\tau} \) is the timing correction applied by the timing synchronization algorithm. Assuming the channel fading is sufficiently slow, such that \( h_{ji}(t_n) \approx h_{ji}(nT) \pm h_{ji}(n) \), the resulting samples are given by

\[
y_j(n) = \sum_{i=1}^{N_i} h_{ji}(n) \sum_{n'} x_i(n') g(nT - n'T + \epsilon) + \eta_j(n),
\]

where \( \eta_j(n) \) denotes the samples of the matched filtered noise \( \eta_j(t) = \hat{y}_i(t) * \hat{g}(t) \), which are uncorrelated if sampled at symbol rate. We can re-write (3) as

\[
y_j(n) = \sum_{i=1}^{N_i} h_{ji}(n)x_i(n) + \eta_j(n),
\]

where \( x_i(n) \) are the intersymbol interference (ISI)-equivalent encoded symbols giving a sampling error \( \epsilon \),

\[
x_i(n) = \sum_{n'} x_i(n') g(nT - n'T + \epsilon).
\]

Consider the samples for \( n = lN_c, \ldots, (l+1)N_c - 1 \), corresponding to time slots \( k = 0, \ldots, N_c - 1 \) within a single code block. We assume that the timing error \( \epsilon \) is constant for the duration of one code block. Similarly, we assume quasi-static fading, where \( h_{ji}(lN_c) \approx h_{ji}(l + 1)N_c - 1 \approx h_{ji}(l) \). For block \( l \), the output samples in (4) can be expressed by a \( N_c \times N_c \) matrix \( Y_l \)

\[
Y_l = H_lX_{\epsilon,l} + N_l,
\]

where \( H_l \) and \( N_l \) denote the channel state and noise matrices, respectively. The quantity \( X_{\epsilon,l} \) denotes the \( N_c \times N_c \) matrix of symbols \( x_{i,n}(n) \). Using (5), we express \( X_{\epsilon,l} \) as

\[
X_{\epsilon,l} = \sum_n X_{\epsilon,n} G_{\epsilon,n},
\]

where \( G_{\epsilon,n} \) is a \( N_c \times N_c \) Toeplitz matrix given by

\[
G_{\epsilon,n} = \begin{bmatrix}
g_{-n,N_c} & g_{-n,N_c+1} & \cdots & g_{-n,N_c+2} 
g_{-n,N_c-1} & g_{-n,N_c} & \cdots & g_{-n,N_c+1} 
g_{-n,N_c-2} & g_{-n,N_c-1} & \cdots & g_{-n,N_c} 
\vdots & \vdots & \ddots & \vdots 
g_{-n,N_c+1} & g_{-n,N_c+2} & \cdots & g_{-n,N_c+3}
\end{bmatrix},
\]

where we denote the pulse shape samples by \( g_{n,k} \).

Finally, the detection variables for each information symbol \( m = 0, \ldots, N_s - 1 \) within block \( l \) are given by [5]

\[
s_{m} = ||H_l||^{-2} \left[ \Re\{ tr(\bar{Y}_l^H H_l A_m) \} - i \Im\{ tr(\bar{Y}_l^H H_l B_m) \} \right],
\]

where \( tr(\cdot) \) denotes the trace operator, superscript \( H \) is the Hermitian transpose operator and \( ||H|| \) is the Frobenius norm of \( H_l \). We note that while, strictly speaking, (9) represents Maximum Likelihood (ML) detection when no timing error is present, we assume the above expression is an approximation to ML detection and verify this assumption via simulations. The projection of \( s_{m} \) onto the signal constellation then forms the data decisions denoted by \( \hat{d}_m \).

III TIMING ERROR DETECTOR DESIGN

In [3], it was shown that for a 2-transmit antenna OSTBC, a TED in the form of \( \hat{\epsilon} = a_0s_1 - a_1s_0 \) results in an S-curve

\[
E\{ \hat{\epsilon} \} = g_{\epsilon,1}^2 - g_{\epsilon}^2.
\]

We will refer to \( C(g_{\epsilon,1}^2 - g_{\epsilon}^2) \), where \( C \) is a constant, as the Timing Error Measurement (TEM). We point out that for the TED in [3], the TEM is independent of the channel state, thus giving robustness in poor channel conditions. We will define a TED whose TEM is independent of the channel fading process as a robust TED. As will be shown, for higher order OSTBC, \( E\{ \hat{\epsilon} \} \) is composed of a dominant TEM term and a bias, which is rational and has a numerator that is dependent on the products of fading variables. It will be shown that the numerator of the bias will average out to zero in the iterative operation of the timing loop. Such TED’s will be referred to as quasi-robust.

A. Conditions for TED Robustness

In deriving TED’s for higher order OSTBC, we consider a general expression in the form a linear combination of products \( a_{m,n}^s s_{m,n} \) and \( a_{m,n}^r s_{m,n} \), that is

\[
\hat{\epsilon}_l = \Re\left( \sum_k \alpha_k a_{m,k}^l s_{m,k}^l + \beta_k a_{m,k}^r s_{m,k}^r \right),
\]

where \( \{ m_{\alpha,k}, n_{\alpha,k}, m_{\beta,k}, n_{\beta,k} \} \) are the indices of the data symbols and decision variables within a block \( l \) to be chosen. In (10) we ignore the imaginary component of the estimator. We evaluate \( E\{ \hat{\epsilon}_l \} \), where the expectation is taken over the data and the noise, for the general form of the estimator in (10). In solving for \( E\{ \hat{\epsilon}_l \} \), we evaluate the individual components of the summation in (10), specifically \( E\{ a_{m,n}^s s_{m,n} \} \) and \( E\{ a_{m,n}^r s_{m,n} \} \). The derivation is presented in the Appendix. Assuming the data, AWGN and the channel fading are independent from each other, one obtains

\[
E\{ a_{m,n} s_{m} \} = \rho ||H||^{-2} \times
\]

\[
\left\{ \begin{array}{c}
\Re\{ \text{tr}\left( A_m G_l^H A_l^H - B_m G_l^H B_l^H \right) \} \Re\{ H_l^H H_l \} \\
+j \Im\{ \text{tr}\left( A_m G_l^H B_l^H - B_m G_l^H A_l^H \right) \} \Im\{ H_l^H H_l \}
\end{array} \right\}
\]

and

\[
E\{ a_{m,n} r_{m} \} = \rho ||H||^{-2} \times
\]

\[
\left\{ \begin{array}{c}
\Re\{ \text{tr}\left( A_m G_l^H A_l^H + B_m G_l^H B_l^H \right) \} \Re\{ H_l^H H_l \} \\
-j \Im\{ \text{tr}\left( A_m G_l^H B_l^H + B_m G_l^H A_l^H \right) \} \Im\{ H_l^H H_l \}
\end{array} \right\},
\]

1The expression \( g_{\epsilon,1}^2 - g_{\epsilon}^2 \) has been referred to in literature [4] as the difference of threshold crossings.
where \( \rho \) is a constellation-dependent constant \( \rho = E\{\text{Re}(a_i)^2\} = E\{\text{Im}(a_i)^2\} \). In (11) and (12) we have dropped the block index \( l \) and simplified \( G_{r,n} \) to \( G_n \) for \( n = 0 \). Using (11) and (12), one obtains the expectation of the TED in (10) given by

\[
E\{\hat{\epsilon}\} = \rho \|H\|^2 \text{tr } \{ \Gamma \Re (H^H H) \},
\]

where we have defined a matrix \( \Gamma \), which is dependent on the coefficient set \( \{\alpha_k, \beta_k, m_{\alpha,k}, n_{\alpha,k}, m_{\beta,k}, n_{\beta,k}\} \) chosen in (10), as

\[
\Gamma = \sum_k \left( \alpha_k (A_{m_{\alpha,k}, n_{\alpha,k}} G^H_{\alpha,n,k} - B_{m_{\alpha,k}} G^H_{\alpha,n,k}) + \beta_k (A_{m_{\beta,k}, n_{\beta,k}} G^H_{\beta,n,k} + B_{m_{\beta,k}} G^H_{n_{\beta,k}}) \right) \tag{14}
\]

Consider the case where \( \Gamma \) in (14) has the form of

\[
\Gamma = f(G_r I + D),
\]

where

1. \( f(G_r) \) is a scalar function of \( G_r \) that returns a TEM
2. \( D \) is an antisymmetric matrix.

Then, using (13)

\[
E\{\hat{\epsilon}\} = \rho \|H\|^2 \text{tr } \{ (f(G_r I + D)) \Re (H^H H) \}
= \rho f(G_r),
\]

where we used the fact that \( \Re (H^H H) \) is symmetric and that \( \text{tr } \{ AB \} = 0 \) for symmetric \( A \) and antisymmetric \( B \). Therefore, if coefficients in (10) are selected such that \( \Gamma \) satisfies conditions 1) and 2), the TED returns a valid timing error measurement that is robust to channel fading.\(^2\)

If only condition 1) is satisfied, that is \( D \) is an arbitrary matrix with zeros on the main diagonal, then

\[
E\{\hat{\epsilon}\} = \rho f(G_r) + \hat{\epsilon},
\]

where \( \hat{\epsilon} \), which is dependent on \( H \), will be referred to as the TEM bias. One can show using (16) that

\[
\hat{\epsilon} = \rho \|H\|^2 \sum_{m=1}^{N_i} \sum_{n=1}^{N_i} \sum_{j=1}^{N_c} d_{mni} \Re (h^*_j h_{jm}) ,
\]

where we used \( d_{mni} \) to denote the \((m, i)\)th entry of \( D \). We note that, for uncorrelated channels, the expectation over \( H \) of the numerator of (18) is zero, and thus the effect of the bias will be small resulting in a quasi-robust TED.

In designing a TED for a specific OSTBC, we aim to select coefficients \( \{\epsilon_k, \beta_k, m_{\epsilon,k}, n_{\epsilon,k}, m_{\beta,k}, n_{\beta,k}\} \) to produce a TEM \( f(G_r) \) of \( g_{\epsilon}^m - g_{\epsilon}^n \). In doing so we note that the integer code matrices in (14) act to shuffle the rows and columns of \( G_r^H \). Thus, the aim is to force the elements \( g_{\epsilon}^m - g_{\epsilon}^n \), located adjacent to the main diagonal of \( G_r \), to the main diagonal of \( \Gamma \) for \( k = 1 \) and \( k = 2 \) respectively. Since the composition of the code matrices \( A_{n_2}, B_{n_2} \) in (1) varies for different OSTBC’s, the procedure has to be done separately for each code.

\(^2\)In other words, the \( \text{tr}\{\cdot\} \) operator in (16) returns a full-diversity estimate of \( \epsilon \), which removes any dependence on \( H \).

B. Examples of TED’s
As proposed in [3], a TED for 2-transmit antenna OSTBC (Alamouti encoding) has the form of

\[
\hat{\epsilon} = \Re (a_0 s_1 - a_1 s_0),
\]

which corresponds to \( \alpha_1 = 1, \alpha_2 = -1, m_{\alpha,1} = n_{\alpha,2} = 1, m_{\alpha,2} = n_{\alpha,1} = 0 \) and \( \beta_1 = 0 \) for all \( k \). For this case, the matrix \( \Gamma \) in (14) can be shown to be

\[
\Gamma = 2 \begin{bmatrix} g_{\epsilon}^1 \quad 0 \\ 0 \quad g_{\epsilon}^2 \end{bmatrix}
\]

that is, \( f(G_r) = 2(g_{\epsilon}^1 - g_{\epsilon}^2) \), and \( D = 0 \). Thus, consistent with the results in [3], the S-curve of the TED in (19) is given by

\[
E\{\hat{\epsilon}\} = 2 \rho (g_{\epsilon}^1 - g_{\epsilon}^2),
\]

that is a robust timing estimate with \( \hat{\epsilon} \) = 0.

A number of \( N_r = 3 \) OSTBC encoders have been presented in literature, such as [6] (Equations 7.4.8, 7.4.9) and [7] (Equation 3.49). Consider the code in Equation 3.49 in [7], that is

\[
X = \begin{bmatrix} a_0 & a_1^* & a_2^* & 0 \\ -a_0 & a_1^* & 0 & -a_2^* \\ a_2 & 0 & a_0^* & a_1^* \end{bmatrix}
\]

Referring to (14), we note that, for \( k = 1 \), selecting \( \alpha_1 = \beta_1 = 1 \) with \( n_{\alpha,1} = n_{\beta,1} = 1 \) and \( m_{\alpha,1} = m_{\beta,1} = 0 \) will cause matrices \( A_{n_1}, B_{n_1} A_{n_2}^H \) and \( B_{n_2}^H \) to force the main diagonal of \( \Gamma \) to be \( \{2g_{\epsilon}^1, -2g_{\epsilon}^1, 2g_{\epsilon}^2\} \). Similarly, for \( k = 2 \), choosing \( \alpha_2 = \beta_2 = -1 \) with \( n_{\alpha,2} = n_{\beta,2} = 0 \) and \( m_{\alpha,2} = m_{\beta,2} = 1 \), contributes \( \{-2g_{\epsilon}^1, 2g_{\epsilon}^1, -2g_{\epsilon}^2\} \). Subtracting the \( k = 2 \) term from the \( k = 1 \) term, which is equivalent to a TED in the form of

\[
\hat{\epsilon} = \Re (a_1 s_1 - a_2 s_0 - a_0^* s_1)
= 2(\epsilon_{n_1}^0 R + \epsilon_{n_0}^0 R),
\]

where we use the superscript \( R \) to denote the real part, results in \( \Gamma \) given by

\[
\Gamma = 2 \begin{bmatrix} g_{\epsilon}^1 - g_{\epsilon}^1 & -2g_{\epsilon}^1 & 2g_{\epsilon}^3 & -g_{\epsilon}^1 \\ 2g_{\epsilon}^1 & g_{\epsilon}^1 - g_{\epsilon}^1 & 2g_{\epsilon}^2 & -g_{\epsilon}^2 \\ g_{\epsilon}^3 - 2g_{\epsilon}^1 & 2g_{\epsilon}^2 & -g_{\epsilon}^1 - g_{\epsilon}^1 \end{bmatrix},
\]

that is \( f(G_r) = 2(g_{\epsilon}^1 - g_{\epsilon}^2) \) and

\[
D = 2 \begin{bmatrix} 0 & -2g_{\epsilon}^1 & 2g_{\epsilon}^3 & -g_{\epsilon}^1 \\ 2g_{\epsilon}^1 & 0 & 2g_{\epsilon}^2 & -g_{\epsilon}^2 \\ g_{\epsilon}^3 - 2g_{\epsilon}^1 & 2g_{\epsilon}^2 & 0 & 0 \end{bmatrix}
\]

Thus, the S-curve for the TED in (22) is given by

\[
E\{\hat{\epsilon}\} = 2 \rho (g_{\epsilon}^1 - g_{\epsilon}^1) + \hat{\epsilon},
\]

where the TEM bias can be shown to be

\[
\hat{\epsilon} = \|H\|^2 \rho \sum_{j=1}^{N_r} \{2 (g_{\epsilon}^m - g_{\epsilon}^n) \Re (h^*_j h_{jm}) - (g_{\epsilon}^m - g_{\epsilon}^n - g_{\epsilon}^n) \Re (h^*_j h_{jm}) \}.
\]
As another example we give the TED for the code given by Equation 7.4.8 in [6], that is

\[
X = \begin{bmatrix}
0 & a_1 & -a_2 \\
0 & a_0 & a_2^* & a_1^* \\
-a_1^* & -a_2 & a_0 \\
0 & a_2 & a_1 \\
\end{bmatrix}. \tag{27}
\]

Using a similar approach as for the code in (22), a valid timing error estimate can be obtained via

\[
\dot{\epsilon} = R(a_{28}s_1 - a_1s_2 - a_2^*s_1 + a_1^*s_2) \\
= 2(a_1^*s_2 - a_2^*s_1), \tag{28}
\]

where the superscript \( f \) refers to the imaginary part of its argument. The S-curve for the TED is given by (25) with \( \delta \) given by (26).

Codes for 4-transmit antenna OSTBC can easily be obtained by appending an appropriate row to the \( N_t = 3 \) codes. For a 4-transmit antenna code based on (21)

\[
X = \begin{bmatrix}
a_0 & a_1^* & a_2^* & 0 \\
-a_1^* & 0 & a_2^* & a_1^* \\
-a_2 & 0 & a_0 & a_1^* \\
0 & a_2 & a_1 & a_0 \\
\end{bmatrix}, \tag{29}
\]

a timing estimate is given by the same expression as its \( N_t = 3 \) counterpart in (22). The S-curve is once again given by (25) with the TEM bias

\[
\delta_e = \|H\|^{-2}2\rho \sum_{j=1}^{N_c} [2(\epsilon^{f}_{j-2} - \epsilon^{f}_{j})R(h_{j3}^*h_{j2} - h_{j3}h_{j2}) \\
- (\epsilon^{f}_{j-1} - \epsilon^{f}_{j-3} + \epsilon^{f}_{j})R(h_{j3}h_{j1} + h_{j3}h_{j2})]. \tag{30}
\]

Similarly, in the case of a \( N_t = 4 \) code based on (27), that is

\[
X = \begin{bmatrix}
a_0 & a_1 & -a_2 \\
0 & a_0 & a_2^* & a_1^* \\
-a_1^* & -a_2 & a_0 & a_1^* \\
0 & a_2 & a_1 & a_0 \\
\end{bmatrix}, \tag{31}
\]

the same TED can be used as in (28). The TED estimate is that in (25), with the TEM bias given by

\[
\delta_e = \|H\|^{-2}2\rho \sum_{j=1}^{N_c} [2(\epsilon^{f}_{j-2} - \epsilon^{f}_{j})R(h_{j3}^*h_{j2} + h_{j3}h_{j1}) \\
- (\epsilon^{f}_{j-1} - \epsilon^{f}_{j-3} + \epsilon^{f}_{j})R(h_{j3}h_{j1} + h_{j3}h_{j2})]. \tag{32}
\]

IV TIMING ERROR CORRECTION

Having described the estimation of the timing error, we now outline the process of timing correction. To minimize the effects of noise, the timing error estimate \( \dot{\epsilon}_i \) is first passed through a first-order IIR filter

\[
\dot{\epsilon}_i'(t) = \alpha\dot{\epsilon}_i(t - \Delta t) + (1 - \alpha)\dot{\epsilon}_i(t), \tag{33}
\]

where the constant \( \alpha \) is set according to the 3 dB bandwidth required by the loop. The resulting filtered timing measurement \( \dot{\epsilon}_i' \) is then used to adjust the sampling phase. If \( \dot{\epsilon}_i' \) exceeds some predefined threshold value \( \epsilon_{th} \), the timing correction \( \dot{\tau}_i \) is adjusted by a fraction of the symbol interval \( T/Q \), depending on the polarity of the error estimate. In practice, this procedure can be implemented by means of a bank of polyphase filters [8].

V SIMULATION RESULTS

We present simulation results evaluating the performance of the TED’s for 3- and 4-transmit antenna OSTBC described in Section III. While the theory in Sections (II) and (III) assumed perfect channel state information at the receiver (CSI), the simulation results presented here include cases for CSIR as well as channel estimation by means of pilot symbol assisted modulation (PSAM), the details of which are described in [1]. SER results are described followed by the analysis of the range of timing drift successfully tracked by the timing loop.

For the purpose of channel estimation, the data was organized into frames consisting of a known orthogonal pilot block \( X_p \), spanning \( L_p = 4 \) time slots, followed by \( N_b = 3 \) OSTBC blocks each of \( N_c = 4 \) symbol intervals. The resulting pilot spacing of \( L_f = 16 \) is adequate for the estimation of channel with normalized Doppler frequency up to \( f_DT = 0.03 \). The data was encoded using (21) and (29) for 3- and 4-transmit antenna cases, respectively. The resulting data streams were filtered by a RRC filter with a rolloff of \( \beta_{RF} = 0.35 \). We note that the simulations were implemented with a resolution of \( T/8 \), that is filtering at both the transmitter and the receiver was done using waveforms sampled at that rate.

The channel fading was assumed to be Rayleigh distributed, with a normalized Doppler frequency of \( f_D T = 0.01 \). We assume that the receiver has performed coarse timing acquisition, which would typically be done using a training sequence. The timing drift was simulated by perturbing the sampling phase \( \tau_i \). In order to add a random component to the timing drift, the interval between timing slips, measured in symbol intervals and denoted by \( \tau_r \), was modeled by a Gaussian random variable, with a mean of \( \bar{\tau} \) and with a variance of \( \sigma_{\tau_r}^2 = 0.1\bar{\tau} \). The drift direction was random and equiprobable, while the step size fixed to \( T/8 \). The resulting mean timing error bandwidth, normalized to the symbol duration \( T \), is given by

\[
\bar{B}_T = \frac{T}{8\bar{\tau}}. \tag{34}
\]

In the case of PSAM channel estimation, the pilots were recovered by decimating the received data stream. The minimum mean square error estimate of the channel states for the \( \bar{k} \)th pilot, denoted by \( \alpha_{\bar{k}}(\bar{k}) \), is given by [1]

\[
\alpha_{\bar{k}}(\bar{k}) = \frac{y_{\bar{k}}(n)X_p^H}{\|y_p\|^2}, \tag{34}
\]

where \( y_{\bar{k}}(n) = [y_{\bar{k}1}(k-1)L_f + 1, \ldots, y_{\bar{k}1}(k-1)L_f + L_p] \) and \( \|y_p\|^2 \) is the square of the norm of the pilots sequences carried by each row of \( X_p \). The channel state for the data portion of the frames was obtained by interpolating channel states from \( K = 9 \) pilot blocks using a Wiener filter [1]. The estimated channel
state information was then used to decode the data according to (9).

Timing estimation was done using the TED given by (22), which is suitable for the 3-transmit antenna code given by (21) and the 4-transmit antenna extension given by (29). Since the focus of the investigation is the tracking performance of the detector, the timing estimation was done without the knowledge of the data symbols at the receiver. Hence the data symbols $a_m$ in (22) were replaced by their estimates $\hat{a}_m$. The TED estimate was filtered according to (33) with $\alpha = 0.9$, with the output determining the timing correction as described by Section IV. The error threshold was set to $\epsilon_{th} = 0.25$.

### A. SER Performance

Figures 2 and 3 show QPSK SER plots for the 3- and 4-transmit antenna codes. The figures include TED tracking performance for a receiver with CSIR and one where the channel state was estimated via PSAM. Also provided are two reference curves: ideal timing and channel knowledge, and ideal timing with PSAM channel estimation. The mean timing drift bandwidth was fixed to $\bar{B}_T = 1 \times 10^{-4}$. Results corresponding to PSAM receivers take into account SNR overhead due to pilot insertion.

![Figure 2: QPSK SER for $N_t = 3$.](image)

![Figure 3: QPSK SER for $N_t = 4$.](image)

The results demonstrate that the receiver is able to track the timing variation with a very small performance drop resulting from the timing correction. By observing the reference curves, it is clear that for most part, the performance loss is due to channel estimation. The performance loss at very low SNR, especially for $N_r = 1$, is most likely a result of noisy timing error estimates and can be remedied by further increasing the threshold value $\epsilon_{th}$ in the low SNR region.

### B. Performance as a Function of Timing Bandwidth

We evaluate the performance as a function of timing drift $\bar{B}_T$. Figures 4 and 5 show SER as a function of $\bar{B}_T$ for $N_t = 3$ and $N_t = 4$ codes with $N_r = 2$ receiver operating at $E_s/N_0 = 8$ dB and 10 dB. Both CSIR and PSAM estimation ($K = 9, L_f = 16$) cases are shown. In the case of CSIR, the system is able to track timing up to $\bar{B}_T = 1 \times 10^{-3}$. For PSAM-based channel estimation, the range decreases to just below $\bar{B}_T = 1 \times 10^{-3}$. The reason for the difference in performance between the perfect channel knowledge and PSAM is two-fold. First, is the effect of channel estimation error on the TED output. Secondly, the delay associated with PSAM interpolation causes a delay in data decoding which in turn results in outdated timing information. This effect is more pronounced for faster timing drift, as seen by Figures 4 and 5.

### VI Conclusion

We described a method for the design of low complexity TED’s for a general OSTBC system. Conditions for timing estimates robust to channel fading were presented as well as examples of TED’s for 3- and 4-transmit antenna codes. We have evaluated the performance of the TED’s in a receiver operating in tracking mode. Both perfect channel knowledge and PSAM-based channel estimation cases were examined. We have shown that the SER performance exhibits negligible degradation due to timing synchronization. In addition, it was shown that the proposed TED’s are able to track timing drift up to approximately
Using the fact that $\Re\mathbf{X}^H$ can be expressed as $\mathbf{X}^H\mathbf{s}$, this section details the derivation of (11) and (12). We begin by noting that using (6), (7) and (9), the decision variables $s_m^l$ can be expressed as

$$s_m^l = \|\mathbf{H}_l\|^2 - 2\text{tr}\left\{\mathbf{A}_m\Re(\mathbf{X}^H_{l+1} \mathbf{H}_{l+1})\right\} - j\mathbf{B}_m\Im(\mathbf{X}^H_{l+1} \mathbf{H}_{l+1}) \right\} + \eta'_l,$$  
(35)

where we have used the fact that $\text{tr}(\mathbf{AB}) = \text{tr}(\mathbf{BA})$ and $\Re(\text{tr}(\cdot)) = \text{tr}(\Re(\cdot))$, $\Im(\text{tr}(\cdot)) = \text{tr}(\Im(\cdot))$. The noise term $\eta'_l$ in (35) is given by

$$\eta'_l = \|\mathbf{H}_l\|^2 - 2\text{tr}\left\{\Re(\mathbf{N}^H_l \mathbf{H}_l \mathbf{A}_m) - j\Im(\mathbf{N}^H_l \mathbf{H}_l \mathbf{B}_m) \right\}.$$
(36)

Using (7), (8) and (35), we have

$$E\{a_n^l s_m^n\} = \|\mathbf{H}_l\|^2 - 2 \times \text{tr}\left\{\mathbf{A}_m \sum_{n'} G_{n',m}^H E\{a_n^l \Re(\mathbf{X}^H_{l+1} \mathbf{H}_{l+1})\} \right\} - j\mathbf{B}_m \sum_{n'} G_{n',m}^H E\{a_n^l \Im(\mathbf{X}^H_{l+1} \mathbf{H}_{l+1})\} \right\}.$$
(37)

Since data symbols $a_n^l$ are used to encode only the $l$th block $\mathbf{X}_l$, $E\{a_n^l \Re(\mathbf{X}^H_{l+1} \mathbf{H}_{l+1})\} = E\{a_n^l \Im(\mathbf{X}^H_{l+1} \mathbf{H}_{l+1})\} = 0$ for $n' \neq 0$. Therefore (37) simplifies to

$$E\{a_n^l s_m^n\} = \|\mathbf{H}_l\|^2 - 2 \times \text{tr}\left\{\mathbf{A}_m \mathbf{G}^H_{n',l} E\{a_n^l \Re(\mathbf{X}^H_{l+1} \mathbf{H}_{l+1})\} \right\} - j\mathbf{B}_m \mathbf{G}^H_{n',l} E\{a_n^l \Im(\mathbf{X}^H_{l+1} \mathbf{H}_{l+1})\} \right\},$$
(38)

where we have dropped the zero-valued subscript $n'$ from $\mathbf{G}_{n',l}$ for notational convenience. In addition, we can now remove the code block index $l$, with the understanding that $a_n, s_m, \mathbf{X}$ and $\mathbf{H}$ refer to the block used to for estimation of the timing error. Using the fact that $\Re(\mathbf{AB}) = \Re(\mathbf{A})\Re(\mathbf{B}) - \Im(\mathbf{A})\Im(\mathbf{B})$ and $\Im(\mathbf{AB}) = \Im(\mathbf{A})\Re(\mathbf{B}) + \Re(\mathbf{A})\Im(\mathbf{B})$, we now expand (38) to

$$E\{a_n s_m\} = \|\mathbf{H}\|^2 - 2 \times \text{tr}\left\{\mathbf{A}_m \mathbf{G}^H_{n,l} E\{a_n \Re(\mathbf{X}^H \mathbf{H})\} \right\} - j\mathbf{B}_m \mathbf{G}^H_{n,l} E\{a_n \Im(\mathbf{X}^H \mathbf{H})\} \right\} \Re\mathbf{X}^H + j\mathbf{X}^H \Im\mathbf{H} \right\}.$$
(39)

Using (1), one can show that

$$E\{\Re(a_n \mathbf{X}^H)\} = \rho \mathbf{A}^H_n,$$
$$E\{\Im(a_n \mathbf{X}^H)\} = -j \rho \mathbf{B}^H_n,$$
(40)

where we have defined a constellation-dependent constant $\rho = E\{(\Re(a_n))^2\} = E\{(\Im(a_n))^2\}$. Substituting (40), and noting that $\Re(j \rho \mathbf{B}^H_n) = 0$ and $\Im(\rho \mathbf{A}^H_n) = 0$, one can simplify (39) to the final result given by (11). The derivation of $E\{a_n s_m\}$ follows the same line of reasoning as for $E\{a_n^l s_m^n\}$, resulting in the solution given in (12).

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**REFERENCES**


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**Figure 5:** SER vs $B_T$ for $N_t = 4$. $B_T = 10^{-3}$ with a reduction in range due to PSAM.