Game-Theoretic Model for Collaborative Protocols in Selfish, Tariff-Free, Multihop Wireless Networks

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***Abstract*—Traditional networks are built on the assumption that network entities cooperate based on a mandatory network communication semantic to achieve desirable qualities such as efficiency and scalability. Over the years, this assumption has been eroded by the emergence of users that alter network behavior in a way to benefit themselves at the expense of others. At one extreme, a malicious user/node may eavesdrop on sensitive data or deliberately inject packets into the network to disrupt network operations. The solution to this generally lies in encryption and authentication. In contrast, a rational node acts only to achieve an outcome that he desires most. In such a case, cooperation is still achievable if the outcome is to the best interest of the node. The node misbehaviour problem would be more pronounced in multihop wireless networks like mobile ad hoc and sensor networks, which are typically made up of wireless battery-powered devices that must cooperate to forward packets for one another. But, cooperation may be hard to maintain as it consumes scarce resources such as bandwidth, computational power and battery power. This paper applies game theory to achieve collusive networking behavior in such network environments. In this work, pricing, promiscuous listening and mass punishments are avoided altogether. Our model builds on recent work in the field of Economics on the theory of imperfect private monitoring for the dynamic Bertrand oligopoly, and adapts it to the wireless multihop network. The model derives conditions for collusive packet forwarding, truthful routing broadcasts and packet acknowledgments under a lossy, wireless, multi-hop environment, thus capturing many important characteristics of the network layer and link layer in one integrated analysis that has not been achieved previously. Finally, we provide a proof of the viability of the model under a theoretical wireless environment.**

I. Introduction

Traditional networks assume that network entities or nodes can be designed to have well-defined behaviors and coordinate accordingly to ensure certain network goals are met. The goals which generally arise from the interest of the network operator or the network users at large, can be the optimized use of network resources or the Quality of Service (QoS) provided to the end users. These goals, however, may not be commonly shared by individual end user who would always prefer to have better network access, even at the expense of other users. Such a selfish behavior has been reported on rogue TCP sources that do not respond to Explicit Congestion Notification (ECN)[1].

The increasingly popular wireless networks are much more vulnerable to node misbehavior than the traditional wired networks, especially the infrastructureless wireless networks like Mobile Ad Hoc NETworks (MANETs) and Wireless Sensor Networks (WSNs) which do not depend on any wired backbone but on members of the network to route packets for one another wirelessly, over multiple hops. Wireless multihop networking is also used to provide access to nodes that are beyond the direct communication range of access points connected to the wired infrastructure. One example of such applications is the rooftop networks[2].

We focus on the problem of selfish behaviour in MANETs as there is a potential for such behaviour to occur in the emerging 4^{th} generation networks where communications is envisaged to span multihop wireless links, across nodes that may subscribe to different providers. Selfish behaviour and competition at the medium access control layer have been studied by [3] and [4]. In the network layer, the assumption of cooperative relaying of packets among nodes to reach destinations that are beyond the wireless transmission range is no longer valid when nodes exhibit selfish behavior. Helping other nodes consumes precious resources, such as battery power, which is costly and non-beneficial to a node, and without suitable incentives to encourage nodes to cooperate, most existing protocols that assume cooperation are likely to fail. Pioneering work on mitigating node misbehaviour in the routing layer ([5], [6], [7] and [8]) have highlighted the problem of selfishness and proposed basically two approaches to solve the problem – pricing and watchdog cum punishment. Subsequent efforts have not deviated far from these approaches but try to align towards game theory.

Adopting pricing as a solution in [9], [10] and [11] gives rise to the reliance on a central bank or a tamper-proof counter, which limits the practicability especially for a purely infrastructureless network. Punishment methods based on repeated games, proposed by [12], [13], [14] and [15], require the monitoring of transmission activities in the neighborhood, usually through promiscuous listening. Depending on the protocol layer of interest, it is typically unviable for a computationally resource-limited node to process all packets overheard on a high data rate link. Due to the difficulty of coordinating punishment in a multi-hop environment, it has been neglected and without this coordination, punishments and deviations become indistinguishable. Another major drawback in many punishment schemes is the need for the whole or a large portion of the network to participate in the punishment of one deviating node making it too severe, inefficient and opens a security hole for denial of service (DoS) attacks. Considering the unreliable nature of the wireless link, and that most reported work considered only isolated components of the protocol stack, an integrated approach addressing both routing and packet forwarding has been proposed by [16]. Despite the increasing application of game theory in wireless multihop networks, the available results do not adequately model the wireless multihop environment.

In this paper, we apply the theory of imperfect private monitoring in game theory, and through the adaptation and re-interpretation of Aoyagi's game of imperfect private monitoring and communication for the Bertrand oligopoly[17], transform the problem into a wireless multihop game model. This model can account for packet errors, buffer overflows, packet forwarding, packet acknowledgements and routing information dissemination, all of which are important and essential characteristics of multihop wireless networks. In section II, we provide a brief overview of imperfect private monitoring based on Aoyagi's model, highlighting salient points relevant to our discussion. In section III, we present the model for a wireless multihop network, and this followed by the validation of the model in section IV based on a theoretical wireless environment. We summarize our contributions and conclude in section V.

II. Imperfect Private Monitoring

In imperfect public monitoring, the players observe a common signal in each period which is an inaccurate indication of the actions taken by them. An example is an economic model of collusion between firms[18]. Each firm secretly chooses its production level and they observe a common market price. The market price is a good but imperfect indicator because of fluctuations in demand levels. No such common signal exists in wireless communications, and thus wireless devices can only rely on locally (privately) available measurements. Game theory models pertaining to imperfect private monitoring are, however, relatively recent, and particularly hard to formulate. The difficulty in private monitoring lies in the lack of recursive game structure and the need to use statistical inference on other players' actions. Using the same example as above, in this case, the firms engage in secret price-cutting. Market price is no longer a good public signal and the firms rely on observing its own (private) sales volume, which is also imperfect due to demand fluctuations. An interesting class of such games relies on communication[17][19][20]. At each stage of the game, the players publicize an indication of their private signals. There is no constraint on what a player can broadcast, but whatever that is sent, will be acted upon by all other players. The equilibrium is constructed such that truthful reporting is sustained, and punishment strategies depend solely on the history of the reports communicated publicly. The analysis is thus simplified to the case of public monitoring.

A. Aoyagi's Game for Dynamic Bertrand Oligopoly

Aoyagi's game is a repeated game with correlated private signals and communication between players[17]. In an oligopoly, the products of the sellers are undifferentiated to the buyers. If one seller lowers its selling price, the other seller's demand would be negatively affected. The problem in this game is that pricing signals are not reflective of the actual price offered by the other sellers. Sellers may publish a price yet provide secret price cutting to customers privately, and hence cannot constitute a publicly observable signal. The basic idea is to introduce communication between the players. At the end of each stage, the players are to reveal their private signals. Rational players would attempt to lie if it is profitable and the equilibrium has to be built such that everyone has the incentive to tell the truth. The equilibrium can be constructed based only on the publicly observable history of communication and the analysis becomes similar to the perfect public equilibriums in the case of public monitoring.

B. Game Model

Quoting [17], the model definition is: "The set I of $n(\geq 2)$ firms produce and sell products over infinitely many periods. In every period t, firm i chooses price p_I^t from the set $\Re_+$ of non-negative real numbers, and then privately observes its own demand $d_i^t \in \Re_+$ whose probability distribution depends on the price profile $p_t = (p_1^t, ..., p_n^t)$ of all firms. Denote the demand profile in period t by $d_t = (d_1^t, ..., d_n^t)$. We suppose the $d_1^t, ..., d_n^t$ are independent, and have identical probability distribution $P(\cdot \mid p)$ conditional on the price profile p."

The game operates in collusion and punishment phases. In the collusion phase, the price profile $p^* = (p_1^*, ..., p_n^*)$ is to be sustained. After each period, every firm i is to make a public report r_i^t. Let b_i represent any arbitrary report rule and $\hat{b}_i$ represent the report rule based on the threshold $m_i(p_i)$:

$$\hat{b}_i(p_i, d_i) = \begin{cases} 1 & \text{if} \quad d_i \geq m_i(p_i) \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

For each set of n reports, $r = (r_1, \ldots, r_n) \in \{0,1\}^n$, let $s(r) = 0$ if r is unanimous, that is, $r_1 = \ldots = r_n$, and $s(r) = 1$ otherwise. If $s(r) = 0$, they continue to collude in the next period; otherwise, punishment begins. Therefore, unanimous reports are desirable for all players. The probability of unanimous reports conditioned on d_i is given by the following equation, where $a_i^* = (p_i^*, \hat{b}_i)$:

$$\begin{aligned} &P(s(r) = 0 \mid d_i, p_i, b_i, a_{-i}^*) \\ &\quad = \quad P(\underset{j \neq i}{min}(d_j - m_j^*) \geq 0 \mid p_i, d_i, p_{-i}^*) b_i(d_i) \\ &\qquad\quad + P(\underset{j \neq i}{max}(d_j - m_j^*) < 0 \mid p_i, d_i, p_{-i}^*)(1 - b_i(d_i)). \end{aligned} \tag{2}$$

The threshold m_i^* is defined as the threshold when the probability of unanimous profiles is maximized under the collusive price profile of p^* and has the following property:

$$\begin{aligned} m_i^* \in arg \underset{m_i \in \Re_+}{max} \{ &P(\underset{j \neq i}{min}(d_j - m_j^*) \geq 0, d_i \geq m_i \mid p^*) \\ &+ \; P(\underset{j \neq i}{max}(d_j - m_j^*) < 0, d_i < m_i \mid p^*) \}. \end{aligned} \tag{3}$$

The game follows the T-segmented grim trigger strategy, which divides the repeated game into T separate component games, with each component game being independent of each other. The t-th component game, out of a total of T component games, consists of periods $t, T+t, 2T+t, \ldots$. The game starts in the collusion phase and stays in the collusion phase until the report profiles are not unanimous. When this happens, it reverts to the punishment phase. The overall average payoff in each component game is then given by $\upsilon_i(\delta) = (1-\delta^T)g_i^* + \delta^T P(s(r)=0 \mid a^*)\upsilon_i(\delta)$, where $\delta \in [0,1)$ is the common discount factor for all firms, δ^T is the effective discount factor for firm i for a component game with T segments and g_i^* is the stage payoff. When all firms collude by playing $a_i^* = (p_i^*, \hat{b}_i)$, the probability of having non-unanimous report profile is given by:

$$\alpha = P(\min_{j\in I}(d_j - m_j^*) < 0 \le \max_{j\in I}(d_j - m_j^*) \mid p^*), \quad (4)$$

and since $P(s(r)=0 \mid a^*) = 1-\alpha$, the payoff can be simplified to $\upsilon_i(\delta) = \frac{(1-\delta^T)g_i^*}{1-\delta^T(1-\alpha)}$. On the other hand, when an arbitrary firm i deviates by unilaterally adopting price p_i while following the reporting rule $a_i = (p_i, \hat{b}_i)$, such that, $g_i(p_i, p_{-i}^*) > g_i^*$ during any collusion period within any component game, the probability of non-unanimous reporting, $\beta_i(p_i)$, and the payoff gained from this deviation, $\upsilon_i(\delta)$, are given by:

$$\begin{aligned}\beta_i(p_i) = &P(\min_{j\neq i}(d_j - m_j^*) < 0, d_i \ge m_i(p_i) \mid p_i, p_{-i}^*) \\ &+ P(\max_{j\neq i}(d_j - m_j^*) \ge 0, d_i < m_i(p_i) \mid p_i, p_{-i}^*) \quad (5)\end{aligned}$$

$$\begin{aligned}\upsilon_i(\delta) = &(1-\delta^T)g_i(p_i, p_{-i}^*) \\ &+ \delta^T P(s(r)=0 \mid p_i, \hat{b}_i, a_{-i}^*)\upsilon_i(\delta). \quad (6)\end{aligned}$$

Hence, the maximum payoff that can be gained is given by:

$$\upsilon_i(\delta) = (1-\delta^T)\bar{g}_i + \delta^T P(s(r)=0 \mid p_i, \hat{b}_i, a_{-i}^*)\upsilon_i(\delta) \quad (7)$$

where $\bar{g}_i = \sup_{p_i \in \Re_+} g_i(p_i, p_{-i}^*)$, $P(s(r)=0 \mid p_i, \hat{b}_i, a_{-i}^*) \le 1-\beta_i$ and $\beta_i = \inf\{\beta_i(p_i) : g_i(p_i, p_{-i}^*) > g_i^*\}$. The result is such that to support collusion, the following inequalities should be satisfied so that no deviation is profitable:

$$\begin{aligned}&(1-\delta^T)\bar{g}_i + \delta^T P(s(r)=0 \mid p_i, \hat{b}_i, a_{-i}^*)\upsilon_i(\delta) \\ &\quad \le (1-\delta^T)g_i^* + \delta^T P(s(r)=0 \mid a^*)\upsilon_i(\delta) \quad (8)\end{aligned}$$

$$\frac{\delta^T}{1-\delta^T}(\beta_i - \alpha)\upsilon_i(\delta) \ge \bar{g}_i - g_i^*. \quad (9)$$

III. Wireless Multihop Game

Analogies of Aoyagi's problem can be drawn to the wireless multihop MANET problem. We draw analogy between prices, *p*, to packet loss probability, and private demand signals, *d*, to the received packet count from a flow, *r*. The received packet count is a local observation that is a random variable where fluctuations can be caused by traffic source variations and various sources of random packet losses. The packet loss probability is a collective quantity of errors caused by buffer overflows and numerous sources of wireless transmission errors such as signal fading and collisions. In this game, the relay nodes may be tempted to quietly drop packets to conserve energy instead of relaying them. The difficulty in identifying the selfish nodes is that losses, intentional or unintentional, are indistinguishable to observing nodes. Despite the analogies, Aoyagi's model cannot be directly applied as it requires public reporting of private signals. Global sharing of reports is difficult to accomplish in a wireless multihop network without having to periodically flood the network. Instead, we adopt regional reporting and punishment.

A region is defined as the overlapping reception range of two adjacent relaying nodes of a flow. Punishment of a node can be triggered by the observation of non-unanimous reports from its upstream or downstream regions. The upstream region of a node consists of itself, the next upstream node and any node that is able to observe them. The downstream region is similarly defined. Nodes that are out of these two regions are unable to administer punishment on this node because they cannot receive two reports from either region for comparison. Nevertheless, the regions may overlap. Our subsequent analysis will modify Aoyagi's model to reflect the change from network-wide to regional punishments.

A. Modelling Multihop Characteristics

Considering a single flow, the probability of unanimous report profile (Eqn. (2)) when all nodes adopt the threshold reporting strategy is reduced to unanimity of reports between a node and its immediate upstream and downstream nodes:

$$\begin{aligned}&P(s(r)=0 \mid d_i, p_i, b_i, a_{-i}^*) \\ &= P(\min_{j=\{i-1,i+1\}}(d_j - m_j^*) \ge 0 \mid p_i, d_i, p_{-i}^*)b_i(d_i) \quad (10) \\ &+ P(\max_{j=\{i-1,i+1\}}(d_j - m_j^*) < 0 \mid p_i, d_i, p_{-i}^*)(1-b_i(d_i)).\end{aligned}$$

During collusion, the probability of unanimous reporting is maximized (Eqn. (3)) with every neighboring node following the collusive threshold m_i^*, where $i \in I$:

$$\begin{aligned}m_i^* \in &arg \max_{m_i \in \Re_+}\{P(\min_{j=\{i-1,i+1\}}(d_j - m_j^*) \ge 0, d_i \ge m_i \mid p^*) \\ &+ P(\max_{j=\{i-1,i+1\}}(d_j - m_j^*) < 0, d_i < m_i \mid p^*)\}. \quad (11)\end{aligned}$$

Based on m_i, it is assumed that there is positive correlation among nodes with regards to the received packet count (demand) and packet loss probability (price) [17]. This assumption, among neighbouring nodes, is expressed as follows:

Assumption 1: For each $i \in I$ and $p_i \in \Re_+$, there exists $m_i(p_i) \in [0, \infty]$ such that

$$\begin{aligned}&P(\min_{j=\{i-1,i+1\}}(d_j - m_j^*) \ge 0 \mid d_i, p_i, p_{-i}^*) \\ &\quad \ge P(\max_{j=\{i-1,i+1\}}(d_j - m_i^*) < 0 \mid d_i, p_i, p_{-i}^*) \quad (12)\end{aligned}$$

$$\begin{aligned}&P(\min_{j=\{i-1,i+1\}}(d_j - m_j^*) \ge 0 \mid d_i, p_i, p_{-i}^*) \\ &\quad < P(\max_{j=\{i-1,i+1\}}(d_j - m_i^*) < 0 \mid d_i, p_i, p_{-i}^*) \quad (13)\end{aligned}$$

for $P(\cdot \mid p_i, p_{-i}^*)$-a.e. $d_i \ge m_i(p_i)$ and $P(\cdot \mid p_i, p_{-i}^*)$-a.e. $d_i < m_i(p_i)$ respectively. (a.e.: common mathematical abbreviation for "almost everywhere".)

When all nodes collude by adopting $a_i^* = (p_i^*, \hat{b}_i)$, the probability of having non-unanimous report profile for the neighborhood of node i is modified from Eqn. (4) to give Eqn. (14) where the scope of the unanimous report profile has been reduced from global to local/regional.

$$\alpha_i = P(\min_{j=\{i-1,i+1\}}(d_j - m_j^*) < 0 \leq \max_{j=\{i-1,i+1\}}(d_j - m_j^*) \mid p^*) \tag{14}$$

The probability of non-unanimous report profile when node i alone deviates by dropping packets quietly (which is analogous to firm i secretly cutting its price), while following the reporting rule $\hat{b}_i$, is given by:

$$\beta_i(p_i) = P(\min_{j=\{i-1,i+1\}}(d_j - m_j^*) < 0, d_i \geq m_i(p_i) \mid p_i, p_{-i}^*) + P(\max_{j=\{i-1,i+1\}}(d_j - m_j^*) \geq 0, d_i < m_i(p_i) \mid p_i, p_{-i}^*). \tag{15}$$

B. Periodic Punishment Approach

Dividing a protocol game into components, like the T-segmented grim trigger strategy adopted in Aoyagi's game, would require tight time synchronization that is usually avoided in distributed systems. Instead, a T-segmented Tit-For-Tat strategy [14] is adopted, which divides the game into infinitely repeating stages, within which a stage lasts for T periods. The strategy played in the t-th stage depends on the report at the end of the $(t-1)$-th stage. The game begins in collusion for the first stage, and if the previous report is unanimous, the game continues to the next stage in collusion; otherwise, punishment occurs. Thus, the game payoff is:

$$\begin{aligned}
\upsilon_i(\delta) = (1-\delta)\{&[g_i^* + \delta g_i^* + \ldots + \delta^{T-1} g_i^*] + \\
&P(s(r)=0 \mid a^*)[\,\delta^T g_i^* + \delta^{T+1} g_i^* + \ldots + \delta^{2T-1} g_i^*\,] + \\
&P(s(r)=0 \mid a^*)P(s(r)=0 \mid a^*)\times \\
&\quad [\,\delta^{2T} g_i^* + \delta^{2T+1} g_i^* + \ldots + \delta^{3T-1} g_i^*\,] + \\
&P(s(r)=1 \mid a^*)P(s(r)=0 \mid a^*)\times \\
&\quad [\,\delta^{2T} g_i^* + \delta^{2T+1} g_i^* + \ldots + \delta^{3T-1} g_i^*\,] + \ldots\} \\
\upsilon_i(\delta) = (1-\delta)&\sum_{t=0}^{T-1} \delta^t g_i^* + \gamma\delta^T \upsilon_i(\delta)\times \\
&\quad [1 + (\alpha\delta^T) + (\alpha\delta^T)^2 + (\alpha\delta^T)^3 + \ldots]
\end{aligned}$$

$$\upsilon_i(\delta) = (1-\alpha\delta^T) g_i^* \tag{16}$$

where $\gamma = P(s(r)=0 \mid a^*)$ and $\alpha = P(s(r)=1 \mid a^*) = 1-\gamma$ are respectively the probability of unanimous and non-unanimous report profile during collusion.

C. Condition for Efficient Collusion

We now derive the conditions that will encourage nodes to continue colluding. The maximum payoff obtained from deviations, consisting of expected stage payoffs that a node will receive if reports are unanimous, or otherwise, is:

$$\begin{aligned}
\bar{\upsilon}_i(\delta) = (1-\delta)\{&[\bar{g}_i + \delta\bar{g}_i + \ldots + \delta^{T-1}\bar{g}_i] \\
&+ P(s(r)=0 \mid p_i, \hat{b}_i, a_{-i}^*) \\
&\quad \times [\,\delta^T \bar{g}_i + \delta^{T+1}\bar{g}_i + \ldots + \delta^{2T-1}\bar{g}_i\,] \\
&+ P(s(r)=0 \mid p_i, \hat{b}_i, a_{-i}^*)P(s(r)=0 \mid a^*) \\
&\quad \times [\,\delta^{2T}\bar{g}_i + \delta^{2T+1}\bar{g}_i + \ldots + \delta^{3T-1}\bar{g}_i\,] \\
&+ P(s(r)=1 \mid p_i, \hat{b}_i, a_{-i}^*)P(s(r)=0 \mid a^*) \\
&\quad \times [\,\delta^{2T}\bar{g}_i + \delta^{2T+1}\bar{g}_i + \ldots + \delta^{3T-1}\bar{g}_i\,] + \ldots\} \\
=(1-\delta)&\sum_{t=0}^{T-1}\delta^t \bar{g}_i + \left[\bar{\gamma}\delta^T + \bar{\alpha}\gamma\delta^{2T}\sum_{t=0}^{\infty}(\alpha\delta^T)^t\right]\upsilon_i(\delta)
\end{aligned}$$

$$\bar{\upsilon}_i(\delta) = (1-\delta^T)\,\bar{g}_i + \left[\bar{\gamma}\delta^T + \frac{\bar{\alpha}\gamma\delta^{2T}}{1-\alpha\delta^T}\right]\upsilon_i(\delta) \tag{17}$$

where $\bar{\gamma} = P(s(r)=0 \mid p_i, \hat{b}_i, a_{-i}^*)$ and $\bar{\alpha} = 1-\bar{\gamma} = P(s(r)=1 \mid p_i, \hat{b}_i, a_{-i}^*)$ are respectively the probability of unanimous and non-unanimous profiles during deviation. To support collusion, the following inequalities (18), (19) and (20), must be satisfied so that any deviation is not profitable, and hence undesirable:

$$\begin{aligned}
(1-\delta^T)\,g_i + \left[\gamma\delta^T + \frac{\alpha\gamma\delta^{2T}}{1-\alpha\delta^T}\right]\upsilon_i(\delta) \\
\geq (1-\delta^T)\bar{g}_i + \left[\bar{\gamma}\delta^T + \frac{\bar{\alpha}\gamma\delta^{2T}}{1-\alpha\delta^T}\right]\upsilon_i(\delta) \\
\left[\gamma\delta^T + \frac{\alpha\gamma\delta^{2T}}{1-\alpha\delta^T} - \bar{\gamma}\delta^T - \frac{\bar{\alpha}\gamma\delta^{2T}}{1-\alpha\delta^T}\right]\upsilon_i(\delta) \\
\geq (1-\delta^T)(\bar{g}_i - g_i)
\end{aligned}$$

$$\frac{\delta^T}{1-\alpha\delta^T}(\beta_i - \alpha)\upsilon_i(\delta) \geq \bar{g}_i - g_i^* \tag{18}$$

where $\bar{\alpha} \geq \beta_i$. To ensure that a node does not deviate to a strategy that has a lower gain per stage than that of the collusive strategy, but achieves a higher overall gain because the deviated strategy has a higher chance of getting unanimous reports than the collusive one, and consequently suffers from fewer punishments, we assume the following [17]:

Assumption 2: For each $i \in I$, $\alpha \leq inf_{p_i \in \Re_+}\beta_i(p_i)$.

As a result, Eqns. (14) and (18) become:

$$\begin{aligned}
\upsilon_i(\delta) = (1-\alpha\delta^T)g_i^* > g_i^* - \epsilon \\
\delta^T < \frac{\epsilon}{\alpha g_i^*}
\end{aligned} \tag{19}$$

$$\begin{aligned}
\delta^T(\beta_i - \alpha)(g_i^* - \epsilon) \geq (1-\alpha\delta^T)(\bar{g}_i - g_i^*) \\
\delta^T \geq \frac{(\bar{g}_i - g_i^*)}{(\beta_i - \alpha)(g_i^* - \epsilon) + \alpha(\bar{g}_i - g_i^*)}
\end{aligned} \tag{20}$$

where $\epsilon > 0$ is any small number. Combining the inequalities, the following condition should be satisfied for deviation to be unprofitable:

$$max_{i\in I}\,\frac{(\bar{g}_i-g_i^*)}{(\beta_i-\alpha)(g_i^*-\epsilon)+\alpha(\bar{g}_i-g_i^*)}\leq\delta^T<min_{i\in I}\,\frac{\epsilon}{\alpha g_i^*}$$
$$max_{i\in I}\,\frac{(\bar{g}_i-g_i^*)}{(\beta_i-\alpha)(g_i^*-\epsilon)+\alpha(\bar{g}_i-g_i^*)}<min_{i\in I}\,\frac{\epsilon}{\alpha g_i^*}$$
$$max_{i\in I}\,\frac{(\bar{g}_i-g_i^*)/\epsilon}{(\beta_i/\alpha-1)(g_i^*-\epsilon)+(\bar{g}_i-g_i^*)}<min_{i\in I}\,\frac{1}{g_i^*}$$
$$min_{i\in I}\left[1+(\frac{\beta_i}{\alpha}-1)\frac{(g_i^*-\epsilon)}{(\bar{g}_i-g_i^*)}\right]\epsilon<max_{i\in I}\,g_i^* \quad (21)$$

IV. Game Model Validation

In this section, we apply the wireless multihop game model in a theoretical environment to prove that the model is feasible. We assume the traffic source follows a Poisson distribution and the wireless impairments are collectively modeled at each link by a Binomial distribution, both of which are common and frequently assumed statistical models for network analysis. The probability distribution of the number of packets s generated by the source of a flow follows a Poisson distribution given by $P(s=x)=\frac{\lambda^x e^{-\lambda}}{x!}$ where λ is the mean number of packets generated, while wireless transmission errors are modeled as a loss probability ρ_t. This includes impairments such as propagation loss, signal fading and packet collisions.

A. Modelling Private Observations

The private observations in the oligopoly economic model refer to the private demand levels observed by a firm. The equivalent in the wireless multihop network scenario is the number of packets received by a node. Based on the above assumptions, the probability distribution of the number of packets, d, received by a node subjected to wireless impairments with a loss probability of ρ_t is given by:

$$\begin{aligned}P(d)=&\sum_{y=0}^{\infty}P(s=y)\binom{y}{d}(1-\rho_t)^d\rho_t^{(y-d)}\\=&\frac{e^{-\lambda(1-\rho_t)}[\lambda(1-\rho_t)]^d}{d!}\end{aligned} \quad (22)$$

which shows that binomially distributed wireless errors do not alter the packet distribution characteristics at the next hop other than lowering the mean arriving packet count. The choice of binomial distributed errors thus has the advantage of creating symmetry at every node. Next, by assuming that the relaying node maintains a dedicated M/M/1 queue of k packets for the flow, congestion can result in packet loss with loss probability $l_c=\chi^k(1-\chi)/(1=\chi^{k+1})$ where χ is the system load, and given congestion loss probability ρ_c , $P(d)$ becomes:

$$P(d)=\frac{e^{-\lambda(1-\rho_t)(1-\rho_c)}[\lambda(1-\rho_t)(1-\rho_c)]^d}{d!}. \quad (23)$$

Aggregating the local congestion and transmission loss rate $1-p_i=(1-\rho_{i,t})(1-\rho_{i,c})$, the packet received probability distribution at node i, with $\Lambda_i=\prod_{j=0}^{i-1}(1-p_j)$, is given by:

$$P(d_i|p_0,p_1,...,p_{i-1})=\frac{[\lambda\Lambda_i]^{d_i}e^{-\lambda\Lambda_i}}{d_i!}. \quad (24)$$

At the end of a stage in the game, if node i received a total of d_i packets, and the (aggregated) packet loss probability it adopted is p_i, the packet received probability of its downstream node $i+1$, conditioned on this knowledge, is given by:

$$P(d_{i+1}\mid d_i,p_i)=\binom{d_i}{d_{i+1}}(1-p_i)^{d_{i+1}}(p_i)^{d_i-d_{i+1}} \quad (25)$$

On the other hand, the packet receive probability of its upstream node $i-1$ conditioned that node i received a total of d_i packets, and the packet loss probability it adopted is p_i is given by:

$$\begin{aligned}P(d_{i-1}\mid d_i,p_0,p_1,...,p_{i-1})&=\frac{P(d_{i-1},d_i\mid p_0,p_1,...,p_{i-1})}{P(d_i\mid p_0,p_1,...,p_{i-1})}\\&=\frac{(\lambda\Lambda_{i-1}p_{i-1})^{d_{i-1}-d_i}e^{-(\lambda\Lambda_{i-1}p_{i-1})}}{(d_{i-1}-d_i)!}\end{aligned} \quad (26)$$

B. The Reporting Strategy

The collusive reporting threshold is the threshold whereby the probability of uniform reporting is maximized when all members are in collaboration. Using Eqn. (11), we get:

$$\begin{aligned}&\sum_{d_{i-1}=m_{i-1}^*}^{\infty}\frac{(\lambda\Lambda_{i-1}^*)^{d_{i-1}}e^{-(\lambda\Lambda_{i-1}^*)}}{d_{i-1}!}\\&\quad\times\sum_{d_i=m_i}^{d_{i-1}}\binom{d_{i-1}}{d_i}(1-p_{i-1}^*)^{d_i}(p_{i-1}^*)^{d_{i-1}-d_i}\\&\quad\times\sum_{d_{i+1}=m_{i+1}^*}^{d_i}\binom{d_i}{d_{i+1}}(1-p_i^*)^{d_{i+1}}(p_i^*)^{d_i-d_{i+1}}\\&+\sum_{d_{i-1}=0}^{m_{i-1}^*-1}\frac{(\lambda\Lambda_{i-1}^*)^{d_{i-1}}e^{-(\lambda\Lambda_{i-1}^*)}}{d_{i-1}!}\\&\quad\times\sum_{d_i=0}^{m_i-1}\binom{d_{i-1}}{d_i}(1-p_{i-1}^*)^{d_i}(p_{i-1}^*)^{d_{i-1}-d_i}\\&\quad\times\sum_{d_{i+1}=0}^{m_{i+1}^*-1}\binom{d_i}{d_{i+1}}(1-p_i^*)^{d_{i+1}}(p_i^*)^{d_i-d_{i+1}}\end{aligned}$$

where the first term is the probability of reporting a ’1’ (high) and the second term is the probability of reporting a ’0’ (low). Each term consists of nested cumulative receive packet probabilities of the next node given that a certain packet count has been received at a previous node. Given that the threshold m_i, like the received packet count, is a positive integer, the amount of deviation in the probability of unanimous reports in the presence of the smallest positive deviation (i.e. value of 1) in reporting threshold is given by:

$$
\begin{aligned}
&P\,(\min_{j=\{i-1,i+1\}}(d_j-m_j^*)\geq 0, d_i\geq m_i \mid p^*)\\
&\quad+\quad P\,(\max_{j=\{i-1,i+1\}}(d_j-m_j^*)<0, d_i<m_i \mid p^*)\\
&-\Bigg\{\sum_{d_{i-1}=m_{i-1}^*}^{\infty}\frac{(\lambda\Lambda_{i-1}^*)^{d_{i-1}}e^{-(\lambda\Lambda_{i-1}^*)}}{d_{i-1}!}\\
&\qquad\times\binom{d_{i-1}}{m_i}(1-p_{i-1}^*)^{m_i}(p_{i-1}^*)^{d_{i-1}-m_i}\\
&\qquad\times\sum_{d_{i+1}=m_{i+1}^*}^{m_i}\binom{m_i}{d_{i+1}}(1-p_i^*)^{d_{i+1}}(p_i^*)^{m_i-d_{i+1}}\\
&\quad+\sum_{d_{i-1}=0}^{m_{i-1}^*-1}\frac{(\lambda\Lambda_{i-1}^*)^{d_{i-1}}e^{-(\lambda\Lambda_{i-1}^*)}}{d_{i-1}!}\\
&\qquad\times\binom{d_{i-1}}{m_i}(1-p_{i-1}^*)^{m_i}(p_{i-1}^*)^{d_{i-1}-m_i}\\
&\qquad\times\sum_{d_{i+1}=0}^{m_{i+1}^*-1}\binom{m_i}{d_{i+1}}(1-p_i^*)^{d_{i+1}}(p_i^*)^{m_i-d_{i+1}}\Bigg\}.
\end{aligned}
$$

Therefore, the increase in unanimous probability when node i deviates from the reporting threshold positively by 1 unit is given by:

$$
\begin{aligned}
\Delta_i =\ &\frac{(\lambda\Lambda_i^*)^{m_i}e^{-(\lambda\Lambda_i^*)}}{(m_i^*)!}\Bigg\{\\
&\sum_{d_{i-1}=0}^{m_{i-1}^*-1}\frac{(\lambda\Lambda_{i-1}^*p_{i-1}^*)^{d_{i-1}-m_i}e^{-(\lambda\Lambda_{i-1}^*p_{i-1}^*)}}{(d_{i-1}-m_i)!}\\
&\quad\times\sum_{d_{i+1}=0}^{m_{i+1}^*-1}\binom{m_i}{d_{i+1}}(1-p_i^*)^{d_{i+1}}(p_i^*)^{m_i-d_{i+1}}\\
&-\sum_{d_{i-1}=m_{i-1}^*}^{\infty}\frac{(\lambda\Lambda_{i-1}^*p_{i-1}^*)^{d_{i-1}-m_i}e^{-(\lambda\Lambda_{i-1}^*p_{i-1}^*)}}{(d_{i-1}-m_i)!}\\
&\quad\times\sum_{d_{i+1}=m_{i+1}^*}^{m_i}\binom{m_i}{d_{i+1}}(1-p_i^*)^{d_{i+1}}(p_i^*)^{m_i-d_{i+1}}\Bigg\}
\end{aligned}
\tag{27}
$$

To analyze Δ_i, we first define Y_{i-1} and Y_{i+1} as follows:

$$
\begin{aligned}
Y_{i-1} &= \sum_{d_{i-1}=0}^{m_{i-1}^*-m_i-1}\frac{(\lambda\Lambda_{i-1}^*p_{i-1}^*)^{d_{i-1}}e^{-(\lambda\Lambda_{i-1}^*p_{i-1}^*)}}{(d_{i-1})!}\\
&=1-\sum_{d_{i-1}=m_{i-1}^*-m_i}^{\infty}\frac{(\lambda\Lambda_{i-1}^*p_{i-1}^*)^{d_{i-1}}e^{-(\lambda\Lambda_{i-1}^*p_{i-1}^*)}}{(d_{i-1})!}\\
Y_{i+1} &= \sum_{d_{i+1}=0}^{m_{i+1}^*-1}\binom{m_i}{d_{i+1}}(1-p_i^*)^{d_{i+1}}(p_i^*)^{m_i-d_{i+1}}\\
&=1-\sum_{d_{i+1}=m_{i+1}^*}^{m_i}\binom{m_i}{d_{i+1}}(1-p_i^*)^{d_{i+1}}(p_i^*)^{m_i-d_{i+1}}
\end{aligned}
$$

and plot the various combinations as shown in Figure 1. Both Y_{i+1} and Y_{i-1} decreases as m_i increases. Hence, as m_i increases, the increase in unanimous probability slows down

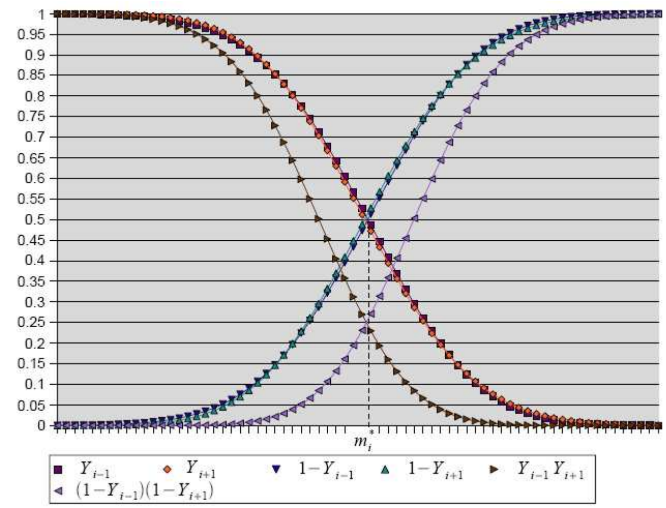

Fig. 1. Optimum Cutoff Reporting

and reaches a peak, before it starts to decrease. The peak occurs when $\Delta_i = 0$. This happens when $Y_{i+1} = Y_{i-1} = 0.5$, which means $m_{i-1}^* - m_i = \lambda\Lambda_{i-1}^* p_{i-1}^*$ and $m_{i+1}^* = m_i(1-p_i^*)$ are at the medians. With the function $\Delta_i = 0$ having a unique solution at $m_i = m_i^* = \lambda\Lambda_i^*(\forall i \in I)$, we can conclude that $m_i^* = \lambda\Lambda_i^*(\forall i \in I)$ is the reporting threshold whereby the probability of unanimous reports is maximized when all nodes report packet loss probability p^* during collusion.

C. Correlated Receive Packet Count Signal

When the receive packet count is “positively correlated” across nodes, there exists a single-crossing property of conditional probabilities (Eqns. (12) and (13)). By Assumption 1, it means that the conditional probabilities of other nodes unanimously reporting a ’1’ (high) or ’0’ (low) increases or decreases, respectively, as the received packet count, d_i , of node i increases. In other words, the higher the receive packet count that node i locally detected, the more likely it is for other nodes to unanimously report a ’1’ (high) and vice versa. From Eqns. (25) and (26), we determine the combined probability of its neighbors receiving packets equaling d_{i-1} and d_{i+1}, conditioned on the event that the node i itself has received d_i packets and is adopting a loss rate of p_i, while other nodes are in collusion:

$$
\begin{aligned}
&P(d_{i-1}, d_{i+1} \mid d_i, p_i, p_{-i}^*)\\
&\qquad= P(d_{i-1}, d_i, p_i, p_{-i}^*)P(d_{i+1} \mid d_i, p_i, p_{-i}^*)\\
&\qquad= \frac{(\lambda\Lambda_{i-1}^*p_{i-1}^*)^{(d_{i-1}-d_i)}e^{-(\lambda\Lambda_{i-1}^*p_{i-1}^*)}}{(d_{i-1}-d_i)}\\
&\qquad\quad\times\binom{d_i}{d_{i+1}}(1-p_i)^{d_{i+1}}p_i^{d_i-d_{i+1}}
\end{aligned}
\tag{28}
$$

The conditional probabilities of neighboring nodes reporting ’1’ (high) or ’0’ (low) are respectively, as follows:

$$P(\min_{j=\{i-1,i+1\}}(d_j - m_j^*) \geq 0 \mid d_i, p_i, p_{-i}^*)$$
$$= \sum_{d_{i-1}=m_{i-1}^*}^{\infty} \frac{(\lambda\Lambda_{i-1}^* p_{i-1}^*)^{(d_{i-1}-d_i)} e^{-(\lambda\Lambda_{i-1}^* p_{i-1}^*)}}{(d_{i-1}-d_i)}$$
$$\times \sum_{d_{i+1}=m_{i+1}^*}^{d_i} \binom{d_i}{d_{i+1}} (1-p_i)^{d_{i+1}} p_i^{d_i - d_{i+1}} \quad (29)$$

$$P(\max_{j=\{i-1,i+1\}}(d_j - m_j^*) < 0 \mid d_i, p_i, p_{-i}^*)$$
$$= \sum_{d_{i-1}=0}^{m_{i-1}^*-1} \frac{(\lambda\Lambda_{i-1}^* p_{i-1}^*)^{(d_{i-1}-d_i)} e^{-(\lambda\Lambda_{i-1}^* p_{i-1}^*)}}{(d_{i-1}-d_i)}$$
$$\times \sum_{d_{i+1}=0}^{m_{i+1}^*-1} \binom{d_i}{d_{i+1}} (1-p_i)^{d_{i+1}} p_i^{d_i - d_{i+1}} \quad (30)$$

Letting $H_{i-1}(d_i) = \sum_{d_{i-1}=m_{i-1}^*}^{\infty} \frac{(\lambda\Lambda_{i-1}^* p_{i-1}^*)^{(d_{i-1}-d_i)} e^{-(\lambda\Lambda_{i-1}^* p_{i-1}^*)}}{(d_{i-1}-d_i)!}$

and $H_{i+1}(d_i, p_i) = \sum_{d_{i+1}=m_{i+1}^*}^{d_i} \binom{d_i}{d_{i+1}} (1-p_i)^{d_{i+1}} p_i^{d_i-d_{i+1}}$, we note that both $H_{i-1}(d_i)$ and $H_{i+1}(d_i, p_i)$ increase as d_i increases, and consequently, Eqn. (29) increases while Eqn. (30) decreases, exhibiting "positive correlation" of receive signal levels across nodes. When all nodes collude, $d_i = m_i^* = \lambda\Lambda_i^*(\forall i \in I)$ is a crossing point. The median of H_{i+1} occurs at $m_i^*(1-p_i^*) = m_{i+1}^*$ giving it a value of 0.5. Similarly, the median of H_{i-1} occurs at $\lambda\Lambda_{i-1}^* p_{i-1}^* = m_{i-1}^* - m_i^*$ giving it a value of 0.5.

D. Highest Unanimity at Collusion

Assumption 2 describes a condition whereby deviation will always increase non-unanimous reports and consequently increases the likelihood of punishments. A node therefore does not have the incentive to play a strategy that has a lower payoff than the collusive strategy. From Eqns. (14) and (15), together with Eqn. (11), and defining A_i and $B_i(p_i)$ as shown below, we get:

$$\alpha_i = P(\min_{j\in\{i-1,i,i+1\}}(d_j - m_j^*) < 0 \leq \max_{j\in\{i-1,i,i+1\}}(d_j - m_j^*) \mid p^*)$$
$$= 1 - \max_{m_i \in \Re_+}\{P(\min_{j=\{i-1,i+1\}}(d_j - m_j^*) \geq 0, d_i \geq m_i \mid p^*)$$
$$+ P(\max_{j=\{i-1,i+1\}}(d_j - m_j^*) < 0, d_i < m_i \mid p^*)\}$$
$$= 1 - P(\min_{j=\{i-1,i,i+1\}}(d_j - m_j^*) \geq 0 \mid p^*)$$
$$- P(\max_{j=\{i-1,i,i+1\}}(d_j - m_j^*) < 0 \mid p^*)$$
$$= 1 - A_i \quad (31)$$

$$\beta_i(p_i) = P(\min_{j\in\{i-1,i+1\}}(d_j - m_j^*) < 0, d_i \geq m_i(p_i) \mid p_i, p_i^*)$$
$$+ P(\max_{j\in\{i-1,i+1\}}(d_j - m_j^*) \geq 0, d_i < m_i(p_i) \mid p_i, p_i^*)$$
$$= 1 - P(\min_{j=\{i-1,i+1\}}(d_j - m_j^*) \geq 0, d_i \geq m_i(p_i) \mid p_i, p_{-i}^*)$$
$$- P(\max_{j=\{i-1,i+1\}}(d_j - m_j^*) < 0, d_i < m_i(p_i) \mid p_i, p_{-i}^*)$$
$$= 1 - B_i(p_i) \quad (32)$$

and Assumption 2 can be reformulated as:

$$\text{For each } i \in I, \qquad A_i \geq \sup_{p_i \in \Re_+} B_i(p_i) \quad (33)$$

The collusive probability of unanimous reporting, A_i, occurs when nodes adopt the collusion loss rate (price) p_i^* and threshold reporting strategy of cutoff value m_i^*. The threshold is obtained from the crossing point of Eqns. (29) and (30) and has the highest probability with respect to any other cutoff values as shown Section IV-B. A node therefore has no incentive to deviate from the collusive cutoff reporting threshold of m_i^* when adopting p_i^*. Nevertheless, a node may decide to deviate from its agreed packet loss rate p_i^* to a loss rate of p_i and choose a different threshold $m(p_i)$ to maximize the deviated unanimous reporting probability B_i.

The relationship between the various distributions are shown in Figure 2 which plots $H_{i+1}(d_i, p_i^*)$, $1 - H_{i+1}(d_i, p_i^*)$, $P(d_i \mid p_i, p_{-i}^*)H_{i-1}(d_i)$ and $P(d_i \mid p_i, p_{-i}^*)(1 - H_{i-1}(d_i))$. When node i adopts the collusive strategy of p_i^*, the two curves $H_{i+1}(d_i, p_i^*)$ and $1 - H_{i+1}(d_i, p_i^*)$ intersect at m_i^* as shown by the thick black lines in Figure 2. These curves shift to the right as p_i increases above p_i^*, and to the left as p_i decreases below p_i^*. On the other hand, the curves $P(d_i \mid p_i, p_{-i}^*)H_{i-1}(d_i)$ and $P(d_i \mid p_i, p_{-i}^*)(1 - H_{i-1}(d_i))$ are independent and invariant of p_i. We observe that the curves $P(d_i \mid p_i, p_{-i}^*)H_{i-1}(d_i)$ and $P(d_i \mid p_i, p_{-i}^*)(1 - H_{i-1}(d_i))$ appear to be symmetrical about m_i^*, which is exactly point where $H_{i+1}(d_i, p_i^*)$ and $1 - H_{i+1}(d_i, p_i^*)$ also symmetrical about the vertical line at m_i^*. It is not surprising since the packet arrival probability distribution function is a Poisson distribution that can be approximated to a Normal distribution that is symmetric about its mean m_i^*. Similarly, $H_{i-1}(d_i)$ and $1 - H_{i+1}(d_i, p_i^*)$ are

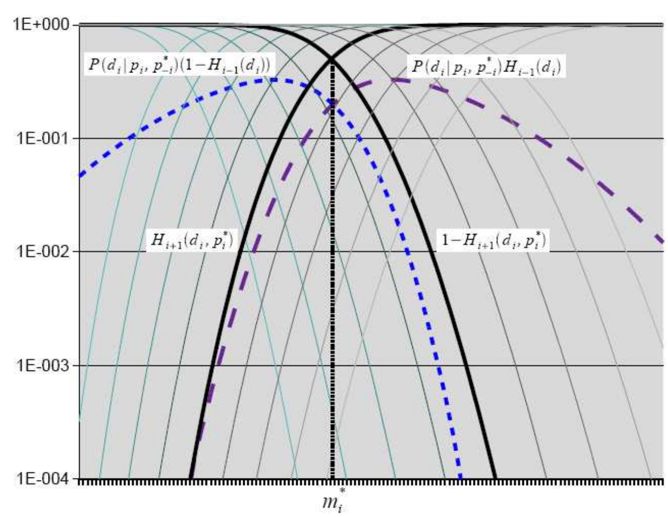

Fig. 2. Graphical Evaluation of Unanimous Probability

approximately symmetrical about m_i^* so that the product $P(d_i \mid p_i, p_{-i}^*)H_{i-1}(d_i)$ and $P(d_i \mid p_i, p_{-i}^*)(1 - H_{i-1}(d_i))$ are symmetrical about m_i^*. The symmetrical property helps to simplify the proof of Assumption 2 (Eqn. (33)) without going into complex mathematical calculations.

To prove that no deviation is profitable by validating Eqn. (33), we first express A_i (which is only a special case when $m_i^* = m(p_i^*)$) and B_i, as follows:

$$\begin{aligned} A_i = &\sum_{d_i=m_i^*}^{\infty} P(d_i \mid p^*)P(\min_{j=\{i-1,i+1\}}(d_j - m_j^*) \geq 0 \mid d_i, p^*) \\ &+ \sum_{d_i=0}^{m_i^*-1} P(d_i \mid p^*)P(\min_{j=\{i-1,i+1\}}(d_j - m_j^*) < 0 \mid d_i, p^*) \\ = &\sum_{d_i=m_i^*}^{\infty} P(d_i \mid p^*)H_{i-1}(d_i)H_{i+1}(d_i, p_i^*) \\ &+ \sum_{d_i=0}^{m_i^*-1} P(d_i \mid p^*)(1 - H_{i-1}(d_i))(1 - H_{i+1}(d_i, p_i^*)) \end{aligned} \tag{34}$$

$$\begin{aligned} B_i = &\sum_{d_i=m(p_i)}^{\infty} P(d_i \mid p_i, p_{-i}^*)P(\min_{j=\{i-1,i+1\}}(d_j - m_j^*) \geq 0 \mid d_i, p_i, p_{-i}^*) \\ &+ \sum_{d_i=0}^{m(p_i)-1} P(d_i \mid p_i, p_{-i}^*)P(\min_{j=\{i-1,i+1\}}(d_j - m_j^*) < 0 \mid d_i, p_i, p_{-i}^*) \\ = &\sum_{d_i=m(p_i)}^{\infty} P(d_i \mid p_i, p_{-i}^*)H_{i-1}(d_i)H_{i+1}(d_i, p_i) \\ &+ \sum_{d_i=0}^{m(p_i)-1} P(d_i \mid p_i, p_{-i}^*)(1 - H_{i-1}(d_i))(1 - H_{i+1}(d_i, p_i)) \end{aligned} \tag{35}$$

where the probabilities $H_{i-1}(d_i)$ and $H_{i+1}(d_i, p_i)$ are obtained from Section IV-C and $P(d_i \mid p_i, p_{-i}^*)$, which is the packet arrival probability distribution function at node i, is evaluated in Eqn. (24).

A_i (Eqn. (34)) comprises a lower summation and a higher summation. The lower summation, which is the probability of having a unanimous ’0’ (low) report, consists of the summation of the product of the decreasing black line and the dotted (blue) curve from the lower limit to $m_i^* - 1$, in Figure 2. The higher summation, which is the probability of a unanimous ’1’ (high) report, consists of the summation of the product of the increasing black line and the dashed (purple) curve from m_i^* to the upper limit.

Similarly, B_i (Eqn. (35)) consists of a lower summation and upper summation of the same pair of lines except for $p_i \neq p_i^*$, when the black lines move away from the line of symmetry at m_i^*. The cutoff point of the lower and higher summations is no longer optimum at m_i^*. Regardless of the choice of cutoff value $m(p_i)$, when the black lines diverge, the overall summation decreases, with one of the summations increasing and the other decreasing in value, until a point when only one summation is dominant and the other is reduced to zero. At that point, the value B_i has the lowest possible probability equaling the area under the dotted (blue) curve or the dashed (purple) curve.

Hence, we have analytically illustrated that as p_i deviates from p_i^*, the probability of unanimous reporting decreases and B_i will always be lower than A_i, thus proving Assumption 2. Figure 3 further demonstrates the changes in unanimous probabilities as node i adopts different p_i. The probability is maximized when $p_i = p_i^* = 0.1$.

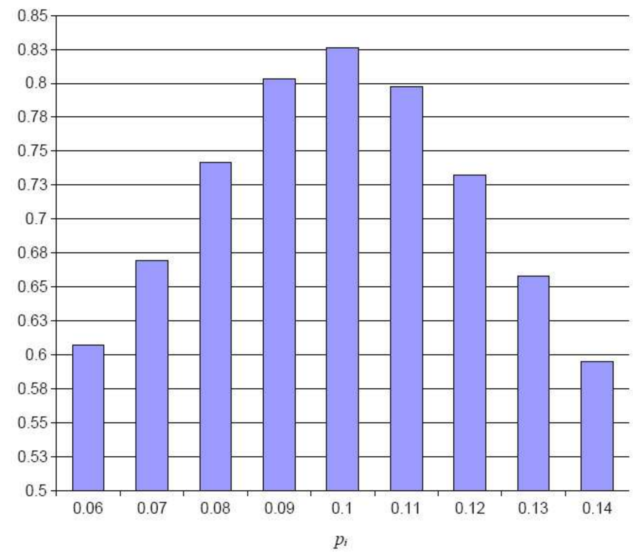

Fig. 3. Unanimous Probability at Various Packet Loss Rates

In this section, we have derived an optimum reporting strategy for the wireless multihop model in this environment. We have shown that correlation of received packet counts (Assumption 1) exists, proving that threshold reporting is part of the equilibrium strategy in such an environment. We have also shown that abiding by the agreed packet loss probability ensures maximum probability of unanimous report profiles (Assumption 2). This ensures that the nodes will only deviate to strategies that have higher short-term gains. With Assumptions 1 and 2 satisfied, punishment strategies are simplified to those applicable to regular (public and perfect) repeated games which include the T-segmented grim trigger strategy suggested by Aoyagi or an improved wireless punishment strategy provided in Section III-B.

V. Conclusion

In this paper, we focus on the problem of selfish behaviour in wireless multihop networks like MANETs as there is a potential for such behaviour to occur in emerging network scenarios where communications is envisaged to span multihop wireless links, over nodes that may subscribe/belong to different providers. We search for a sustainable network behavior in wireless multihop networks where cooperation comes at a cost. While these (selfish) users have no malicious intent to disrupt network operations, they are rational users that are sometimes constrained by resources which make them less likely to cooperate. They would require incentives or punishments to encourage cooperation and participation in network operations.

Game theory is exploited to analyze an integrated model of transmission losses, buffer overflows, packet acknowledgments, packet forwarding and routing information dissemination, all of which are important characteristics of wireless networks. Specifically, we applied Aoyagi's game of imperfect private monitoring with communication [17] and adapted it to the wireless environment. Our wireless multihop model provides a guiding design principle for protocols that are robust against selfish users. The analysis is not confined to a particular layer, but is designed to capture the overall behavior of a protocol stack.

In this model, relaying nodes establish a mutual agreement on the collusive packet loss probability (combination of transmission losses and buffer overflows) prior to the transmission of a flow. The negotiation of supported packet loss probability is not different from routing broadcasts with QoS or link quality indications. With this threshold, it is optimal for nodes to report a "1" (high) if their received flow rate exceeds their threshold and a "0" (low) if otherwise. These reports are, in fact, packet acknowledgments which we have proven to be truthful. We have further proven that the routing information disseminated is also truthful. The local broadcasting of reports allows the coordination of regional punishments. Nodes in a region hear the reports from two neighboring nodes of a flow, and punishment is administered by these nodes in the next stage if non-unanimous reports have been received.

Lastly, we validated the model in a theoretical wireless environment using well accepted statistical models of packet generation and transmission errors. We have proven by mathematical derivations and analysis that assumptions made in our Wireless Multihop Game model are satisfied in this environment. We have also derived a collusive reporting threshold thereby making the model realizable. Our model is theoretically consistent with game theory and technically practical for a distributed, wireless network. We have also proven that the assumptions made for the model are true under a commonly accepted wireless environment. Typical pitfalls, like coordinated global punishments, are avoided and the model fits nicely into existing MANET protocols, requiring little overheads and modifications.

There are nevertheless limitations to our model. Firstly, synchronized reporting is required although we have relaxed the requirement. We envisage that synchronization may not be required ultimately. Secondly, reports are to be reliably broadcast which may be hard to achieve in wireless networks. Thirdly, the model did not capture the medium arbitration function of the link layer. Finally, we have not answered the question of how the nodes should choose a collaborative packet relay probability which is left for future study.

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