A SCREW SYZYGY WITH APPLICATIONS TO ROBOT SINGULARITY COMPUTATION

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Abstract A syzygy is a relation between invariants. In this paper a syzygy is presented between invariants of sequences of six screws under the action of the Euclidean group. This relation is useful in simplifying the computation of the determinant of a robot Jacobian and hence can be used to investigate the singularities of robot manipulators.

Keywords: Robot jacobians, determinants, singularities.

1. Introduction

Methods for determining singularities of serial and parallel manipulators generally involve a combination of exact or symbolic algebraic manipulation, to obtain as simple a form for the Jacobian or its determinant as possible, together with algorithms from linear algebra and numerical analysis to obtain good approximate solutions. The extent to which the former methods are effective depends on properties of the Euclidean geometry that describes the motion of the rigid components of manipulators and on the specific architectural features of a given manipulator, such as partitioning or symmetry.

In this paper we derive a new relation or syzygy among the Euclidean invariants of sequences of screws that leads to a simplification in the calculation of the Jacobian determinant for 6-dof manipulators. This is illustrated by some specific applications.

2. Isometries and Invariants

The motion of a rigid link, in particular the end-effector of a robot manipulator or platform relative to its home configuration, is described mathematically by a distance and orientation-preserving transformation of 3-space: a proper Euclidean isometry. The collection of these isometries forms a Lie group, denoted $SE(3)$. For a given choice of coordinate system such an isometry is represented by a rotation about the origin together with a translation and so $SE(3)$ is a product of the rotation group $SO(3)$ and the translation group $\mathbb{R}^3$, though the composition of two Euclidean isometries is not component-wise or direct but 'semi-direct'. There is a 6×6 matrix representation of an isometry:

$$H = \begin{pmatrix} R & 0 \\ TR & R \end{pmatrix} \tag{1}$$

where R is a 3×3 rotation matrix, T a 3×3 skew-symmetric 'translation' matrix and the composition of two isometries is given by matrix multiplication. For details see for example Selig, 2005.

If we chose a different initial coordinate system in 3-space then there would be a different but related matrix representation of a given isometry. Specifically, if the change of coordinates itself is represented by $A \in SE(3)$ then H transforms to $H' = AHA^{-1}$; H and H' are said to be congruent.

Considering just rotations for the moment, an infinitesimal rotation at the home configuration is a skew-symmetric matrix

$$\Omega = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix} \tag{2}$$

and these form a 3-dimensional space. Once again an orthogonal change of coordinates A (fixing the origin) results in a transformation of Ω to $\Omega' = A\Omega A^{-1}$. This is known as the *adjoint action* of the rotation group $SO(3)$. This action is exactly the same as the effect of transforming the 3-vector $\boldsymbol{\omega} = (\omega_x, \omega_y, \omega_z)^T$ by the orthogonal transformation A. In particular, invariants of the adjoint action of $SO(3)$ are simply invariants of its ordinary action on $\mathbb{R}^3$. Hence, writing Q for the 3×3 identity matrix, the quadratic form $\boldsymbol{\omega}^T Q \boldsymbol{\omega}$ is an invariant of the adjoint action, while $\boldsymbol{\omega}_1^T Q \boldsymbol{\omega}_2$ is an invariant of pairs of infinitesimal rotations. In the theory of Lie groups, the quadratic form $\boldsymbol{\omega}^T Q \boldsymbol{\omega}$ is called the *Killing form* of the adjoint action.

Generalising to $SE(3)$, the infinitesimal isometries now form a 6-dimensional space. Its elements are combinations of infinitesimal rotations Ω (2) and infinitesimal translations $\mathbf{v}$. The infinitesimal isometries are *twists*, though frequently referred to as *screws* which, properly, are the collections of all non-zero multiples of a given (non-zero) twist. However, we use the common term 'screws' here for the 6-vectors

$\mathbf{s} = (\boldsymbol{\omega}^T, \mathbf{v}^T)^T$. The adjoint action of $SE(3)$ on its Lie algebra is given by the matrix multiplication:

$$\begin{pmatrix} R & 0 \\ TR & R \end{pmatrix} \begin{pmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{pmatrix} \tag{3}$$

The Killing form is given by $\mathbf{s}_1^T Q_\infty \mathbf{s}_2$ where Q_∞ is the 6×6 symmetric matrix given below. It is degenerate, having only rank 3. There is a second, non-degenerate, invariant bilinear form for the adjoint action of $SE(3)$, namely the *reciprocal product* $\mathbf{s}_1^T Q_0 \mathbf{s}_2$:

$$Q_0 = \left(\begin{array}{c|c} 0 & I_3 \\ \hline I_3 & 0 \end{array} \right), \qquad Q_\infty = \left(\begin{array}{c|c} 2I_3 & 0 \\ \hline 0 & 0 \end{array} \right)$$

Unlike the Killing form Q for $SO(3)$ however the reciprocal product, regarded as a quadratic form, the *Klein form*, is not positive definite. The ratio of the two quadratic forms is the *pitch* of a screw:

$$p(\mathbf{s}) = \mathbf{s}^T Q_0 \mathbf{s} / \mathbf{s}^T Q_\infty \mathbf{s}.$$

The set of screws for which $\mathbf{s}^T Q_\infty \mathbf{s} \neq 0$ but $p(\mathbf{s}) = \mathbf{s}^T Q_0 \mathbf{s} = 0$ can be identified with the set of lines in 3-space, the corresponding line being the axis of the screw. The coordinates of $\boldsymbol{\omega}, \mathbf{v}$ are the Plücker coordinates of the line. The quadric hypersurface $\mathbf{s}^T Q_0 \mathbf{s} = 0$ (projectively, Klein's quadric) includes the subspace $\mathbf{s}^T Q_\infty \mathbf{s} = 0$ of screws of pitch infinity, i.e. infinitesimal translations. Pairs of screws for which $\mathbf{s}_1^T Q_0 \mathbf{s}_2 = 0$ are called *reciprocal*. From the manipulator point of view, screws of pitch zero and infinity correspond to the motions associated with revolute and prismatic joints respectively.

3. The Syzygy

As motivation, we begin by looking at a syzygy among invariants of the group of rotations $SO(3)$. Weyl, 1946 (Theorem 2.9.A) asserts that the invariants of the standard representation of $SO(3)$ (i.e. its action on vectors in $\mathbb{R}^3$) are generated by the scalar product of pairs of vectors and the scalar triple product of triples of vectors.

There is one form of syzygy between these invariants—that is to say, they are not *algebraically* independent. Consider three arbitrary vectors $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3 \in \mathbb{R}^3$ and write them as columns of a matrix 3×3 matrix

$$M = \left(\begin{array}{c|c|c} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{array} \right),$$

Then $\det M = \mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3)$ is a basic invariant. We have

$$M^T M = \begin{pmatrix} \mathbf{a}_1 \cdot \mathbf{a}_1 & \mathbf{a}_1 \cdot \mathbf{a}_2 & \mathbf{a}_1 \cdot \mathbf{a}_3 \\ \mathbf{a}_2 \cdot \mathbf{a}_1 & \mathbf{a}_2 \cdot \mathbf{a}_2 & \mathbf{a}_2 \cdot \mathbf{a}_3 \\ \mathbf{a}_3 \cdot \mathbf{a}_1 & \mathbf{a}_3 \cdot \mathbf{a}_2 & \mathbf{a}_3 \cdot \mathbf{a}_3 \end{pmatrix} \tag{4}$$

and the matrix entries are themselves invariants. In the terminology of Section 2, the left-hand side could be written as M^TQM. Taking the determinant of both sides of (4) gives the syzygy

$$\det(M)^2 = \det\left(\mathbf{a}_i \cdot \mathbf{a}_j\right),$$

representing a polynomial identity between the basic invariants, of degree 6 in the coordinates of the vectors. Given the identification of the standard representation with the adjoint representation for $SO(3)$, we may also think of these as invariants and a syzygy among invariants of the adjoint action.

By analogy we now exhibit an $SE(3)$ syzygy. Begin with six screws $\mathbf{s}_1, \ldots, \mathbf{s}_6$. The reciprocal product $\mathbf{s}_i^T Q_0 \mathbf{s}_j$ of any pair is certainly invariant. Combine the screws into the 6×6 matrix

$$J = \left(\; \mathbf{s}_1 \mid \mathbf{s}_2 \mid \mathbf{s}_3 \mid \mathbf{s}_4 \mid \mathbf{s}_5 \mid \mathbf{s}_6 \; \right).$$

The adjoint action (3) of $SE(3)$ on the individual screws gives rise to an action on J and since $\det R = 1$, the determinant of J is also an invariant. Now form the product

$$J^T Q_0 J = \begin{pmatrix} \mathbf{s}_1^T Q_0 \mathbf{s}_1 & \mathbf{s}_1^T Q_0 \mathbf{s}_2 & \mathbf{s}_1^T Q_0 \mathbf{s}_3 & \mathbf{s}_1^T Q_0 \mathbf{s}_4 & \mathbf{s}_1^T Q_0 \mathbf{s}_5 & \mathbf{s}_1^T Q_0 \mathbf{s}_6 \\ \mathbf{s}_2^T Q_0 \mathbf{s}_1 & \mathbf{s}_2^T Q_0 \mathbf{s}_2 & \mathbf{s}_2^T Q_0 \mathbf{s}_3 & \mathbf{s}_2^T Q_0 \mathbf{s}_4 & \mathbf{s}_2^T Q_0 \mathbf{s}_5 & \mathbf{s}_2^T Q_0 \mathbf{s}_6 \\ \mathbf{s}_3^T Q_0 \mathbf{s}_1 & \mathbf{s}_3^T Q_0 \mathbf{s}_2 & \mathbf{s}_3^T Q_0 \mathbf{s}_3 & \mathbf{s}_3^T Q_0 \mathbf{s}_4 & \mathbf{s}_3^T Q_0 \mathbf{s}_5 & \mathbf{s}_3^T Q_0 \mathbf{s}_6 \\ \mathbf{s}_4^T Q_0 \mathbf{s}_1 & \mathbf{s}_4^T Q_0 \mathbf{s}_2 & \mathbf{s}_4^T Q_0 \mathbf{s}_3 & \mathbf{s}_4^T Q_0 \mathbf{s}_4 & \mathbf{s}_4^T Q_0 \mathbf{s}_5 & \mathbf{s}_4^T Q_0 \mathbf{s}_6 \\ \mathbf{s}_5^T Q_0 \mathbf{s}_1 & \mathbf{s}_5^T Q_0 \mathbf{s}_2 & \mathbf{s}_5^T Q_0 \mathbf{s}_3 & \mathbf{s}_5^T Q_0 \mathbf{s}_4 & \mathbf{s}_5^T Q_0 \mathbf{s}_5 & \mathbf{s}_5^T Q_0 \mathbf{s}_6 \\ \mathbf{s}_6^T Q_0 \mathbf{s}_1 & \mathbf{s}_6^T Q_0 \mathbf{s}_2 & \mathbf{s}_6^T Q_0 \mathbf{s}_3 & \mathbf{s}_6^T Q_0 \mathbf{s}_4 & \mathbf{s}_6^T Q_0 \mathbf{s}_5 & \mathbf{s}_6^T Q_0 \mathbf{s}_6 \end{pmatrix}. \tag{5}$$

Each entry in the matrix is invariant, so taking determinants of each side of (5) gives a syzygy of degree 12 in the screw coordinates:

$$\det(J)^2 = -\det\left(\mathbf{s}_i^T Q_0 \mathbf{s}_j\right), \tag{6}$$

the negative sign coming from the fact that $\det(Q_0) = -1$.

4. Applications

The key point is that the syzygy (6) enables us to find the determinant of a manipulator Jacobian, at least up to sign, by finding the determinant of a *symmetric* matrix. In particular, singularity detection, that is checking whether $\det J = 0$, is feasible. In general this leads to faster computation. Moreover in many particular cases the matrix in (5) has a nice form, reflecting the intrinsic geometry of the manipulator.

4.1 Fast computation. In general the determinant of a positive definite symmetric matrix can be found using Cholesky decomposition, this algorithm is known to be about twice as fast as LU decomposition which would be the appropriate method for a non-symmetric matrix, see Press *et al*, 1992, Section 2.9. Unfortunately the matrix of reciprocal products is not necessarily positive definite so another algorithm must be used, for example LDLT decomposition, Bunch and Parlett, 1971, which is still twice as fast as LU decomposition.

A simple count of the number of arithmetic operations involved in a symbolic algebraic expression of the determinant, using Maple say, gives 3600 multiplications for a general 6×6 determinant but only 2252 for a symmetric matrix. Moreover, if the joints are revolute or prismatic, that is if the screws satisfy $\mathbf{s}_i^T Q_0 \mathbf{s}_i = 0$, $i = 1, \ldots, 6$, then the diagonals in the matrix in (5) are all zero and the number of multiplications is reduced to 765. Similarly, additions are also reduced. Of course, the matrix of reciprocal products has to be determined first, but given the simplicity of Q_0, that involves only six multiplications per entry so 126 in all, by symmetry.

4.2 Wrist-partitioned serial manipulators In many cases, the matrix of reciprocal products has a special form, further simplifying calculation of the Jacobian determinant and hence robot singularities. Consider the singularities of a wrist-partitioned serial 6R robot (see, for example, Stanišić and Engelberth, 1988; Hayes et al., 2002). Not only are the computations accomplished easily using the syzygy, but the approach yields a simple proof of the types of singularity that can occur. Notice that it is not even necessary to set up a particular coordinate frame.

Since the last three joints of the robot comprise a wrist, the joint axes all meet at the wrist centre. This means that the last three screws are mutually reciprocal and hence the bottom right-hand corner of the matrix of reciprocal products is zero. Since the top right and bottom left corners of the matrix are the transpose of each other the expansion of the determinant leads to

$$\det(J) = \pm \det \begin{pmatrix} \mathbf{s}_1^T Q_0 \mathbf{s}_4 & \mathbf{s}_1^T Q_0 \mathbf{s}_5 & \mathbf{s}_1^T Q_0 \mathbf{s}_6 \\ \mathbf{s}_2^T Q_0 \mathbf{s}_4 & \mathbf{s}_2^T Q_0 \mathbf{s}_5 & \mathbf{s}_2^T Q_0 \mathbf{s}_6 \\ \mathbf{s}_3^T Q_0 \mathbf{s}_4 & \mathbf{s}_3^T Q_0 \mathbf{s}_5 & \mathbf{s}_3^T Q_0 \mathbf{s}_6 \end{pmatrix}.$$

If we are only interested in singularities the sign ambiguity is not important. Setting this 3×3 determinant to zero means that the columns (or rows) of the matrix must be linearly dependent. Column dependence can be expressed by the equations

$$\begin{aligned} \lambda \mathbf{s}_1^T Q_0 \mathbf{s}_4 + \mu \mathbf{s}_1^T Q_0 \mathbf{s}_5 + \nu \mathbf{s}_1^T Q_0 \mathbf{s}_6 &= 0 \\ \lambda \mathbf{s}_2^T Q_0 \mathbf{s}_4 + \mu \mathbf{s}_2^T Q_0 \mathbf{s}_5 + \nu \mathbf{s}_2^T Q_0 \mathbf{s}_6 &= 0 \\ \lambda \mathbf{s}_3^T Q_0 \mathbf{s}_4 + \mu \mathbf{s}_3^T Q_0 \mathbf{s}_5 + \nu \mathbf{s}_3^T Q_0 \mathbf{s}_6 &= 0, \end{aligned} \qquad (7)$$

for some non-zero constants λ, μ and ν. These equations can be solved in two ways. First, if the last three joints are linearly dependent, that is

$$\lambda\mathbf{s}_4 + \mu\mathbf{s}_5 + \nu\mathbf{s}_6 = \mathbf{0}$$

for some λ, μ, ν, then clearly (7) is satisfied. This occurs if and only the three screw axes $\mathbf{s}_4$, $\mathbf{s}_5$ and $\mathbf{s}_6$ are coplanar and constitutes a *wrist singularity*. Likewise, if the rows of the matrix are linearly dependent then for some α, β, γ not all zero

$$\begin{aligned} \alpha\mathbf{s}_1^T Q_0\mathbf{s}_4 + \beta\mathbf{s}_2^T Q_0\mathbf{s}_4 + \gamma\mathbf{s}_3^T Q_0\mathbf{s}_4 &= 0 \\ \alpha\mathbf{s}_1^T Q_0\mathbf{s}_5 + \beta\mathbf{s}_2^T Q_0\mathbf{s}_5 + \gamma\mathbf{s}_3^T Q_0\mathbf{s}_5 &= 0 \\ \alpha\mathbf{s}_1^T Q_0\mathbf{s}_6 + \beta\mathbf{s}_2^T Q_0\mathbf{s}_6 + \gamma\mathbf{s}_3^T Q_0\mathbf{s}_6 &= 0, \end{aligned} \tag{8}$$

and the same reasoning leads to a *shoulder singularity* of the first three joints when they fail to span a 3-dimensional subspace. Whether or not such a singularity can occur depends on the screw systems determined by the first three joints and so, in turn, on the design parameters of the shoulder.

The second kind of solution is most easily derived from equations (8) which are satisfied if and only if there is a screw $\alpha\mathbf{s}_1+\beta\mathbf{s}_2+\gamma\mathbf{s}_3$ reciprocal to each of $\mathbf{s}_4$, $\mathbf{s}_5$ and $\mathbf{s}_6$, and hence to all screws in the screw system $\mathcal{S}$ spanned by the last three joints. Since their axes intersect in the wrist centre, $\mathcal{S}$ is a type IIA ($p = 0$) 3-system, which is self-reciprocal (Gibson and Hunt, 1990). Hence the screw $\alpha\mathbf{s}_1 + \beta\mathbf{s}_2 + \gamma\mathbf{s}_3$ lies in $\mathcal{S}$ so it must also be a line through the wrist centre. In general, the $\mathbf{s}_1$, $\mathbf{s}_2$ and $\mathbf{s}_3$ form a 3-system corresponding to a projective 2-plane which will intersect the Klein quadric in a conic. Each point on the conic represents a line in the screw system and it is well known that such a set of lines form the regulus of a hyperboloid. If any of these lines passes through the wrist centre then the configuration of the robot is singular. In some cases the only possibilities are when the wrist centre lies on the first or second joint axis.

However, there are some more complex situations. Consider the PUMA manipulator where the first and second joint axes meet and the second and third joint axes are parallel. In this case, the conic in the screw system degenerates into a pair of lines. One of these corresponds to the lines in 3-space parallel to the second and third joint axes, the other to the set of lines meeting both the first and second joint axes. The PUMA then has singularities if its wrist centre lies in either the plane containing the first and second joints or the plane containing the second and third joints.

4.3 Serial manipulators with self-reciprocal subassemblies. In the previous example an important point was the fact that a subassembly of three consecutive joints form a self-reciprocal screw system. Such systems are necessarily 3-systems and must consist only of lines. The fact that the system is self-reciprocal ensures that the matrix of reciprocal products contains a diagonal 3×3 block of zeros. Besides the IIA ($p=0$) 3-systems met in the example above, there are two other such systems: IIC ($p=0$) systems and the IID system. These three screw systems are also Lie subalgebras; this is important because it means that the screw system is invariant under motions of the manipulator and hence the the matrix of reciprocal products will contain a block of zeros in every position of the manipulator. There is another 3-system which is a subalgebra, the IIC ($p\neq0$) system, but it is not self-reciprocal.

The IID system contains only screws of infinite pitch, that is infinitesimal translations. Cartesian-type and gantry robots contain subassemblies of prismatic joints which generate this screw system. However their singularities are already easy to analyse and we do not pursue this case.

The IIC ($p=0$) systems can be generated by three parallel lines. Hence a robot containing three consecutive parallel revolute joints satisfies this requirement. Such designs are rare in commercially available robots at the moment. Examples are the original Cincinnati Milacron T3 and the Telequipment MA2000 in which joints $2, 3$ and 4 are parallel. Expanding the determinant of the matrix of reciprocal products gives

$$\det(J) = \pm\det\begin{pmatrix} \mathbf{s}_1^T Q_0 \mathbf{s}_2 & \mathbf{s}_2^T Q_0 \mathbf{s}_5 & \mathbf{s}_2^T Q_0 \mathbf{s}_6 \\ \mathbf{s}_1^T Q_0 \mathbf{s}_3 & \mathbf{s}_3^T Q_0 \mathbf{s}_5 & \mathbf{s}_3^T Q_0 \mathbf{s}_6 \\ \mathbf{s}_1^T Q_0 \mathbf{s}_4 & \mathbf{s}_4^T Q_0 \mathbf{s}_5 & \mathbf{s}_4^T Q_0 \mathbf{s}_6 \end{pmatrix}.$$

By similar arguments to Section 4.2 we have three types of singularity. First, when $\mathbf{s}_2$, $\mathbf{s}_3$ and $\mathbf{s}_4$ are linearly dependent, the three parallel joints must be coplanar and the robot has an *elbow singularity.* Second, it may be possible for the other three joints $\mathbf{s}_1$, $\mathbf{s}_5$ and $\mathbf{s}_6$ to be linearly dependent, though this will depend on the disposition of these joints. Thirdly, the robot will be singular if a screw from the 3-system determined by $\mathbf{s}_1$, $\mathbf{s}_5$ and $\mathbf{s}_6$ lies in the screw system determined by the three parallel joint axes. For example this can happen in the T3 design if the last joint is parallel to joints 2, 3 and 4.

The IIC ($p=0$) systems can also be generated by a revolute joint and two prismatic joints or two parallel revolute joints and a prismatic joint. These do not seem to correspond to subassemblies of any design of robot that has been proposed.

4.4 Gough–Stewart platforms. For Gough-Stewart platforms it is well known that the rows of the Jacobian matrix can be found from the lines joining the passive spherical joints, Merlet, 2000. Indeed if $\mathbf{s}_i$ is the unit line joining the passive spherical joints on the ith leg then the corresponding row in the Jacobian matrix will be $\mathbf{s}_i^T Q_0$. Writing K for this Jacobian and using $Q_0^2 = I$, the syzygy gives

$$-\det(K)^2 = \det(\mathbf{s}_i^T Q_0 \mathbf{s}_j).$$

Since the screws $\mathbf{s}_i$ are lines, the diagonal of the matrix of reciprocal products will consist of zeros. In the 3–3 design pairs of legs meet at the passive spherical joints alternately on the base and the platform. In this way there are just three passive spherical joints on each. In this case consecutive leg axes are reciprocal, so that the matrix of reciprocal products has the form

$$KQ_0K^T = \begin{pmatrix} 0 & 0 & \mathbf{s}_1^T Q_0 \mathbf{s}_3 & \mathbf{s}_1^T Q_0 \mathbf{s}_4 & \mathbf{s}_1^T Q_0 \mathbf{s}_5 & 0 \\ 0 & 0 & 0 & \mathbf{s}_2^T Q_0 \mathbf{s}_4 & \mathbf{s}_2^T Q_0 \mathbf{s}_5 & \mathbf{s}_2^T Q_0 \mathbf{s}_6 \\ \mathbf{s}_3^T Q_0 \mathbf{s}_1 & 0 & 0 & 0 & \mathbf{s}_3^T Q_0 \mathbf{s}_5 & \mathbf{s}_3^T Q_0 \mathbf{s}_6 \\ \mathbf{s}_4^T Q_0 \mathbf{s}_1 & \mathbf{s}_4^T Q_0 \mathbf{s}_2 & 0 & 0 & 0 & \mathbf{s}_4^T Q_0 \mathbf{s}_6 \\ \mathbf{s}_5^T Q_0 \mathbf{s}_1 & \mathbf{s}_5^T Q_0 \mathbf{s}_2 & \mathbf{s}_5^T Q_0 \mathbf{s}_3 & 0 & 0 & 0 \\ 0 & \mathbf{s}_6^T Q_0 \mathbf{s}_2 & \mathbf{s}_6^T Q_0 \mathbf{s}_3 & \mathbf{s}_6^T Q_0 \mathbf{s}_4 & 0 & 0 \end{pmatrix}.$$

The zero-structure of this matrix renders the determinant relatively easy to compute.

5. Conclusions

We have presented a new syzygy among invariants of sets of screws. Using a couple of examples we have shown how this relation enables easy determination and analysis of robot manipulator singularities.

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