Towards a Unified Notion of Kinematic Singularities for Robot Arms and Non-Holonomic Platforms

Andreas Müller and Peter Donelan

Abstract Kinematic singularities are classically defined in terms of the rank of Jacobians of associated maps, such as forward and inverse kinematic mappings. A more inclusive definition should take into account the Lie algebra structure of related tangent spaces. Such a definition is proposed in this paper, initially for serial manipulators and non-holonomic platforms. The definition can be interpreted as a change in the number of successive infinitesimal motions required for the system to reach an arbitrary configuration in the vicinity of the given configuration. More precisely, it is based on the filtration of a controllability distribution.

Key words: Kinematic singularity, filtration, non-holonomic

1 Introduction

The singularities of holonomic mechanisms are fairly well-understood, and there is a well-established concept for holonomic mechanisms [4, 10]. There is yet no established notion for non-holonomic systems. It is thus instructive to point out some common features of the output singularities of holonomic serial manipulators (SM) and the input singularities of non-holonomic platforms.

It is known that, in a forward kinematic singularity, the complexity of infinitesimal motions that a SM has to perform in order to reach nearby configurations increases. This is reflected by a drop of rank of the forward kinematics Jacobian. Furthermore, the nesting level of Lie brackets of the instantaneous joint screws necessary to generate the Lie algebra corresponding to the motion subgroup of the kinematic chain increases [6, 12]. This is a non-generic phenomenon, and singularities form closed dense subspaces in the configuration space. The SM can be regarded as a driftless kinematic control system, and the Lie bracketing determines the accessibility algebra of this control system. An unconstrained holonomic SM is always kinematically controllable.

Also a non-holonomic platform can modeled as a driftless kinematic control problem. It is configuration controllable if it is completely non-holonomic, i.e. there is no integral manifold defined by the constraints. Controllability is ensured by the Lie algebra rank condition, i.e. when the nested Lie brackets of the control vector fields span the accessibility algebra. Even if this local property holds, the complexity of the infinitesimal motions necessary to reach nearby configurations may change in configurations that are referred to as singular.

The apparent similarity of the holonomic and non-holonomic systems is discussed in this paper, and a unified definition accounting for both types of systems is proposed. The definition rests on the concept of a filtration of a distribution associated with the kinematic control system, which characterizes the complexity of motion in a singularity. It provides a general framework of SMs and completely non-holonomic systems. This preliminary result can potentially be extended to parallel manipulators.

2 Singularities of Lower-Pair Serial Manipulators

For a serial manipulator (SM) whose joints have n degreees of freedom, denote its joint variables by $\mathbf{q}=(q_1,\ldots,q_n)\in\mathbb{V}^n$, the c-space of the SM. The task of the SM is to position an end-effector (EE) and its task space is a subset of $SE(3)$. The forward kinematic mapping $f:\mathbb{V}^n\rightarrow SE(3)$ relates the configuration $\mathbf{q}\in\mathbb{V}^n$ of the SM to the EE configuration $\mathbf{C}=f(\mathbf{q})\in SE(3)$. The mapping can be expressed as a PoE:

$$f(\mathbf{q})=\exp(\mathbf{Y}_1q_1)\cdot\ldots\cdot\exp(\mathbf{Y}_nq_n)\,\mathbf{A} \tag{1}$$

where $\mathbf{Y}_i\in se(3)$ (the Lie algebra of the Euclidean group) is a twist generating the motion of of joint i, with respect to to the chosen global frame and $\mathbf{A}\in SE(3)$ is the EE pose in the reference configuration $\mathbf{q}=\mathbf{0}$.

The EE twist $\mathbf{V}=(\boldsymbol{\omega},\mathbf{v})\in se(3)$ arising from a trajectory $\mathbf{q}(t)$ through configuration $\mathbf{C}$, in spatial representation, is determined by $\widehat{\mathbf{V}}=\dot{\mathbf{C}}\mathbf{C}^{-1}\in se(3)$, where $\widehat{\mathbf{V}}$ is the matrix representation of $\mathbf{V}\in se(3)$. It is determined in terms of joint velocities by the spatial Jacobian $\mathbf{J}(\mathbf{q}):\mathbb{R}^n\rightarrow se(3)$ as

$$\mathbf{V}(\mathbf{q},\dot{\mathbf{q}})=\mathbf{J}_1(\mathbf{q})\,\dot{q}_1+\ldots+\mathbf{J}_n(\mathbf{q})\,\dot{q}_n=\mathbf{J}(\mathbf{q})\,\dot{\mathbf{q}}. \tag{2}$$

The columns $\mathbf{J}_j$, $j=1,\ldots,n$ of $\mathbf{J}$ are the instantaneous joint screws in configuration $\mathbf{C}$, given by

$$\mathbf{J}_j(\mathbf{q})=\mathbf{Ad}_{g_j}\mathbf{Y}_j. \tag{3}$$

with $g_j(\mathbf{q})=\exp(\mathbf{Y}_1q_1)\cdot\ldots\cdot\exp(\mathbf{Y}_{j-1}q_{j-1})$.

Definition 1 (Singularity—rank criterion). A *(forward) kinematic singularity* of an SM is a critical point $\mathbf{q}\in\mathbb{V}^n$ of the kinematic mapping f, i.e. a configuration where $\operatorname{rank}(\mathbf{J}(\mathbf{q}))<r_{\max}=\min\{n,\dim(\text{task space})\}$.

Since the rank is lower semi-continuous and the assignment is analytic, this is equivalent to saying there is no neighbourhood of $\mathbf{q}$ on which rank $\mathbf{J}$ is constant, whereas at a regular point, the rank will be locally constant and equal to $r_{\max}$.

The vector space of achievable EE twists at a given configuration $\mathbf{q}$ is given by:

$$D_{\mathbf{q}} := \mathrm{im}\,\mathbf{J}(\mathbf{q}) = \mathrm{span}_{\mathbb{R}}(\mathbf{J}_1(\mathbf{q}),\ldots,\mathbf{J}_n(\mathbf{q})). \tag{4}$$

The $\mathbf{J}_j$ are analytic right-invariant vector fields on $SE(3)$ so the assignment $\mathbf{q} \mapsto (\mathbf{J}_1(\mathbf{q}),\ldots,\mathbf{J}_n(\mathbf{q}))$ is smooth and so, at a regular point, $\mathbf{q} \mapsto D_{\mathbf{q}} \subseteq se(3)$ is a smooth map to the Grassmannian of subspaces of $se(3)$ of dimension $r_{\max}$.

An important characteristic of an SM is the vector space of all EE twists the SM can generate at a given configuration $\mathbf{q}$. This is a subspace of the involutive closure of $D_{\mathbf{q}}$. The latter is the Lie algebra, denoted $\overline{D_{\mathbf{q}}} \subseteq se(3)$, generated by all Lie brackets of the joint screws. It can be determined by taking all nested Lie brackets of $\mathbf{J}_j(\mathbf{q})$, so constructed by means of the filtration of $D_{\mathbf{q}}$, which is the sequence of vector spaces $D_{\mathbf{q}}^{i+1} := D_{\mathbf{q}}^{i} + \left[D_{\mathbf{q}}, D_{\mathbf{q}}^{i}\right]$, with $D_{\mathbf{q}}^{1} := D_{\mathbf{q}}$. This terminates with $\overline{D}_{\mathbf{q}} = D_{\mathbf{q}}^{\kappa}$ for some κ. It can be shown for the task space $SE(3)$ that $\kappa \leq 4$ [6]. So, for example, $\overline{D}_{\mathbf{0}} = \mathrm{span}_{\mathbb{R}}(\mathbf{Y}_i, [\mathbf{Y}_i, \mathbf{Y}_j], [\mathbf{Y}_i, [\mathbf{Y}_j, \mathbf{Y}_k]], [\mathbf{Y}_i, [\mathbf{Y}_j, [\mathbf{Y}_k, \mathbf{Y}_l]]]])$.

Moreover, the involutive closure is the same at any $\mathbf{q} \in \mathbb{V}^n$ and we denote this common closure by $\overline{D}$. This follows from the expression (3) for the instantaneous joint screws, invoking the BCH formula. Hence $r_{\max} \leq \min\left(\dim \overline{D}, n\right)$. Moreover, the subgroup G generated by the subalgebra $\overline{D}$ is the smallest $SE(3)$ subgroup comprising all possible EE configurations. Hence $f(\mathbf{q}) \in G$ and $\mathrm{im}\,\mathbf{J}(\mathbf{q}) \subseteq \overline{D}$ for any $\mathbf{q} \in \mathbb{V}^n$, and f can be regarded as a mapping $f : \mathbb{V}^n \to G$. But note that this restricted forward kinematic mapping may still not be surjective if the EE motions do not form a subalgebra, since then $D_{\mathbf{q}} \subsetneq \overline{D}$.

The filtration at $\mathbf{q}$ locally characterizes the process of manipulating the EE when starting at $\mathbf{q}$. The length κ of the filtration is the maximal number of successive infinitesimal joint motions necessary to produce any given (feasible) EE twist. It seems intuitively clear that this number should change at a singularity. The following can be proved using the BCH formula.

Lemma 1. *The filtrations of $D_{\mathbf{q}}$ are identical at all regular configurations $\mathbf{q} \in \mathbb{V}^n$ of the SM, i.e. when $\mathbf{J}(\mathbf{q})$ has full rank. The configuration $\mathbf{q}$ is a kinematic singularity of f if and only if the filtration of $D_{\mathbf{p}}$ is not constant for $\mathbf{p}$ in a neighbourhood of $\mathbf{q}$.*

This leads an alternative definition of kinematic singularities for SM. Denote with κ_0 the length of the filtration at a regular point.

Definition 2 (Singularity—filtration criterion). A configuration $\mathbf{q} \in \mathbb{V}^n$ of an SM is a *forward kinematic singularity* if the length $\kappa(\mathbf{q}) > \kappa_0$.

The condition $\kappa > \kappa_0$ is equivalent to say that the filtration of D is not constant in a neighbourhood of $\mathbf{q}$. Since the filtration reveals the effect of higher-order infinitesimal motions it allows for identification of the joint motions that lead the SM out of a singularity [7, 12].

Example 1: Consider the redundant 7 DOF (anthropomorphic) SM in the reference configuration $\mathbf{q}=\mathbf{0}$ in figure 2. The SM has 7 revolute joints, and its c-space is $\mathbb{V}^n=T^7$. the joint screw coordinate vectors in the reference configuration w.r.t. to the global frame are

$$\mathbf{Y}_{1,3,5,7}=\begin{pmatrix}0\\0\\1\\0\\0\\0\end{pmatrix},\mathbf{Y}_2=\begin{pmatrix}0\\-1\\0\\-L_3-L_5-L_E\\0\\0\end{pmatrix},\mathbf{Y}_4=\begin{pmatrix}0\\1\\0\\L_5+L_E\\0\\0\end{pmatrix},\mathbf{Y}_6=\begin{pmatrix}0\\-1\\0\\-L_E\\0\\0\end{pmatrix}. \tag{5}$$

The Jacobian $\mathbf{J}(\mathbf{0})=(\mathbf{Y}_1,\mathbf{Y}_2,\mathbf{Y}_3,\mathbf{Y}_4,\mathbf{Y}_5,\mathbf{Y}_6,\mathbf{Y}_7)$ has rank $\mathbf{J}(\mathbf{0})=\dim D_{\mathbf{0}}=3$. In regular configurations $\mathbf{q}$ it is rank $\mathbf{J}(\mathbf{q})=6$. The configuration $\mathbf{q}=\mathbf{0}$ is thus a corank 3 singularity, according to definition 1. The Lie brackets

$$[\mathbf{Y}_1,\mathbf{Y}_2]=(1,0,0,0,-L_3-L_5-L_E,0)^T$$
$$[\mathbf{Y}_1,\mathbf{Y}_4]=(-1,0,0,0,L_5+L_E,0)^T$$
$$[\mathbf{Y}_2,\mathbf{Y}_4]=(0,0,0,0,0,-L_3)^T$$

for instance, yield linearly independent vectors not in $D_{\mathbf{0}}$. Hence the vector space $D^2_{\mathbf{0}}=se(3)$ has $\dim D_{\mathbf{0}}=6$. The filtration length at $\mathbf{q}=\mathbf{0}$ is thus $\kappa=2$, and the growth vector is $\rho(\mathbf{0})=(3,6)$. Consequently, the SM may escape from this singularity by first-order motions of joints 1,2, and 4. The filtration length in regular configurations $\mathbf{q}$ is $\kappa_0=1$ since $D_{\mathbf{q}}=SE(3)$. With $\kappa>\kappa_0$ the configuration is a singularity according to Definition 2.

Fig. 1 Kinematic model of a 7 DOF KUKA LWR.

Using the definition $\widehat{\mathbf{V}}=\dot{\mathbf{C}}\mathbf{C}^{-1}$ of spatial velocity, the relation (2) can be written as a right-invariant driftless control system on $SE(3)$

$$\dot{\mathbf{C}}=\left(\widehat{\mathbf{J}}_1u_1+\ldots+\widehat{\mathbf{J}}_nu_n\right)\mathbf{C}. \tag{6}$$

The vector space D serves as the right-trivialized controllability distribution of this control system. Necessary and sufficient for (6) to be locally controllable at $\mathbf{q}$ is that $\overline{D}_{\mathbf{q}}=\overline{D}$ [1,5]. Hence the system (6), and thus the SM, is always locally controllable, even in forward kinematic singularities.

Remark 1. Thus far only holonomic SM were considered. In general, holonomic manipulators are mechanisms comprising closed kinematic loops. This is beyond the scope of this paper, but a note is in order. A mechanism is a physical realization of set of kinematic relations. The mathematical model for the kinematics of a general holonomic mechanism consists of its c-space $V := h^{-1}(\mathbf{0}) \subset \mathbb{V}^n$, where the system of k holonomic constraints arising from closed loops is written as $h(\mathbf{q}) = \mathbf{0} \in \mathbb{R}^k$; the input space $\mathscr{I} \subset \mathbb{R}^m$; and the output space $\mathscr{W} \subset SE(3)$ [8]. These objects are related via the input mapping f_I and the output mapping f_O

$$\mathscr{W} \xleftarrow{f_\mathrm{O}} V \xrightarrow{f_\mathrm{I}} \mathscr{I}\,. \tag{7}$$

The c-space V is an analytic variety when the constraint mapping h is formulated in terms of POEs. Configurations $\mathbf{q}$ where V is not locally a smooth manifold are *c-space singularities*. Configurations $\mathbf{q} \in V$ where the constraint Jacobian $\mathbf{J}_h$ is not constant in a neighbourhood of $\mathbf{q}$ in V are *constraint singularities*. It is important to note that c-space singularities are automatically constraint singularities, but the opposite is not necessarily the case [9]. At c-space singularities the mobility of the mechanism changes. How this affects the possible input and output motions is determined by f_I and f_O, respectively. *Input (output) singularities* are such that the input (output) Jacobian is not constant in a neighbourhood of $\mathbf{q}$ in V. These three types of singularities can occur simultaneously. All possible combinations and their instantaneous phenomenology were reported in [14]. The consequences for the local finite mobility is yet to be explored.

The associated kinematic control system for a closed loop mechanism can be written in implicit form as:

$$\mathbf{J}_h(\mathbf{q})\,\dot{\mathbf{q}} = \mathbf{0}, \qquad \mathbf{J}_\mathrm{I}(\mathbf{q})\,\dot{\mathbf{q}} = \mathbf{u}, \qquad \mathbf{J}_\mathrm{O}(\mathbf{q})\,\dot{\mathbf{q}} = \mathbf{V} \tag{8}$$

where $\mathbf{u}$ denotes an input vector. Suppose that, locally, rank $\mathbf{J}_h = n - \delta_\mathrm{loc}$ (i.e. no c-space singularities, no underconstrained mechanisms). Away from input singularities, the Implicit Function Theorem enables one to locally invert f_I. Then the first two constraint equations in (**??**) can be rewritten in the form $\mathbf{F}(\mathbf{q})\,\mathbf{u} = \dot{\mathbf{q}}$. The problem of input singularities can be circumvented by working directly with the codistribution defined by the constraints. This will not be pursued further here. Rather the purpose has been to signal the connection to non-holonomic systems, which will be investigated next.

3 Singularities of Non-Holonomic Mobile Platforms

Wheeled mobile platforms are frequently used in mobile robotics. The rolling constraint gives rise to non-holonomic constraints so that they can be treated as non-holonomic kinematic control systems. Denote with $\mathbf{x} \in \mathbb{V}^p$ the p coordinates represent the configuration of the system. They are subjected to a system of k non-holonomic Pfaffian constraints

$$\mathbf{A}(\mathbf{x})\,\dot{\mathbf{x}} = \mathbf{0}. \tag{9}$$

Throughout this sectionr (9) is assumed to be completely non-holonomic. Then a configuration $\mathbf{q}$ is a *constraint singularity* if $\mathbf{A}(\mathbf{q})$ is not full rank.

Remark 2. If the constraints were not completely non-holonomic, it would be necessary to determine the integral manifold M passing through $\mathbf{x}$. The configuration $\mathbf{x}$ is a *constraint singularity* iff rank $\mathbf{A}$ is not constant in a neighbourhood of $\mathbf{x}$ in M, adopting the concept of holonomic mechanisms [10]. For completely non-holonomic constraints this simply means that $\mathbf{A}$ is not full rank.

In the following $\mathbf{A}$ is assumed to have full rank k. Then there are $m = n-k$ vector fields $\mathbf{g}_1, \ldots, \mathbf{g}_m$ that span $\ker \mathbf{A}$. They constitute the columns of the orthogonal complement $\mathbf{G}$ of $\mathbf{A}$. This gives rise to the driftless control system

$$\dot{\mathbf{x}} = \mathbf{g}_1(\mathbf{x})\,u_1 + \ldots + \mathbf{g}_m(\mathbf{x})\,u_m = \mathbf{G}(\mathbf{x})\,\mathbf{u}. \tag{10}$$

Usually the inputs $\mathbf{u}$ form a subset of $\mathbf{x}$ consisting of steering and rolling velocities. The associated controllability distribution is $\Delta := \mathrm{span}_{\mathbb{R}}(\mathbf{g}_1, \ldots, \mathbf{g}_m)$, which is also referred to as the constraint distribution [11]. The distribution Δ is regular if it has a locally constant dimension, i.e. if the constraints (9) have locally constant rank.

The similarity to the forward kinematics (2) of holonomic manipulators is obvious, but now (10) describes how input rates affect the system velocity. Now the matrix $\mathbf{G}$ plays the role of an input Jacobian. In case of holonomic mechanisms a drop of rank would necessary and sufficient for an input singularity, and the definitions 1 and 2 are equivalent. With the assumption that $\mathbf{G}$ is full rank, the control system (10) would not have a singularity according to the classical definition 1 in terms of the rank of $\mathbf{G}$. However, if one accepts that a singularity is a configuration in which the kinematic accessibility changes then there are further situations that qualify as singular in case of non-holonomic systems.

Definition 3. The configuration $\mathbf{x}$ is a *kinematic singularity* if the filtration of Δ at $\mathbf{x}$ is not constant in a neighbourhood of $\mathbf{x}$. If additionally Δ is regular, i.e. $\mathbf{G}(\mathbf{x})$ has full rank, the configuration $\mathbf{x}$ is a *non-holonomic kinematic singularity*.

The difference to definition 2 is that $\mathbf{G}$ may have full rank at a non-holonomic singularity. Only for non-holonomic systems can $\mathbf{G}(\mathbf{x})$ be regular but the filtration Δ not regular at $\mathbf{x}$.

Corollary 1. *The set of non-holonomic singularities, Σ_{nh}, is closed in $\mathbb{V}^p$.*

Example 2: Consider the car with two trailers in fig. 2. The $p = 7$ system coordinates are $(x, y, \theta_1, \theta_2, \varphi, \alpha) \in \mathbb{V}^7 = \mathbb{R}^2 \times T^5$. The kinematic control system is

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\varphi} \\ \dot{\alpha} \end{pmatrix} = \begin{pmatrix} R\cos\varphi\cos\theta \\ R\cos\varphi\sin\theta \\ \frac{R}{L}\sin\varphi \\ \frac{R}{L}\sin\varphi - \frac{R}{L_1}\cos\varphi\sin\theta_1 \\ \frac{R}{L_1}\cos\varphi\sin\theta_1 - \frac{R}{L_2}\cos\varphi\cos\theta_1\sin\theta_2 \\ 0 \\ 1 \end{pmatrix} u_1 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} u_2. \tag{11}$$

The accessibility distribution $\Delta = \text{span}\,(\mathbf{g}_1, \mathbf{g}_2)$ is regular, i.e. has constant dimension for all $\mathbf{x} \in \mathbb{V}^7$. Its filtration terminates with the accessibility algebra $\overline{\Delta} = \mathbb{R}^7$, and the system is thus accessible and controllable. As long as $\varphi \neq \pm\frac{n}{2}\pi$ the filtration terminates with growth vector $\rho = (2,3,5,6,7)$. But if the steering angle attains $\varphi = \pm\frac{n}{2}\pi$, the length of the filtration increases by two and the growth vector is $\rho = (2,3,5,5,6,6,7)$. These are non-holonomic singularities: $\Sigma_{\text{nh}} = \{\mathbf{x} | \varphi = \pm\frac{n}{2}\pi\}$. In these singularities the control of the car with two trailers becomes more complex than it is in regular configurations. This is intuitively clear since a steering motion must be performed first so to reorientate the front axis. It can be shown that for each additional trailer, the length of the filtration at a singularity increases by one.

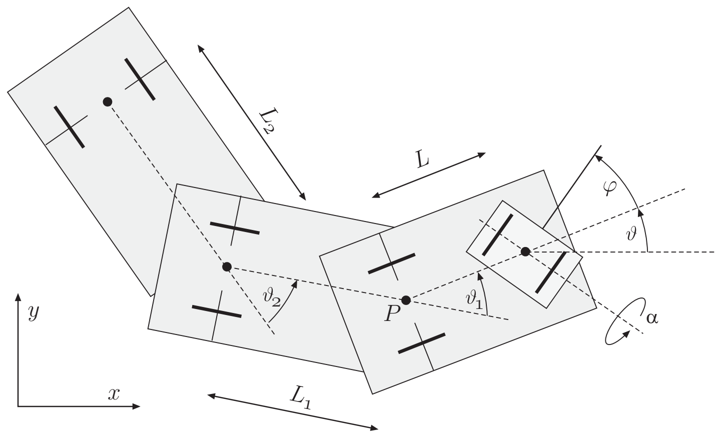

Fig. 2 Kinematic model of a car with two trailers.

4 Discussion and Conclusion

Singularities are kinematic configurations where the kinematic properties change. For SMs this is simply reflected by a rank-deficient forward kinematics Jacobian. This means that the complexity of the infinitesimal motion to reach a point in the vicinity of a singularity increases. Non-holonomic systems possess further critical configurations that qualify as singularities although the input Jacobian is full rank. They are also characterized by an increase of the complexity of the motion. It has been proposed here that the complexity in holonomic and non-holonomic cases is characterized and exemplified by the degree of nesting of Lie brackets necessary to generate the screw algebra defined by the joint screws of an SM, respectively the controllability algebra of a mobile platform. The iteration depth of nested Lie brackets (filtration of distribution) is used as defining property of singularities.

Acknowledgements The first author acknowledges partial support of this work by the Austrian COMET-K2 program of the Linz Center of Mechatronics (LCM).

References

1. Cardetti, F. , Gordina, M.: A note on local controllability on Lie groups. Systems & Control Letters, **57**, 978–979 (2008)
2. Chitour, Y., Jean, F., Trélat, E.: Singular trajectories of driftless and control-affine systems, SIAM J. Control Optim. **47**(2), 1078–1095 (2008)
3. Chitour, Y., Sussmann, H.J..: Motion planning using the continuation method, in:Essays on Mathematical Robotics, J. Ballieul, S.S. Sastry, H.J. Sussmann (eds), Springer Verlag, 91–125 (1998)
4. Donelan, P.: Singularity-theoretic methods in robot kinematics, Robotica, **25**, 641–659 (2007)
5. Jurdevic, V., Sussmann, H.J.: Control Systems on Lie Groups, J. Differential Equations, **12**, 313–329 (1972)
6. Müller, A.: On the manifold property of the set of singularities of kinematic mappings: genericity conditions, ASME J Mech. Robot., **4**(1), (2012)
7. Müller, A.: A proposal for a unified concept of kinematic singularities for holonomic and nonholonomic mechanisms, 36th Mechanisms and Robotics Conference (MECH), ASME 2012 International Design Engineering Technical Conferences, Chicago, IL, 1543–1552 (2012)
8. Müller, A.: On the terminology and geometric aspects of redundantly actuated parallel manipulators, Robotica, **31**(1), 137–147 (2013)
9. Müller, A.: Local kinematic analysis of closed-loop linkages—mobility, singularities, and shakiness, ASME J. Mech. Robot., *in press*, (2016)
10. Müller, A., Zlatanov, D.: Kinematic singularities of mechanisms revisited, IMA Conference on the Mathematics of Robotics, Oxford, UK (2015)
11. Nijmeijer, H. ; van der Schaft, A.J.: Nonlinear dynamical control systems, Springer (1990)
12. Rico, J.M., Gallardo, J., Duffy, J.: A determination of singular configurations of serial nonredundant manipulators, and their escapement from singularities using Lie products, in: Computational Kinematics, J.P. Merlet, B. Ravani (eds.), Kluwer, 143–152 (1995),
13. Tchoń, K.: On kinematic singularities of nonholonomic robotic systems, Proc. RoManSy 13, Zakopane, Poland, 75–84 (2000)
14. Zlatanov, D., Bonev, I.A., Gosselin, C.M.: Constraint singularities as c-space singularities, 8th International Symposium on Advances in Robot Kinematics (ARK 2002), Caldes de Malavella, Spain, 183–192 (2002)
15. Zadarnowska, K., Tchoń, K.: A control theory framework for performance evaluation of mobile manipulators, Robotica, Vol. 25, 2007, pp. 703–715